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Bounds for the asymptotic order parameter of the stochastic Kuramoto model. (English summary)

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This paper deals essentially with sharp estimates for the solution of a transcendental equation involving modified Bessel functions of the first kind. The authors use their expertise in Turán inequalities to prove upper bounds for the order parameter $r(K)$ in the Kuramoto model, which satisfies the aforementioned equation. Furthermore, sharp bounds based on properties of Bessel functions are derived, with the correct claim that no better rational powers can be found for the scaling of r near the critical coupling strength $K = K_c$. These nice results settle some previous mathematical discussions in [L. Bertini, G. Giacomin and K. Pakdaman, J. Stat. Phys. **138** (2010), no. 1-3, 270–290 (p. 278, expression (2.4)); MR2594897] and [B. Sonnenschein and L. Schimansky-Geier, Phys. Rev. E **88** (2013), no. 5, 052111 (p. 3, equation (17)), doi:10.1103/PhysRevE.88.052111], where bounds were already provided without a proof.

It should be said, however, that the original source of the transcendental equation is an integral relation involving the ratio I_1/I_0. As one reads further, around section 2.4, one finds extended discussions of sharp bounds when the index of Bessel functions is generalized to $\nu \geq 1/2$, and this is done without specifying a precise connection with the stationary solutions of the Kuramoto model. From a purely mathematical perspective, some results for general ν can be regarded as new. Towards the end of 2.4, 2.6 and 2.7 one also finds claims based exclusively on numerical experiments.

In general, the first part of this paper is useful and interesting. On the other hand, the proposed extensions do not seem to have a direct counterpart with the models studied in the standard literature [J. A. Acebrón et al., Rev. Mod. Phys. **77** (2005), no. 1, 137–185, doi:10.1103/RevModPhys.77.137]. A comprehensive treatment of Turán-type inequalities for a variety of functions was given in [H. Skovgaard, Math. Scand. **2** (1954), 65–73; MR0063415].

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