Mecánica Clásica
Tarea 2: Pequeñas Oscilaciones

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Nombre del Estudiante: ____

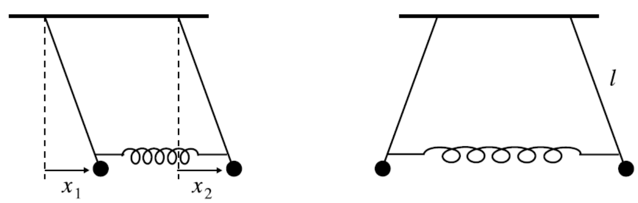
Problema 1 *Coupled pendulums*

Two pendulums of equal mass m and length l are connected by a spiral spring of constant k. They vibrate/oscillate in a plane. Considering small amplitudes:

(a) Find the motion of each mass: $x_1(t)$ and $x_2(t)$.

(b) Determine the oscillation frequency(ies).

The initial conditions are: $x_1(0) = 0$, $x_2(0) = A$, $\dot{x}_1(0) = \dot{x}_2(0) = 0$.



Hint: You can use the normal coordinates $u_1 = x_1 - x_2$ and $u_2 = x_1 + x_2$ to decouple the system.

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Problema 2 *Energy for a damped oscillator*

Derive the expressions for the energy and energy-loss (dE/dt) for the damped oscillator. Additionally, show that in the limit of weak damping ($\omega_0/\beta \to \infty$) the energy of an under-damped oscillator is given by,

$$E(t) = E_0 e^{-2t\beta} \quad \forall \quad E_0 = \frac{1}{2}kA^2 = \frac{1}{2}m\omega_0^2 A^2.$$

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Problema 3 *Undamped driven oscillator*

An undamped oscillator is driven at its resonance frecuency ω_0 by a harmonic force $F = F_0 \text{Sen}\,\omega_0 t$. The initial conditions are $x(t=0)=0$ and $v(t=0)=0$. Determine $x(t)$.

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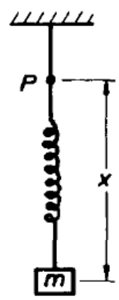
Problema 4 *Hanging mass*

A mass m hangs in equilibrium by a spring which exerts a force $F = -k(x-l)$, where x is the lenght of the spring and l is its lenght when relaxed. At $t=0$ the point of support to which the upper end of the spring is attached begins to oscillate sinusoidally up and down due to a force with amplitude F_0 and angular frequency ω. Show that the equation of motion for $x(t)$ is:

$$x(t) = \frac{A}{\omega_0^2 - \omega^2}\left[\text{Sen}\,\omega t - \frac{\omega}{\omega_0}\text{Sen}\,\omega_0 t\right] + \frac{mg}{k} + l \quad \forall \quad \omega_0 = \sqrt{k/m} \;\; \& \;\; A = F_0/m.$$



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Problema 5 *Jerk force on a damped driven oscillator*

Suppose a jerk force,

$$F = -\gamma \frac{d^3x}{dt^3} \;\; \forall \;\; \gamma = \text{cte.},$$

is applied to the damped drived oscillator subject to a one-dimensional restoring force F_x and a frictional force proportional to the velocity F_f, and a harmonic driving force F_d given by:

$$\begin{aligned} F_x &= -kx \;\; \forall \;\; k > 0, \\ F_f &= -\alpha v \;\; \forall \;\; \alpha > 0, \\ F_d &= F_0 \text{Cos}\,\omega t \;\; \forall \;\; F_0 \;\&\; \alpha = \text{ctes}. \end{aligned}$$

1. Show that the amplitude $D(\omega)$ and phase $\delta(\omega)$ of the steady-state oscillations are given by:

$$\begin{aligned} D(\omega) &= \frac{F_0/m}{\sqrt{4\beta^2\omega^2(1-2\omega^2/\omega_c^2)^2 + (\omega_0^2-\omega^2)^2}} \\ \mathrm{Tg}\,\delta &= \frac{2\beta\omega(1-2\omega^2/\omega_c^2)}{\omega_0^2-\omega^2} \end{aligned}$$

   where $\omega_0^2 = k/m$ and $\omega_c^2 = 4m\beta/\gamma$.

2. Suppose $\gamma > 0$. Show that the amplitude of the steady-state oscillations is increased by the jerk force provided, considering $\omega < \omega_c$, in comparison with the case with $\gamma = 0$.

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