MR3999521 81Q37 81Q15 81Q35
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Quantum waveguides with magnetic fields. (English summary)

Rev. Math. Phys. **31** (2019), *no. 8,* 1950025, 38 *pp.*

The authors offer a complete survey of quantum wires or tubes under the influence of external magnetic fields. Their treatment is based on differential methods and fiber bundles. In full analogy with their previous article [Ann. Henri Poincaré **16** (2015), no. 11, 2535–2568; MR3411741], they derive effective Hamiltonians that "pick up" the contribution of the tube's curvature, together with effective magnetic potentials; this is done in the limit of a vanishing thickness for both weak and moderate fields. In this limit, rigorous proofs via the Hausdorff distance show that the spectrum of the magnetically perturbed system is sufficiently close to that of a curved, but otherwise free configuration. The authors also show an emergent longitudinal—here called horizontal—potential quadratic in B, for both massive and hollow waveguides. The resulting effective Hamiltonians are real, eluding thus the typical time-reversal symmetry breaking imposed by magnetic vector potentials as in, e.g., atomic physics.

This original contribution should be compared with similar, though less formal, treatments of restrictive geometries and the d'Alembert principle in quantum mechanics [H. H. Jensen and H. Köppe, Ann. Phys. **63** (1971), no. 2, 586–591, doi:10.1016/0003-4916(71)90031-5; R. C. T. da Costa, Phys. Rev. A (3) **23** (1981), no. 4, 1982–1987; MR0607422]. It should also be mentioned that the spectral effects of curvature were investigated on more physical grounds in [J. Goldstone and R. Jaffe, Phys. Rev. B **45** (1992), no. 24, 14100, doi:10.1103/PhysRevB.45.14100; S. Bittner et al., Phys. Rev. E **87** (2013), no. 4, 042912, doi:10.1103/PhysRevE.87.042912].

In general, adiabatic perturbation theory constitutes a useful method in the investigation of the low-energy spectrum, as long as the waveguide geometry is smooth enough. In this direction, one may note a few challenges offered by magnetic tubes beyond the adiabaticity paradigm of this article:

- Sharply bent waveguides display binding capabilities that entail nonnegligible couplings with transverse modes. This is in contrast with the ground-state averaging utilized in the present work.
- For hollow tubes, a modification of the spectrum is expected after the introduction of a horizontal magnetic flux. This is a manifestation of the Aharonov-Bohm effect and should be present even if B vanishes at the surface where the electric charge dwells.
- Periodically curved massive waveguides may exhibit a singular (Hofstadter) spectrum under normal incidence of the magnetic flux. This suggests the use of other spectral characterizations, such as the Minkowski or Hausdorff dimension, in contrast to the Hausdorff distance employed here.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.