VIRTUAL CONFERENCE:

"Advanced Computational and Experimental Techniques in Nonlinear Dynamics"

October 26 - 30, 2020

(Zoom Webinar Virtual Conference)

Program

MONDAY 26

"Seeing the Light: Waves and Photons, Complexity and Randomness"

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Light conveys information at multiple scales of space and time, and through multiple variables. We will explore how we have learned to extract information from light over the ages, first with our eyes and brain, and then through the use of different detectors that sense the dynamics of light (waves and photons) in ever more sensitive and delicate ways. As new light sources and detectors have been developed, we have learned how order and disorder can both be useful and how we can learn to control both the temporal and spatial coherence of light. We find that entropy and information measures depend on the scales of detection and measurement, and our perception of what is deterministic chaos and what is noise is governed by the precision and time scale of our observations. Finally, we describe a new form of chaotic dynamics, observed recently in optoelectronic systems, that is named "laminar" chaos to distinguish it from the traditionally studied "turbulent" chaos. This type of dynamics - characterized by irregularly spaced steady states of periodic duration and separated by rapid transitions - occurs in systems with variable time-delays. Laminar chaos could have novel applications in information processing.

“Non-Markovian Stochastic Resonance of Light in a Microcavity”

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Stochastic resonance (SR) is a phenomenon wherein an optimum amount of noise amplifies the response of a nonlinear system to a periodic signal. Originally proposed as an explanation for recurrent ice ages, SR has been extensively observed in physics, chemistry, biology, ecology, psychophysics, climate science, finance, and social science. To date, all observations of SR have been described within the Markov approximation neglecting memory effects. While non-Markovian dynamics have been experimentally observed in various systems and are expected to modify SR, non-Markovian SR has not been experimentally reported. In this talk, I will present the first observation of non-Markovian SR.

We investigate an oil-filled optical microcavity with thermo-optical nonlinearity. This system has memory in its nonlinear response because of the oil's non-instantaneous thermal relaxation. To evidence SR, we drive our oil-filled cavity with a continuous wave laser while periodically modulating the cavity length. The modulation imprints a signal on the laser. We add a controlled amount of Gaussian white noise to the laser, and we measure the transmitted signal-to-noise ratio (SNR) as a function of the added noise variance. SR manifests as a peak in the SNR at a certain noise variance. Our experimental observations are reproduced by numerical simulations based on a recently developed model accounting for the memory time of the nonlinearity. Simulations show how the memory time of the nonlinearity enlarges the SR bandwidth, making it 8 orders of magnitude greater for our oil-filled cavity than for conventional Kerr nonlinear cavities. This memory-enhanced robustness of SR against changes in signal frequency opens new perspectives for harvesting energy from fluctuations across an unprecedentedly large bandwidth.

"The arrow of time across five centuries of classical music"

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Continuity, succession, direction and narrative in musical compositions have been recurrently discussed, amongst others, by musicians and musicologists. Here we study these attributes quantitatively by looking into statistical properties of around 9000 classical musical scores spanning from Renaissance to early Twentieth Century. We determine through time series analysis methods, nonlinear traits, asymmetry, scaling and irreversibility. The arrow of time is dealt with in terms of irreversibility by means of visibility horizons, which can be implemented for short time series. Besides exhibiting correlations among the above traits our study shows that irreversibility is predominant in the compositions along the different musical periods. Furthermore, with our analysis we explore aspects of the process of composition in terms of out of equilibrium thermodynamics. Additionally, the effect of nonlinearity, directionality and dissipation on musical appreciation is also considered. Our work relies on melody, the inclusion of harmony and rhythm is challenging.

"The topological structure of reconstructed flows in latent spaces"

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One of the main objectives in science and engineering is to pro- pose interpretable mathematical models capable of explaining sequences of experimental observations. These models are usually expressed as a set of differential equations. The explosive increase of computing power and data availability in recent years has boosted the development of machine learning algorithms capa- ble of finding these equations directly from the data. This is especially relevant when we do not know the elemental mechanisms that govern the evolution of the system or when the nature of the observations does not allow a direct analytical approach. Recent work shows that this data-driven methodology for discov- ering governing equations can greatly benefit from deep neural networks. These powerful models can be used to transform the experimental data into a new set of coordinates in which the dynamics can be easily expressed. This process is known as embed- ding. In this presentation, I will discuss reconstructed flows using autoen- coders, a neural network architecture widely used for dimensionality reduction in machine learning (1). For most cases, this model is capable of learning proper embeddings of chaotic data. However, remarkably, there are cases where this model does not learn a topologically correct representation of the data, even when its reconstruction error is low. Autoencoders have the potential to drastically improve equation discovery methods, but a warn- ing should be raised since it is not possible to find a correct model if topological invariants computed from the data are not preserved.

1. Chaos 30, 093109 (2020); doi: 10.1063/5.0013714

"A continuous phase transition characterized via chaotic diffusion for two dimensional mappings"

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We characterize a second order phase transition via the chaotic diffusion for a family of Hamiltonian mappings whose angles diverge in the limit of vanishingly action. The investigation is made by using the solution of the diffusion equation. The system is described by a two-dimensional mapping for the variables action, I, and angle, $₩theta$ and controlled by two control parameters: (i) $₩epsilon$, controlling the nonlinearity of the system, particularly a transition from integrable for $₩epsilon=0$ to non-integrable for $₩epsilon₩ne 0 and; (ii) $₩gamma$ denoting the power of the action in the equation defining the angle. For $₩epsilon₩ne 0$ the phase space is mixed and chaos is present in the system leading to a finite diffusion in the action characterized by the solution of the diffusion equation. The analytical solution is then compared to the numerical simulations showing a remarkable agreement between the two procedures. For the chaotic dynamics far apart from the periodic islands, normal diffusion is observed. Our observables allow us to conclude that the transition observed is similar to a continuous phase transition, as discussed in statistical mechanics, since the order parameter goes continuously to zero at the transition while its susceptibility diverges in the same limit.

"Localized Modes in Discrete and Continuous Nonlinear Dirac-Like Equations"

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Emerging photonic structures, modeled in ways the depart from the well known Nonlinear Schroedinger Equation, present novel topological features producing interesting dynamics. Motivated by this, in this talk we will present results of systems governed by discrete and continuous Dirac-like equations.

TUESDAY 27

“Droplets of quantum matter”

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In the course of the past 25 years, several new quantum states of matter have been created in ultracold gases, including Bose-Einstein condensates, degenerate Fermi gases, and Tonks-Girardeau gas of hard-core bosons. A recent addition to this set of fundamental states of quantum matter is the prediction [1] and experimental demonstration [2] of quantum droplets (QDs) built of coherent atomic waves in binary (two-component) BEC. This is an extension of BEC beyond the limits of the usual mean-field (MF, alias semi-classical) approximation, with the averaged action of quantum fluctuations around the MF states leading to drastic changes in static and dynamical properties of the quantum gas. The result may be summarized as an effective quartic nonlinear term competing with the usual cubic MF nonlinearity [1]. A remarkable advantage of QDs is that the beyond-MF quartic self-repulsion makes it possible to stabilize self-trapped three-dimensional (3D) and quasi-2D droplets, which are kept together by the usual cubic MF attraction between two BEC components [1], against the collapse (catastrophic self-compression of the condensate). Further, it was recently predicted that 3D [4] and quasi-2D [3-6] QDs can be created as stable topological states, which carry embedded vorticity. In this connection, it is relevant to stress that prediction and creation of stable 3D and 2D vortex solitons is well known to be an especially challenging problem. The present talk aims to present a brief overview of basic results obtained in this area.

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"Topological transitions in an oscillatory driven liquid crystal cell"

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Matter under different equilibrium conditions of pressure and temperature exhibits different states such as solid, liquid, gas, and plasma. Exotic states of matter, such as Bose-Einstein condensates, superfluidity, chiral magnets, superconductivity, and liquid crystalline blue phases are observed in thermodynamic equilibrium. Rather than being a result of an aggregation of matter, their emergence is due to a change in the topological state of the system.

We will present an investigation of topological states of matter in a system with injection and dissipation of energy. In an experiment involving a liquid crystal cell under the influence of a low-frequency oscillatory electric field, we observe a transition from a non-vortex state to a state in which vortices persist. Depending on the period and the type of the forcing, the vortices self-organize forming square lattices, glassy states, and disordered vortex structures. Based on a stochastic amplitude equation, we recognize the origin of the transition as the balance between stochastic creation and deterministic annihilation of vortices. Our results show that the matter maintained out of equilibrium by means of the temporal modulation of parameters can exhibit exotic states.

"Kaleidoscopic Self-Similar Characteristics of Integral Apollonian Gaskets"

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Apollonian packing of circles will be described using inversions from "grand mirrors", associated with the tangency points of Pappus chains. These chains consist of packing of circles in "arbelos"or shoe makers knife-like shapes in the Apollonian gasket. Reflections though these "Pappus mirrors" are described by a special class of conformal maps that preserve the scaling associated with the hierarchical sets of self-similar packing of circles. These mirrors associate all the hierarchical patterns in the Apollonian gasket to packing of Ford circles - the circles that provide the pictorial representations of fractions. Consequently, the self-similarity of the entire gasket is encoded in the nesting of Ford Apollonians. The asymptotic scalings of curvatures of the circles are found to be given by a class of quadratic irrationals with continued fraction[n + 1 : 1, n] - that is a set of irrationals with period-2 continued fraction consisting of 1 and any another integer n. Intriguingly, the even n hierarchy is found to be described by the Pythagorean tree. Finally, relationship between the number theoretical aspect of this abstract fractal to the iconic Hofstadter butterfly fractal that describes quantum Hall effect in condensed matter physics will also be discussed.

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"Low dimensional models and electrophysiological experiments to study neural dynamics in songbirds"

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Birdsong emerges when a set of highly interconnected brain areas manage to generate a complex output. The similarities between birdsong production and human speech have positioned songbirds as unique animal models for studying learning and production of this complex motor skill.

In this work, we developed a low dimensional model for a neural network in which the variables were the average activities of different neural populations within the nuclei of the song system. This neural network is active during production, perception and learning of birdsong. We performed electrophysiological experiments to record neural activity from one of these nuclei and found that the low dimensional model could reproduce the neural dynamics observed during the experiments. Also, this model could reproduce the respiratory motor patterns used to generate song. We showed that sparse activity in one of the neural nuclei could drive a more complex activity downstream in the neural network. This interdisciplinary work shows how low dimensional neural models can be a valuable tool for studying the emergence of complex motor tasks.

"Semiclassical and quantum phase transitions of two coupled photonic crystal nanocavities"

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Photonic crystal (PhC) nanocavities are optical devices that operate with a few hundred photons due to their small size. For the case of two coupled nanocavities, experiments have shown that they can exhibit spontaneous symmetry breaking and bistable behaviour. Furthermore, due to their low-photon operation, they are considered as a possible experimental realisation of an open Bose–Hubbard dimer model, which is a celebrated fundamental quantum mechanical model that accounts for boson dynamics for only two interacting sites.

Our work focuses on characterising the different dynamics that arise in the semiclassical approximation of the open Bose–Hubbard dimer model. More precisely, we perform a bifurcation analysis of the semiclassical system to delimit regions of different behaviour as different properties of the driving signal (pump power f and detuning $₩delta$) are varied. Specifically, we focus on the existence of multi-stability after symmetry breaking, and the appearance and disappearance of self-pulsations as different bifurcation occurs. As more energy is pumped into the system, a balance between dissipation and driving power creates chaotic attractors and highly complicated dynamics that are predicted by our analysis.

Moreover, we simulate the open Bose–Hubbard dimer model with a quantum-jump method and compare the results with the predictions from the semiclassical equations. We find that the bifurcations that we observe in the semiclassical description create different attractors (equilibria, periodic orbits and chaotic attractors) that guide the temporal fluctuations of the quantum system. This agreement is also observed even if the number of photons simulated is small (in the order of twenties). Our results provide a detailed roadmap of different behaviour that should be observable in the two coupled PhC-nanocavities.

"Vortices to spiral-galaxy-like tracer patterns induced by contact-line shear gradient on Faraday waves"

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In this talk, I will show experimental results of exotic self-organizations of small tracers under the action of longitudinal Faraday waves in a narrow container. With the onset of Faraday waves, steady currents form dividing the interface in small cells given by Faraday-wave symmetries. These streaming currents rotate in each cell and their circulation increases with wave amplitude. This flow drives the tracers to form patterns, whose shapes depend on the Faraday-wave amplitude: from low to high amplitudes we find tracers dispersed on vortices, narrow rotating rings and spiral-galaxy-like patterns. I will describe the main pattern features and characterize the wave and tracers' motion. We show experimentally that the main source of the streaming flow is the time and spatial dependent shear at the wall contact line created by the Faraday wave itself. I will end by presenting a 2D compressible advection model that considers the minimal ingredients present in the Faraday experiment, namely the stationary circulation, the stretching component due to the oscillatory wave and a steady converging field, which combined produce the observed self--organized patterns.

WEDNESDAY 28

"Computation of multicritical points at the onset of convectionin rotating fluid spheres"

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It is known that the onset of convection in rotating fluid spheres and shells usually gives rise to rotating waves, which can travel in the prograde or retrograde direction relative to the frame of reference rotating with the bulk of the fluid. It was discovered recently that axisymmetric periodic regimes can also be preferred at low Prandtl, Pr, and Ekman, E, numbers. These flows are known as torsional because they consist of the superposition of a toroidal vortex, which fills the sphere and reverses its rotation in the meridional plane, and a poloidal component, antisymmetric with respect to the equator, which also changes direction, but with a shift of a quarter of the period with respect to the toroidal field.

In order to determine the parameter space where the torsional flows are the first bifurcated solutions, the curves of double Hopf points corresponding to simultaneous transitions to azimuthal wave numbers m=(0,1), (0,1), (1,1), etc. have been computed. These curves form the skeleton of the bifurcation diagram, separating the regions of different preferred azimuthal wave numbers. Their intersection are triple Hopf points. Several of them have been found. The limit of the double Hopf curves as E decreases, of interest in geophysical and astrophysical problems, has also been studied.

"The Origin of GnRH Pulse Generation: An Integrative Mathematical-Experimental Approach"

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The gonadotropin-releasing hormone (GnRH) pulse generator controls the pulsatile secretion of the gonadotropic hormones LH and FSH and is critical for fertility. The hypothalamic arcuate kisspeptin neurons are thought to represent the GnRH pulse generator, since their oscillatory activity is coincident with LH pulses in the blood; a proxy for GnRH pulses. However, the mechanisms underlying GnRH pulse generation remain elusive. We developed a mathematical model of the kisspeptin neuronal network and confirmed its predictions experimentally, showing how LH secretion is frequency-modulated as we increase the basal activity of the arcuate kisspeptin neurons in vivo using continuous optogenetic stimulation. Our model provides a quantitative framework for understanding the reproductive neuroendocrine system and opens new horizons for fertility regulation.

"Invariant manifolds organising the propagation and containment of dengue"

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Arboviruses such as dengue, zyka and chikungunya are viruses transmitted to humans by mosquitoes. In particular, Aedes aegypti mosquito is the responsible for dengue transmission. In the absence of medical treatments and vaccines, one of the control methods is to introduce Aedes aegypti mosquitoes infected by the bacterium Wolbachia into a population of wild (uninfected) mosquitoes. The goal consists in achieving population replacement in finite time by driving the population of wild females towards extinction while keeping Wolbachia-infected mosquitoes alive. This strategy has several advantages for control of dengue: Wolbachia decreases the virulence of the dengue infection and it reduces the lifespan of the mosquito. Moreover, mating of a female uninfected by Wolbachia and an infected male leads to sterile eggs.

"Effects of state-dependence in the delayed feedback loop driving El Niño"

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Feedback loops in climate systems arise from interactions between various subsystems, such as distinct bodies of water, the atmosphere, land and ice masses. Importantly, each feedback loop takes effect only after an inherent time delay. Such delayed feedback loops give rise to conceptual models in the form of delay differential equations (DDEs), and we consider here a model for the El Niño Southern Oscillation (ENSO) phenomenon. Theory and advanced numerical techniques are well developed for DDEs with constant delays, which is arguably the main reason why delays are generally taken to be constant. However, this is a serious modeling assumption, certainly in climate science.

We present arguments for the state dependence of delays in the ENSO DDE model and then conduct a bifurcation analysis to investigate its effects on the observed dynamics of the system. More specifically, we show that the underlying delay-induced structure of resonance regions may change considerably in the presence of state dependence. More generally, this case study shows that state-dependence of delays is capable of generating entirely new dynamics, and that its effects can be studied effectively with the now available continuation tools.

"Estimate for the Yarkovsky effect for Apophis based on automatic differentiation"

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Apophis is a Near Earth Asteroid (NEA), discovered in 2004, which will approach the Earth on April 13th 2020, with a minimum orbit intersection distance of about 1/6 the Earth-Moon distance. The largest source of uncertainty of its orbit is a dissipative non-gravitational interaction called the Yarkovsky effect, which is difficult to measured. In this talk, I will describe how we have estimated the transversal Yarkovsky parameter and the initial conditions, using precise numerical integrations of the Solar System and Apophis which involve automatic differentiation techniques.

THURSDAY 29

"Numerical methods to study tipping phenomena in highly multistable systems"

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Critical transitions or tipping phenomena are related to sudden changes in the behavior of a dynamical system. This could be either transitions between different coexisting stable states in a multistable system or large deviations from the usual dynamics. Examples for such behavior can be found in different fields of science ranging from mechanical or chemical systems to ecosystem and climate dynamics. Such critical transitions are called tipping phenomena in climate science, regime shifts in ecology or phase transitions in physics. They can happen in various ways: (1) due to bifurcations, i.e. changes in the dynamics when external forcing or parameters are varied extremely slow (2) due to fluctuations which are always inevitable in natural systems, (3) due to rate-induced transitions, i.e. when external forcing changes on characteristic time scale comparable to the time scale of the considered dynamical system and (4) due to shocks or extreme events. We discuss in more detail two numerical approaches to study shock tipping and rate-induced tipping. On the one hand, we demonstrate an algorithm how to compute the minimal fatal shock that measures the closest distance between a stable steady state and its basin boundary in high-dimensional systems like complex networks. This method does not only provide the magnitude of the shock but also its direction. On the other hand, we employ the concept of snapshot attractors to study the dynamics in highly multistable systems, which are subject to a parameter drift. We investigate the impact of chaotic saddles embedded in basin boundaries on the tipping dynamics and find partial tipping and random tipping.

"Rate-Induced Tipping Points: Beyond Classical Bifurcations"

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Many nonlinear systems are subject to continuously changing external conditions. For a system near a stable state (an attractor) we might expect that as external conditions change with time, the stable state will change too. In many cases the system may adapt to changing external conditions and track (adiabatically follow) the moving stable state. However, tracking may not always be possible owing to nonlinearities and feedbacks in the system. So far, the main focus has been on dangerous levels of external conditions (classical bifurcations) where the stable state turns unstable or disappears causing the system to move to a different and often undesired state. We describe this phenomenon as bifurcation-induced tipping or simply B-tipping. However, critical levels are not the only critical factor for tipping. Some systems can be particularly sensitive to how fast the external conditions change and have critical rates: they suddenly and unexpectedly move to a different state if the external input changes too fast. It happens even though the moving stable state never loses stability! We describe this phenomenon as rate-induced tipping or simply R-tipping. Being a genuine non-autonomous instability, R-tipping cannot be captured by the classical bifurcation theory and thus requires an alternative mathematical framework.

In the first part of the talk, we demonstrate R-tipping in a simple ecosystem model where environmental changes are represented by time-varying parameters. We define R-tipping as a critical transition from the herbivore-dominating equilibrium to the plant-only equilibrium, triggered by a smooth parameter shift. We then show how to complement the classical bifurcation diagram with information on nonautonomous R-tipping that cannot be captured by the classical bifurcation analysis. Specifically, we obtain tipping diagrams in the plane of the magnitude and rate of a parameter shift to uncover nontrivial R-tipping phenomena.

In the second part of the talk, we develop a general mathematical framework for R-tipping based on the concepts of thresholds, edge states and suitable compactification of the nonautonomous system. This allows us to define R-tipping in terms of connecting heteroclinic orbits in the compactified system, which greatly simplifies the analysis. We explain the key concept of threshold instability and give rigorous testable criteria for R-tipping to occur in arbitrary dimension.

"Topologies that favor synchronization in energy transmission networks"

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Energy transmission networks (power grids) are a typical example of the study of a system that presents a collective behavior of interconnected dynamical units. Local instabilities in these networks can result in cascade failures and even in blackouts. Power grids can be described my means of complex networks of oscillators, where transmission lines are described by the edges and generators or consumers of energy are represented by nodes. The oscillator model often used in literature to describe the behavior of the generators/consumers is the second order Kuramoto model. In this work, an evolutionary optimization technique is used to generate network topologies that present a relatively small number of edges and favors frequency synchronization as the dynamics of the nodes are given by a second order Kuramoto oscillator. These topologies would be of great interest when building power grids duo to the costs involved in the construction of transmission lines.

"Matching geometric and numerical characteristics of wild chaotic attractors"

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Wild chaotic attractors exhibit chaotic dynamics that cannot be destroyed with small perturbations. Such attractors arise in higher-dimensional systems for which homoclinic tangencies occur robustly in open one-parameter intervals. We investigate a smooth discrete-time system defined by a non-invertible map on the complex plane. A recent bifurcation study of this map suggests that wild chaos arises from a so-called backward critical tangency, which occurs when a critical point, the origin for this map, lies on the unstable manifold of a saddle fixed point. Geometrically, this bifurcation leads to an accumulation of homoclinic and heteroclinic tangencies between manifolds of different dimensions, generating the necessary ingredients for robustness of the tangencies. In this talk, we confirm that the backward critical tangency creates a wild chaotic attractor via a numerical exploration of the Lyapunov exponents associated with this attractor. We compute the two Lyapunov exponents along representative orbits over a range of parameters in a two-parameter plane and investigate sign changes of both the maximum and the sum of the Lyapunov exponents. The latter sign-change indicates existence of wild chaotic attractors, and we discuss the parameter regime for which this zero locus matches the geometric characteristic of a backward critical tangency.

"A restricted four-body problem for the eight figure choreography"

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In this work we introduce a planar restricted four-body problem where a massless particle moves under the gravitational influence due to three bodies following the eight figure choreography, and we explore some symmetric periodic orbits of this system which turns out to be non autonomous. The symmetric periodic orbits (initial conditions) were determined by means of solving some boundary value problems.

"New families of periodic orbits in the 4–body problem emanating from a kite configuration"

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In this talk I will show the existence of families of periodic and quasiperiodic orbits (also called relative periodic orbits) emanating form a kite configuration in the planar four body problem with three equal masses. We introduce a new coordinate system which measures (in the planar four body problem) how far is an arbitrary configuration from a kite configuration. Using these coordinates, and the Lyapunov center theorem, we get families of periodic and quasiperiodic orbits emanating from a kite configuration.

FRIDAY 30

"First stages of thermal convection in rotating spherical fluids at low Prandtl numbers"

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The onset of thermal convection in spherical rotating fluids most commonly appears as azimuthally rotating waves, symmetric with respect to the equator. It was recently found that for Prandtl numbers lower than one the convection can appear as axisymmetric oscillations, almost antisymmetric with respect to the equator (torsional flows) if the ratio from the Prandtl to the Ekman number is order ten. The kinetic energy of these flows propagates latitudinally on the surface of the sphere. Preliminary results of the first stages of the three-dimensional dynamics of the thermal convection in this range of parameters will be presented. It will be shown that when the axisymmetry is broken at a secondary Hopf bifurcation the flows start to drift in the azimuthal direction giving rise to quasi-periodicity, and a double direction of propagation. Several stable patterns of convection with different symmetries have been identified in the range of Rayleigh numbers explored, all of them retaining the torsional motion of the basic velocity field. Particular attention is paid to their dependence on the Rayleigh number, and to the mean values of the zonal flow and of the kinetic energy of the fluid.

“A novel approach to generate attractors with a high number of scrolls”

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In this work, a novel method for increasing the number of scrolls in a hybrid nonlinear switching system is presented. Using the definition of the "Round to the Nearest Integer Function", as a generalization of a Piece-Wise Linear (PWL) function, which is capable of generating up to a thousand of scrolls. An equation that characterizes the grown in the number of scrolls is calculated, which fits to the behavior of the system measured by means of the coefficient of determination, denoted R 2. The proposed equation is based on obtaining as many scrolls as desired, based on the control parameters of the linear operator of the system. The work here presented provides a new approach for the generation and control of a high number of scrolls in a hybrid system. The results are verified for all the scenarios that the equations covers.

"Modeling complex ecological networks in spatially structured environments"

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A few years ago a group of researchers started an interdisciplinary collaboration with the aim of identifying the factors that contribute to changes in the biodiversity of Patagonian ecosystems. In this framework, approaches from diverse disciplines are combined, seeking to understand the evolution of these complex systems through changing scenarios, such as those derived from climate change, environmental degradation, invasion of species, or human activity. The interrelation of such diverse factors makes it necessary to develop mathematical models that allow predicting the effect of each of them on the species involved.

In this talk I will present the metapopulation models of generic predator-prey-competition systems we developed to study this problem. The combined use of ordinary differential equations and stochastic simulations allowed us to obtain the average behavior of relevant variables but also to study the role of fluctuations and spatial correlations. I will summarize some general results obtained from simple systems that help to elucidate which are the determining factors in the extinction or survival of the species.

Our purpose is to move towards increasingly complex systems in successive approaches, in order to design more sustainable socio-environmental dynamics.

"The robustness of synchronization in the Kuramoto model of identical networked nodes"

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The concept of basin of stability leads to the notion of the attractor's robustness: the larger the basin, the less susceptible the attractor is under perturbations. In this sense, the border of the basin is of great importance. It can be smooth, due to stable invariant manifolds of a saddle point, or fractal, usually promoted by the presence of unstable chaotic sets. In all of these cases, there is no clear influence of the attractor on its basin of attraction. The mean point of this talk is to present the role of the eigenvalues of stable equilibrium in the strength of the attractor. This notion suggests that, unlike known, the attractor acts on the size of its basin, i.e., the faster the convergence of dynamics to the attractor, the more robust the attractor is. We present analytical numerical evidence of this phenomenon in the multiple synchronized states of the Kuramoto model composed of N symmetrically coupled nodes.

"Anticontinuous limit for Landau-de Gennes equilibria"

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We study the relation between solutions of the Landau-de Gennes and Oseen-Frank theories for liquid crystals in lattices and graphs. This relation has been examined in the continuum theories for equilibria that minimize the respective energy functionals in the limit of small coupling. We examine the zero coupling limit of the discretized Landau-de Gennes functional for the cases of 2x2 and 3x3 Q-tensors and derive a bifurcation equation on the manifold of critical points and a continuation criterion. The bifurcation equation at the lowest order is the equation of critical points of the discretized Oseen-Frank theory. The 2x2 case is analogous to the problem of continuation of breathers in discrete NLS equations.

"Pattern formation on a finite disk"

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The dynamics of the cubic-quintic Swift-Hohenberg equation over a finite disk with no-flux boundary conditions are studied. We predict the unstable modes of the trivial state using a linear stability analysis. These modes are followed via numerical continuation, revealing a great variety of spatially extended and spatially localized behaviors. Notably, we find solutions localized in the interior as well as solutions localized along the boundary or part of the boundary. Bifurcation diagrams summarizing these results and their stability properties are presented, linking the different solutions. The findings of this study are likely relevant to nonlinear optics, combustion as well as convection.