APPROXIMATING STABLE MANIFOLDS OF A SADDLE SLOW MANIFOLD IN A BURSTING MODEL

Many models of neuronal activity exhibit complex oscillations in response to an input from a stimulus or other neurons in a network. We consider an ordinary differential equation model with two time scales that exhibits dynamics qualitatively similar to neuronal bursting. Without an external stimulus, the model is at rest, that is, at a globally attracting equilibrium. We are interested in understanding its intrinsic excitability, particularly the number of spikes in the transient response caused by applying a short stimulus. We find that the transient response is organised by a separatrix associated with a saddle slow manifold of the system. Based on manifold theory, geometric singular perturbation theory, and the theory of nonautonomous systems, we give a precise definition for this separatrix and design an algorithm to compute it; our computational method is formulated as a two-point boundary value problem and uses continuation to compute the manifold.

In a different parameter regime, the separatrix that determines the transient response is the stable manifold of a saddle-type equilibrium; methods for computing such manifolds are well established. We compare the separatrices and how they organise the transient responses for both cases.