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Semiclassical limit of the Schrödinger-Poisson-Landau-Lifshitz-Gilbert system. (English summary)

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This work deals with an elaborate (Landau-Lifshitz) model of magnetization with transfer terms acting on spin 1/2 charge carriers in materials. This is known as the Schrödinger-Landau-Lifshitz-Gilbert (SLLG) system, which has been so important in the description of spin waves and spintronics. For a review, see [M. Lakshmanan, Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **369** (2011), no. 1939, 1280–1300; MR2774539], and a very simple derivation for the physicists in [M. Krawczyk et al., Adv. Condensed Matter Phys. **2012**, Article ID 764783, doi:10.1155/2012/764783].

The first part of the paper concentrates on the existence of weak and strong solutions to the SLLG system of equations, which exhibits dissipation, non-linearity and an inherent two-level structure via Pauli coupling. Functional-analytic techniques such as Sobolev embedding theorems and Galerkin methods are applied. The second part is devoted to the semiclassical limit of such a complex system: this is done by introducing a scale parameter in the Laplace operator and a Wigner transform acting exclusively on the translational degrees of freedom of densities. As a final result, the authors find a simplified Landau-Lifshitz-Gilbert (LLG) equation for the magnetization plus a Liouville-type flow (diffusionless) with spin for the Wigner function. The existence of solutions to this system is also proved, either strongly or weakly.

The paper is very complete and well motivated. The propositions presented are thorough in their goals. This is especially important if one recalls that LLG models are very diverse, including not only dissipation, but chaos and lattice hopping models as well [for instance, see M. Lakshmanan and A. Saxena, Phys. D **237** (2008), no. 7, 885–897; MR2417078]. Two comments can be made:

• The semiclassical parameter (Planck's constant) is typically included quadratically in the Laplace operator, but for the physicists it should also appear linearly in front of the Pauli term, which corresponds to the spin-magnetization interactions of the present model. As a consequence, the semiclassical limit studied by the authors retains the discrete nature of spin 1/2, while the wavelength of the spin wave is the actual small parameter of the problem: classicality is only of a translational type.

• The Wigner transform is taken directly in terms of spinors. While some authors use this definition as well, one should note that discrete Wigner transforms have also been studied and proved to be useful in a number of contexts; some references for these cases are [A. B. Klimov and S. M. Chumakov, Rev. Mexicana Fís. **48** (2002), no. 4, 317–324; MR1928446; A. B. Klimov, J. L. Romero and H. de Guise, J. Phys. A **50** (2017), no. 32, 323001; MR3673490]. A full Wigner transform in the present problem would include an additional summation over spinor components with a discrete kernel. Perhaps for the authors' purposes, such a Wigner function would not be as tractable as the one presented, but it would be interesting to visualize semiclassical dynamics with other sophisticated tools.

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