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**Spectral analysis of the $2+1$ fermionic trimer with contact interactions. (English summary)**

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As announced in the abstract, the authors qualify the spectrum of a trimer containing two fermions, in the same context as the well-known Efimov effect for bosons. It is proved that the spectrum contains a finite number of bound states for negative scattering lengths (in contrast with the bosonic case) and only positive continuous energies are found for positive scattering lengths. Interesting thresholds for the mass of the third particle appear in the analysis, showing a transition from an empty set of discrete eigenvalues to a finite set. The boundary conditions of contact interactions are incorporated in Fourier space with the so-called charge operator, which is essentially a two-particle Green's function, in the spirit of L. D. Faddeev's work [Soviet Physics. JETP **12** (1960), 1014–1019; MR0119854]. The employed techniques rest heavily on norm inequalities. This work is a very strong formalization of previously obtained results [R. A. Minlos and M. K. Shermatov, Vestnik Moskov. Univ. Ser. I Mat. Mekh. **1989**, no. 6, 7–14, 97; MR1065968], which improves the techniques in [A. Michelangeli and A. Ottolini, Rep. Math. Phys. **79** (2017), no. 2, 215–260; MR3648286] and [A. Michelangeli and C. Schmidbauer, Phys. Rev. A **87** (2013), no. 5, 053601, doi:10.1103/PhysRevA.87.053601]. The introduction can be hardly improved with more references, as it is the case for other works by the same authors [A. Michelangeli and A. Ottolini, op. cit.]. Very detailed proofs are offered, including the explicit evaluation of some integrals in the derivation of inequalities, as well as a variational method and optimality of mass parameters. In particular, the numerical values of the critical masses $m^*$ and $m^{**}$ are of great relevance in the theory of self-adjoint extensions of these operators.

Some side remarks can be made: For example, the physicist's point of view is—and always has been—that fermions require half-integer spin. The fermionic treatment in this work is reduced to orbital antisymmetry and odd angular momentum number (as in other mathematical papers) but one may also ask about the existence of the Efimov effect when the fermions are in a spin singlet, allowing states of even angular momentum in the orbital part. This is not examined in this paper or any other works. Also, for the casual reader, it is worth mentioning the first work by V. N. Efimov [Yad. Fiz. **12** (1970), no. 5, 1080–1091; translated in Sov. J. Nucl. Phys. **12** (1971), no. 5, 589–595]; a similar work by the same author is also available [Phys. Lett. B **33** (1970), no. 8, 563–564, doi:10.1016/0370-2693(70)90349-7].

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*