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Second harmonic Hamiltonian: algebraic and Schrödinger approaches. (English summary)

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This paper deals with a study of Second Harmonic Generation (SHG) under some special parametric conditions that allow quasi-exact solvability of the Schrödinger equation. Such a property is produced by the intentional non-linear deformation of symmetry algebras. It seems that the authors have found an exceptional motivation—i.e., more than a good pretext—to introduce the study of specialized trilinear (anharmonic) Hamiltonians, which translate into sextic potentials in the usual one-dimensional Schrödinger equation. It is important to mention that a long time ago, the papers [J. A. Armstrong et al., Phys. Rev. (2) **127** (1962), no. 6, 1918–1939, doi:10.1103/PhysRev.127.1918] and [N. Bloembergen and P. S. Pershan, Phys. Rev. (2) **128** (1962), 606–622; MR0145826] already showed useful calculations of power output in non-linear dielectrics, and that a good deal of references mentioned in the introduction of the paper under review already tried trilinear Hamiltonians in a quantum-mechanical context. Therefore, most parts of the present paper consist of review material regarding non-linear algebraic deformations, perhaps with the aim of introducing the reader to the formalism. In this direction, a recommendable summary of this topic can be found in work by A. V. Turbiner [in R. L. Anderson et al., *CRC handbook of Lie group analysis of differential equations. Vol. 3*, CRC, Boca Raton, FL, 1996 (Chapter 12); see MR1383090]. Regarding the applicability of the models to SHG, it should be recalled that only some low-lying levels can be retrieved exactly for a specific choice of parameters. Thus, while interesting, the results might be considerably limited when dealing with more general physical situations. Obvious examples come to mind, such as the evolution of infinite superpositions of Fock states, e.g. coherent states, sub-Poissonian thermal distributions, and so on.

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