Isolas of periodic orbits as a source of complexity in molecular lasers with a saturable absorber.

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Outline

(1) Introduction : Chaos in single-mode lasers.

(2) A molecular model for the CO_2 laser with saturable absorber and the 3-level-2-level model.

(3) Isolas of pulsations (spikes) , bifurcation and phase diagrams.

(4) LSA chaotic dynamics.

(5) Conclusions.

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PASSIVE *Q*-SWITCHING OF A CO_2 LASER*

O. R. Wood and S. E. Schwarz
Department of Electrical Engineering and Computer Sciences
University of California
Berkeley, California
(Received 16 June 1967)

Passive *Q*-switching of a CO_2–N_2–He laser has been obtained, using SF_6 gas as the saturable absorber. Peak power is 1 kW, in what appears to be a single transverse mode. This is 200 times the CW level for the same configuration and one-fifth that obtained with a mechanical *Q* switch. Pulse rates are in the range 10^3 to 10^4 pulses per sec. Operation is on a single vibrational-rotational line, unlike the case of CW operation.

1. Chaos in single-mode lasers.

Lasers, operating

with a single longitudinal mode and

with a single transverse mode,

are able to induce chaotic behaviour.

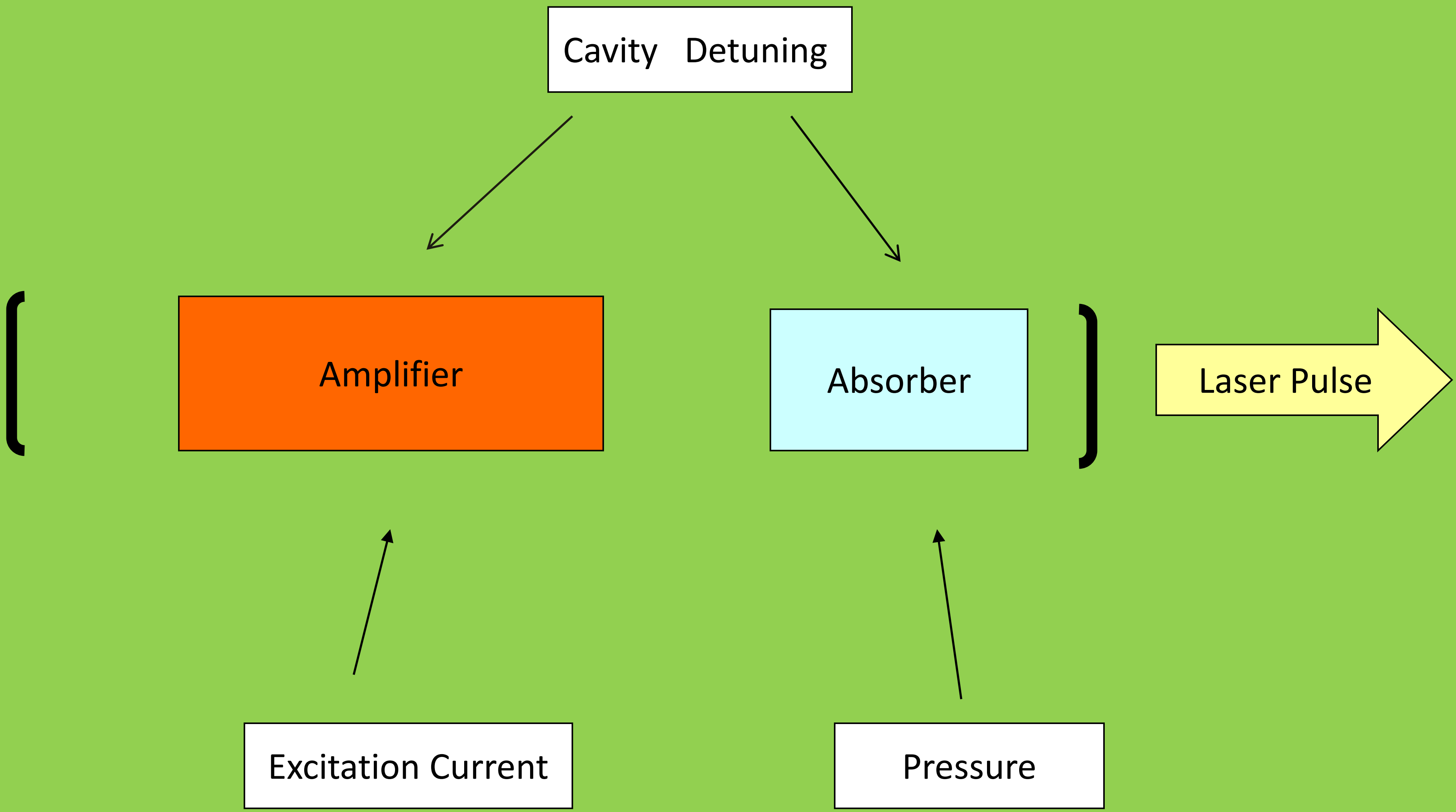
Hybrid laser systems :
lasers with modulated parameters or
electro-optical feedback (Theor.+Exp.)

Consider, however, an all-optical laser system.

Consider a CO_2 Laser with an Intracavity Saturable Absorber (CO_2 LSA).

Main Parameters :

- **Excitation Current in the Amplifier**
- **Detuning of the Laser Cavity Frequency**
- **Pressure in the Absorber**


Cavity Detuning
Amplifier
Absorber
Laser Pulse
Excitation Current
Pressure

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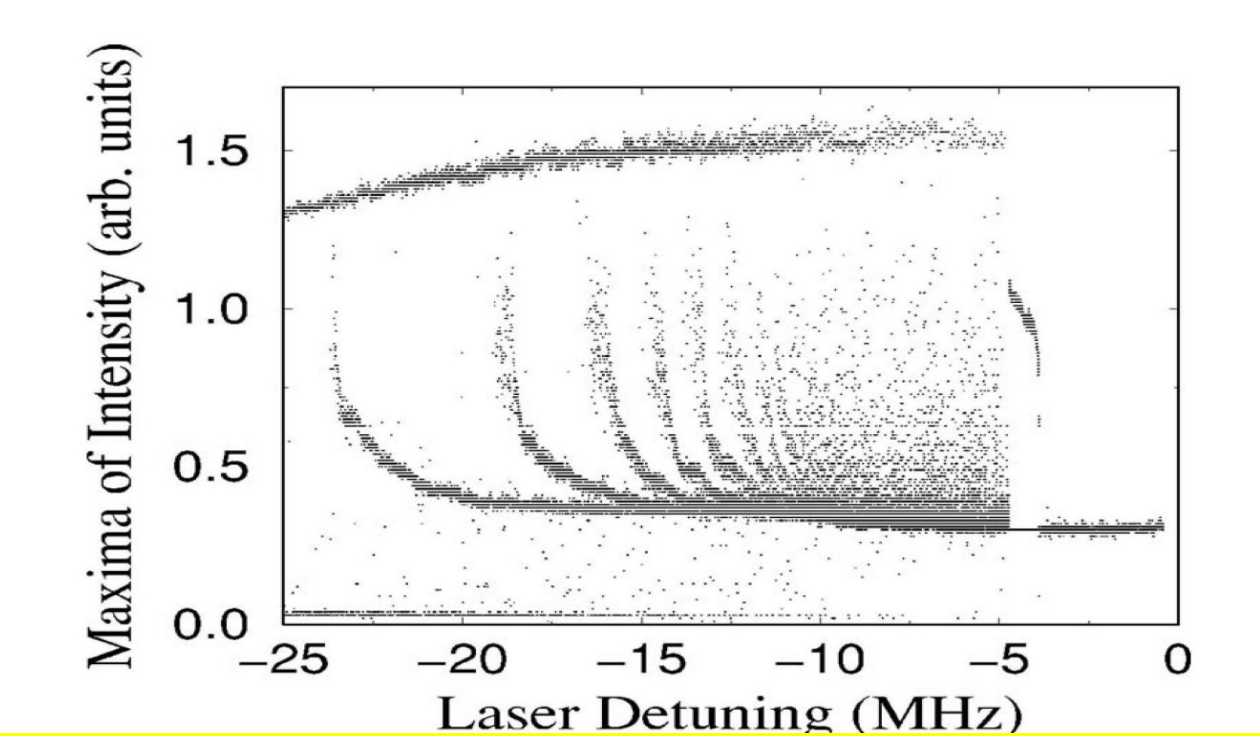
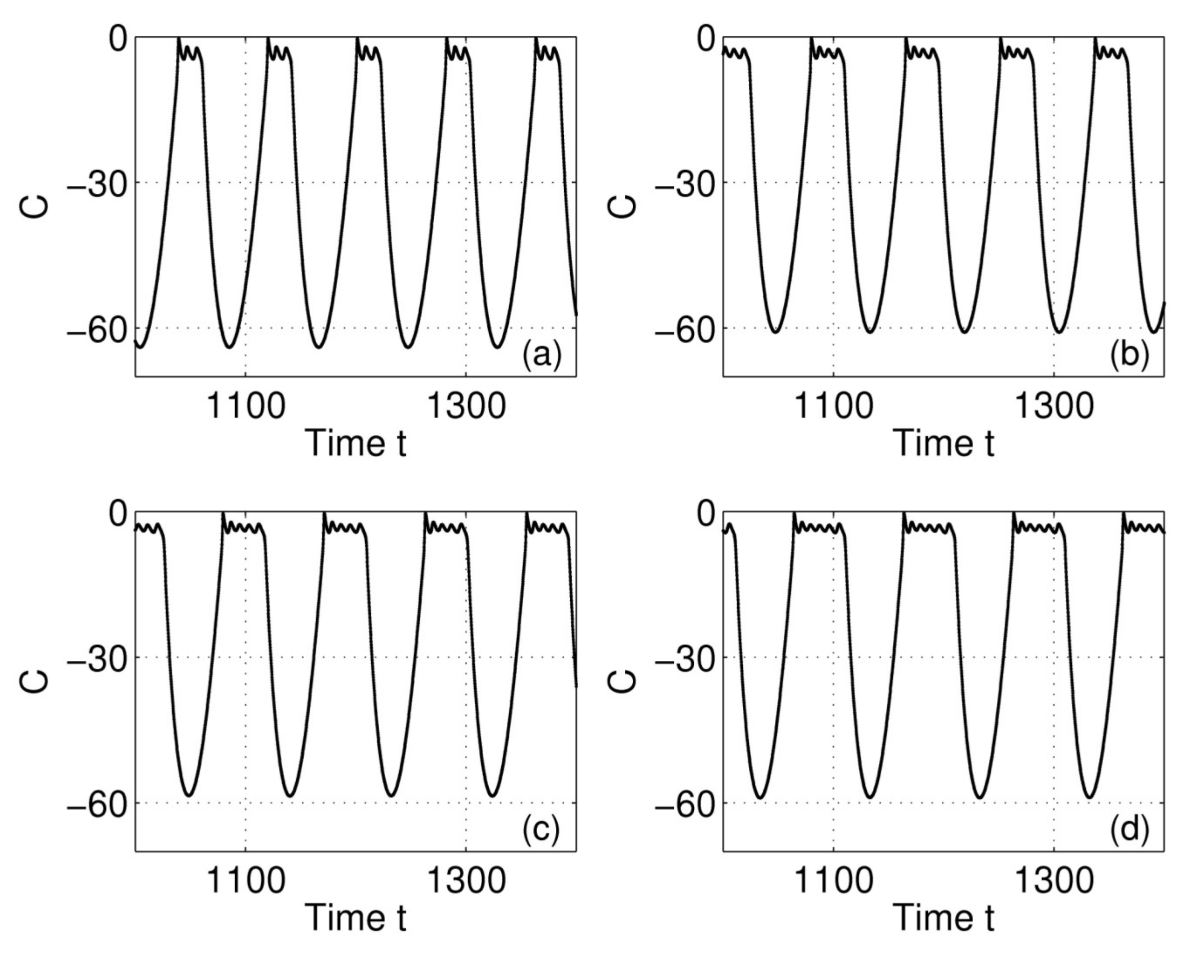
Period-adding cascades of stable periodic orbits (pulsations, spikes) with chaos in the LSA.



H.L.D. de S. Cavalcante and J.R. Rios Leite, Chaos v. 18 , 023107 (2008)



As Q changes

→

Period-Adding

Cascades :

π(n) → π(n+1)

C=ln(I)

Period-adding cascades : $\pi(n) \rightarrow \pi(n+1)$

$\leftrightarrow$

Regions in parameter space where stable $\pi(n)$ coexist according to a certain order.

How do the stability intervals become, eventually, unstable ?

(1)The Periodic Orbits ($\pi(n)$) are

organized along Isolas in parameter space,

, in “all” CO_2 LSA models!

ISOLA : family of periodic orbits that forms
a closed branch (isola),
as a control parameter is changed.

(2) The "characteristic" chaotic
behavior of the LSA is
a complex hopping mechanism
between neighborhoods
of the unstable π(n)
which belong to different isolas,
in "all" CO_2 LSA models !

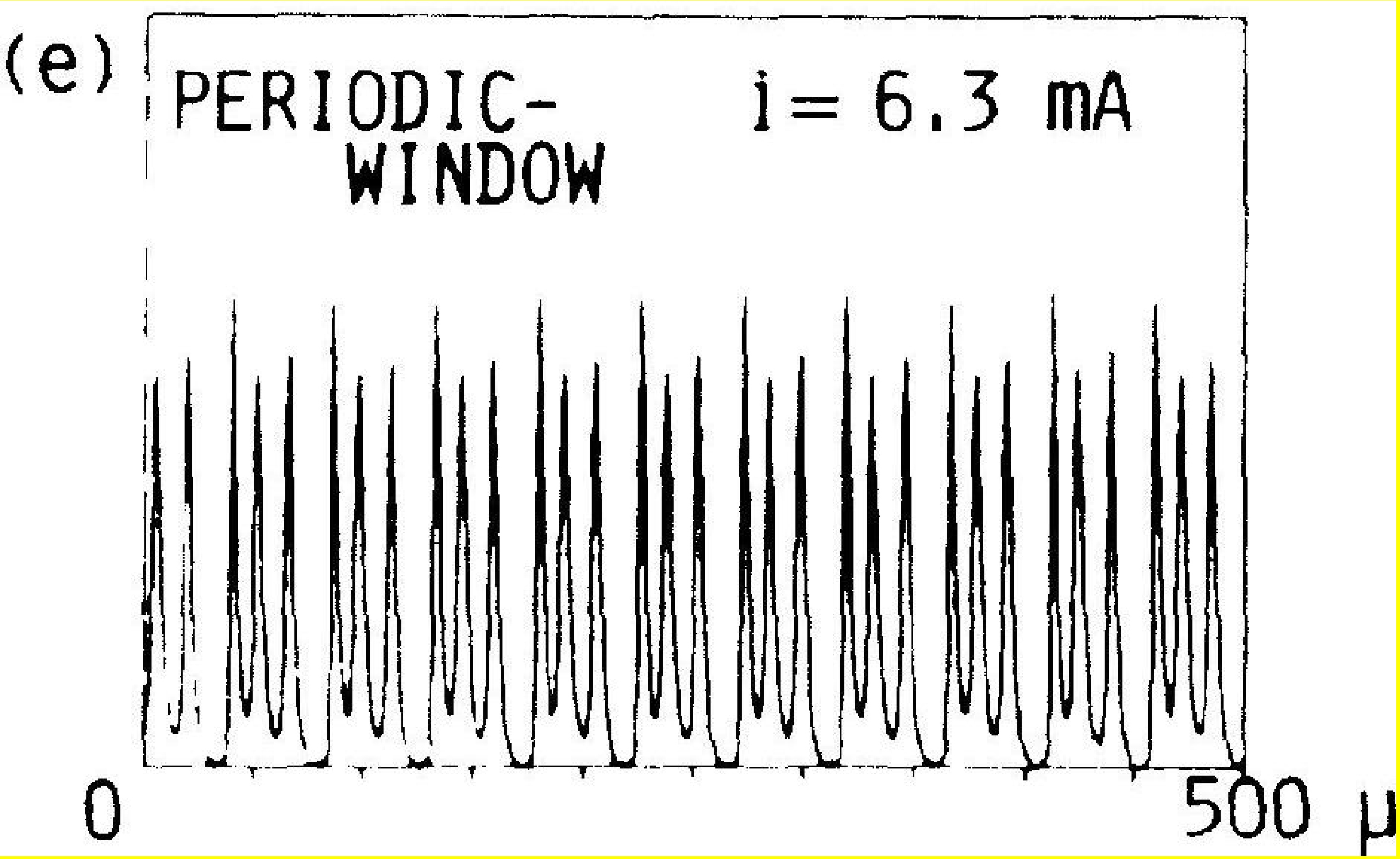
2. CO2 LSA Models

Spiking Behaviour , $\pi(n)$,

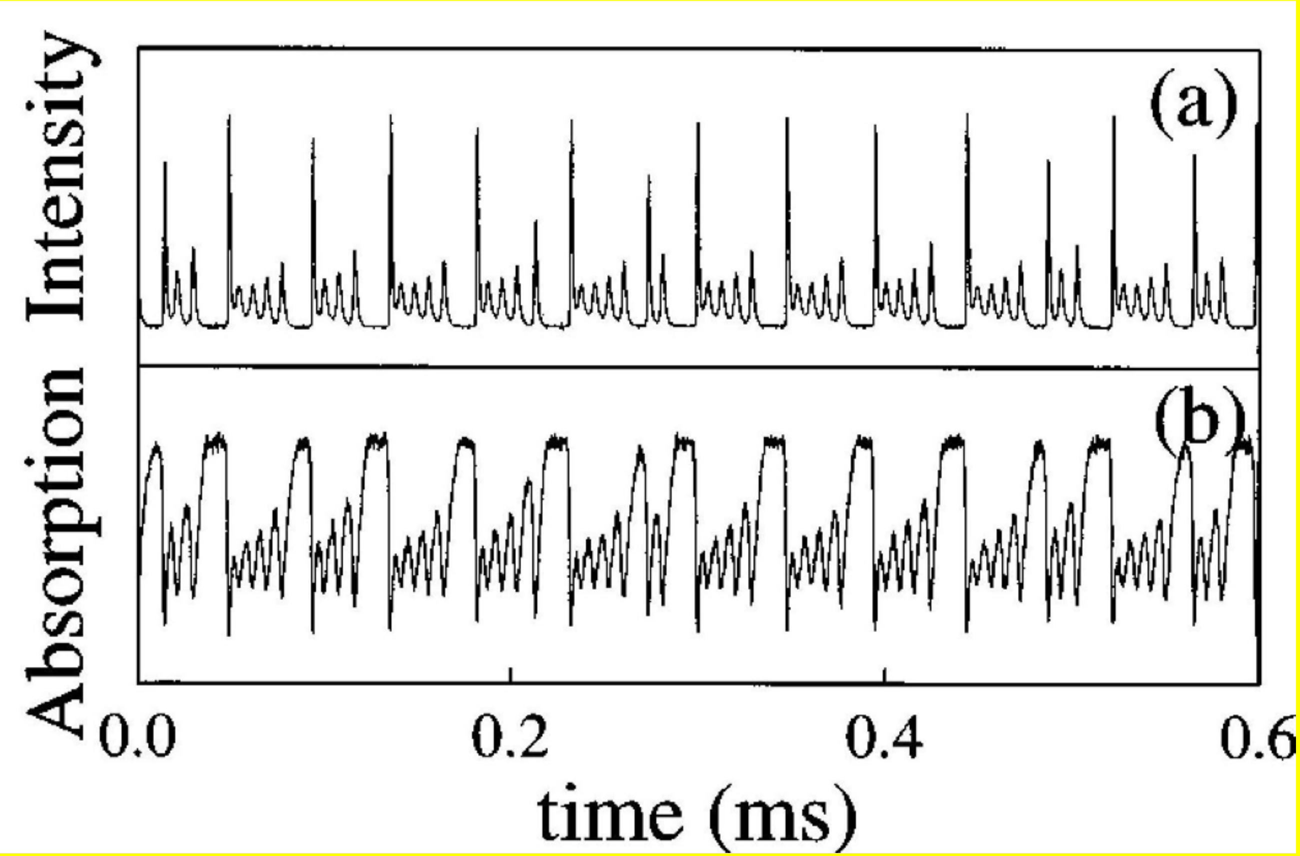
$\leftrightarrow$

Nonlinear Losses of the Absorber

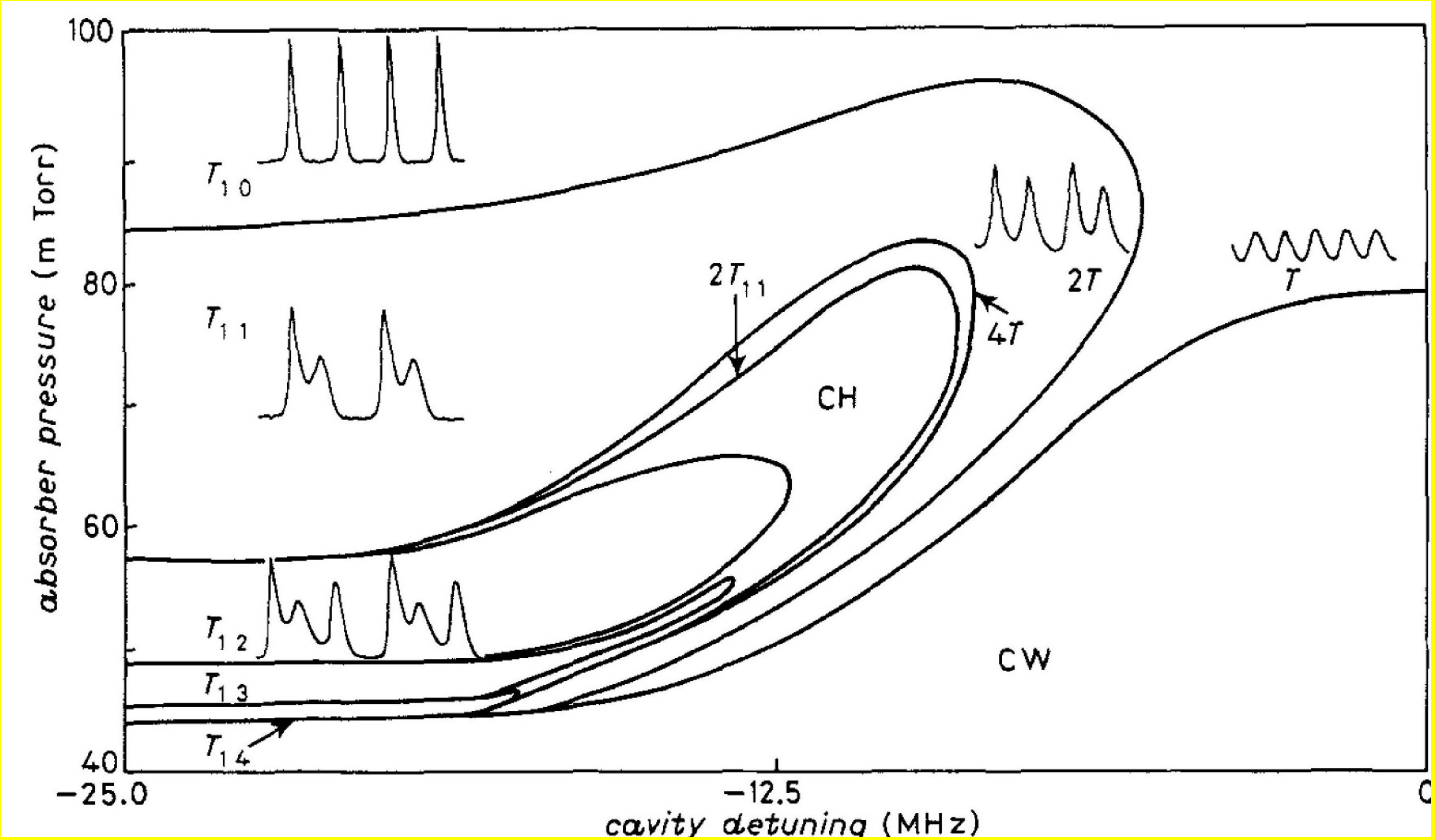
N. B. Abraham et al, Progress in Optics XXV p. 3 (1998).
P. Mandel , Theoretical Problems in Cavity Nonlinear Optics, CUP, NY, (1997)



M. Tachikawa et al, PRL v. 60 p. 2268 (1988)
Parameters : pressure (absorber) & discharge current (amplifier)



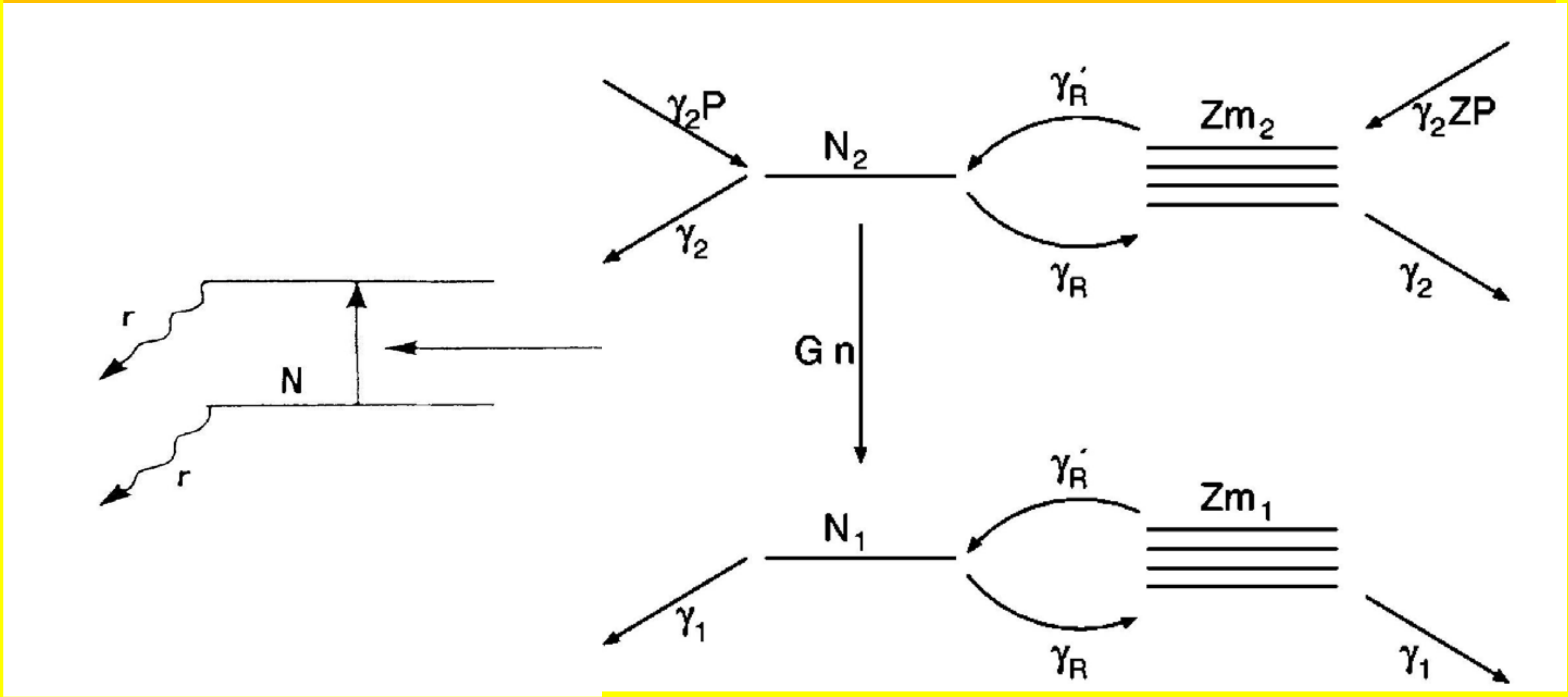
P.C. de Oliveira et al, PRA , v. 55 p. 2463 (1997)
Parameters : Cavity detuning & discharge current gain medium.



D. Dangoisse et al., EPL v. 6 , p. 335 (1988)

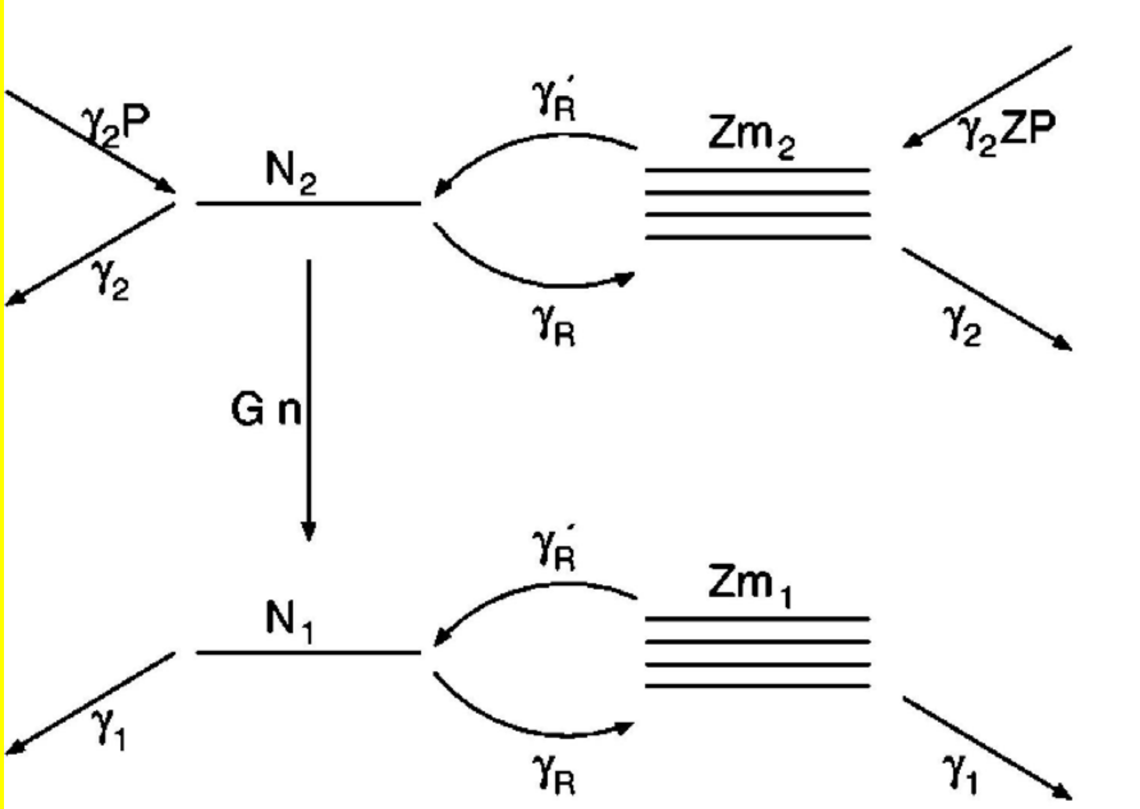
4 Level Model

Absorber Medium   Amplifier Medium



E.J. Doedel, B. Oldeman, C.L. Pando L., IJBC v. 21 p.305 (2011),
C.L. Pando L., PLA v.210 p.391 (1996).

Amplifier (CO2)



I. Burak et al. , IEEE v. 7 p.73 (1971).

The Absorber is assumed to be a
FAST Saturable Absorber (large relaxation rates).

We apply a reduction technique.

(M. Ciofini et al., PRE v. 48 p. 605 (1992),
P. Braza, SIAM 1999,
Y. Kuznetsov, "Elements of Applied Bif. Th."
(the reduction principle).

LSA Equations

$$\frac{dI}{dt} = -I + \frac{(z+1)\Omega}{z}(w - v) - \frac{\alpha I}{1 + 2\beta I} \ ,$$

$$\frac{dv}{dt} = -(\Omega + \gamma_1)v + \Omega w \ ,$$

$$\frac{dw}{dt} = \Omega v - (\Omega + \gamma_2)w + z\gamma_2 Q \ .$$

$$\Omega = \Omega_1 I \ , \qquad \Omega_1 = \frac{z+1}{(z+1)^2 + 2zI/\gamma_{R'}}$$

E.J. Doedel, B. Oldeman, C.L. Pando L., IJBC v. 21 p.305 (2011)

I is the laser intensity within the laser cavity.

V, W are the effective atomic populations

$\Omega(I)$ is a rational function of I (laser intensity), In our CO_2 LSA model we have,

$\Omega \rightarrow \Omega_1 (I)\, I$ ("built-in absorber").

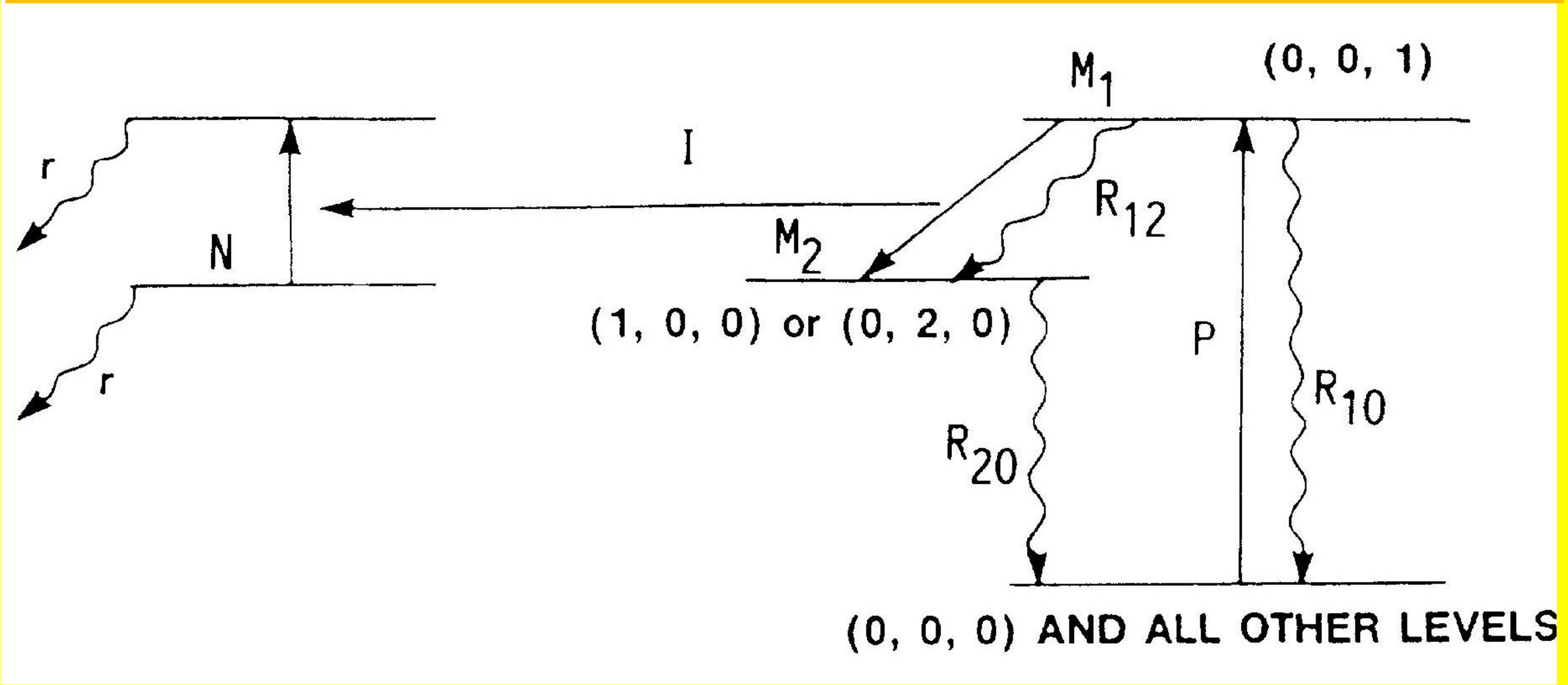
The "3 : 2" level model for the CO_2 LSA
(with a fast absorber) is a limit case.

(M. Tachikawa et al. , Appl. Phys. B, v. 39 p. 83 (1986))

$\Omega \rightarrow \Omega_1 (0)\, I$ (large nonradiative relaxation rates)

3:2 Level Model

Absorber Medium | Amplifier Medium

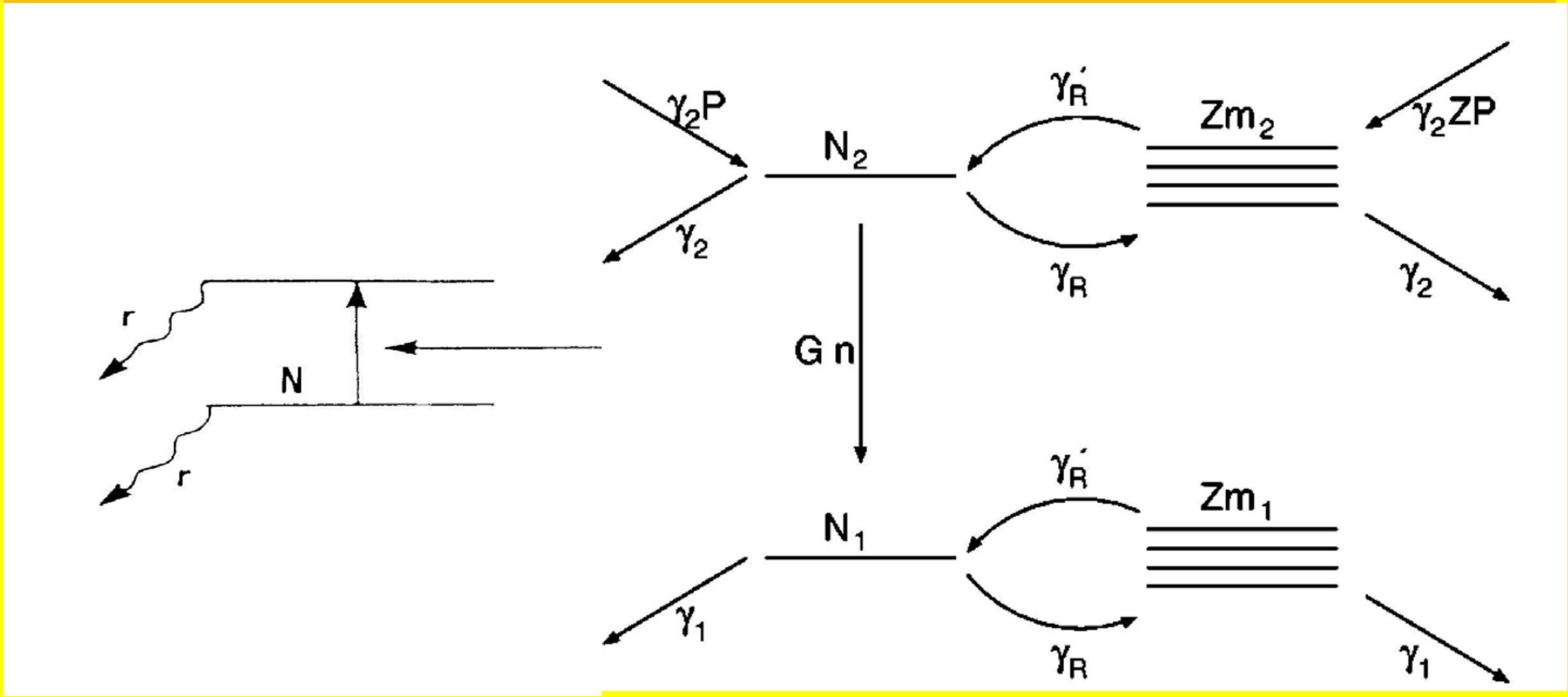


T. Tohei et al, PRA v. 45 p. 5166 (1992)
Parameters : SA pressure & discharge current gain medium

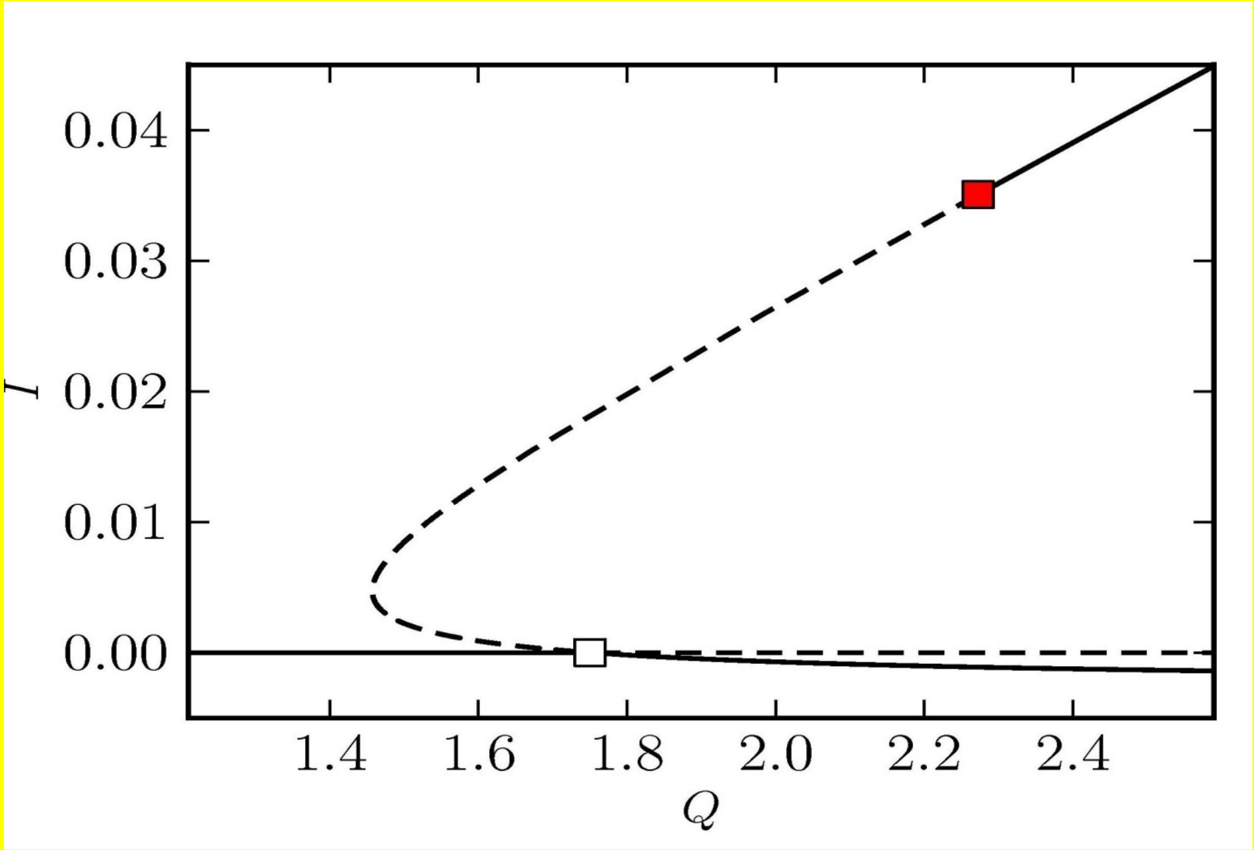
4 Level Model

Absorber Medium

Amplifier Medium



E.J. Doedel, B. Oldeman, C.L. Pando L., IJBC v. 21 p.305 (2011)



4 Level Model

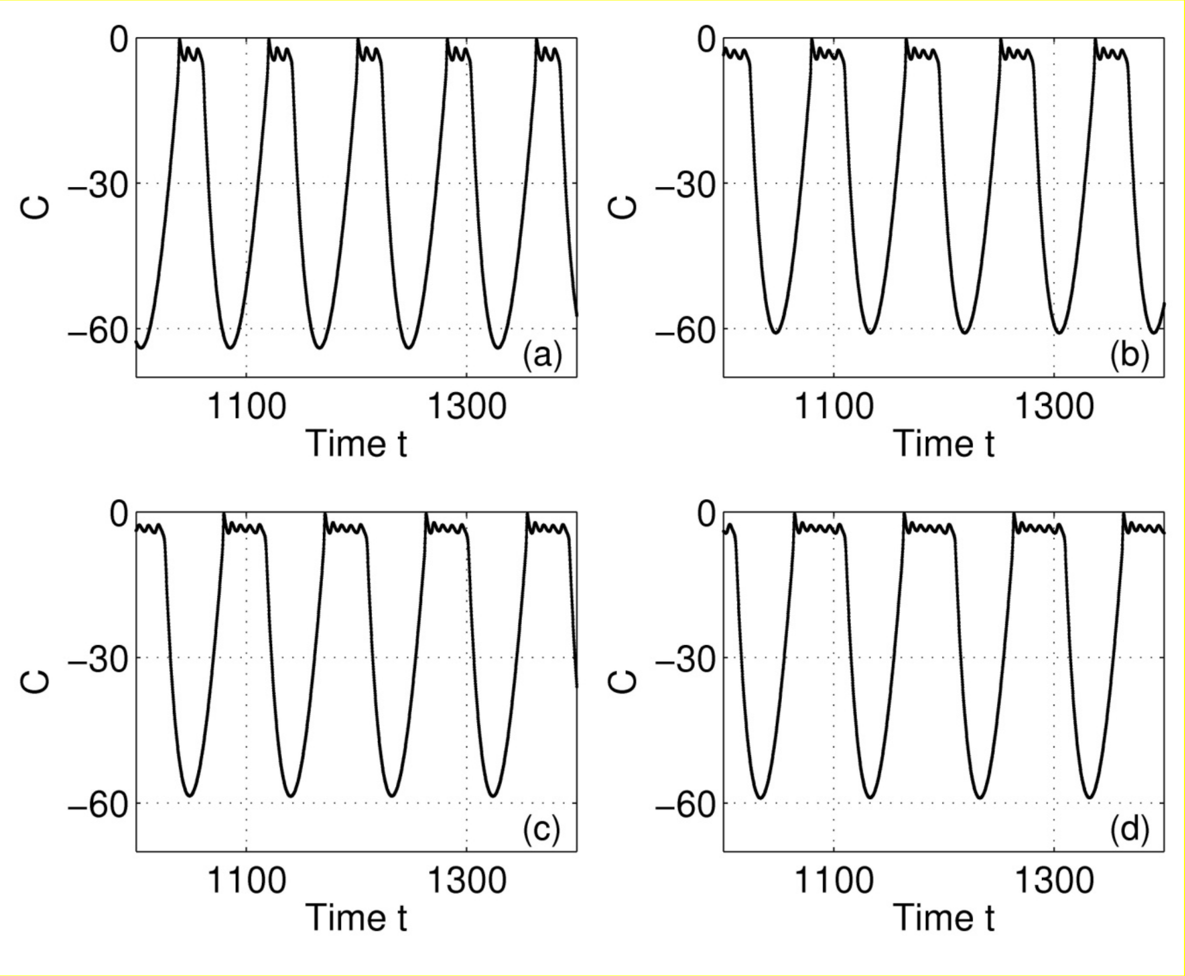
$I=0$ is stable when $Q < 1+\alpha$.

$Q = 1+\alpha$, TC bifurcation (white square)

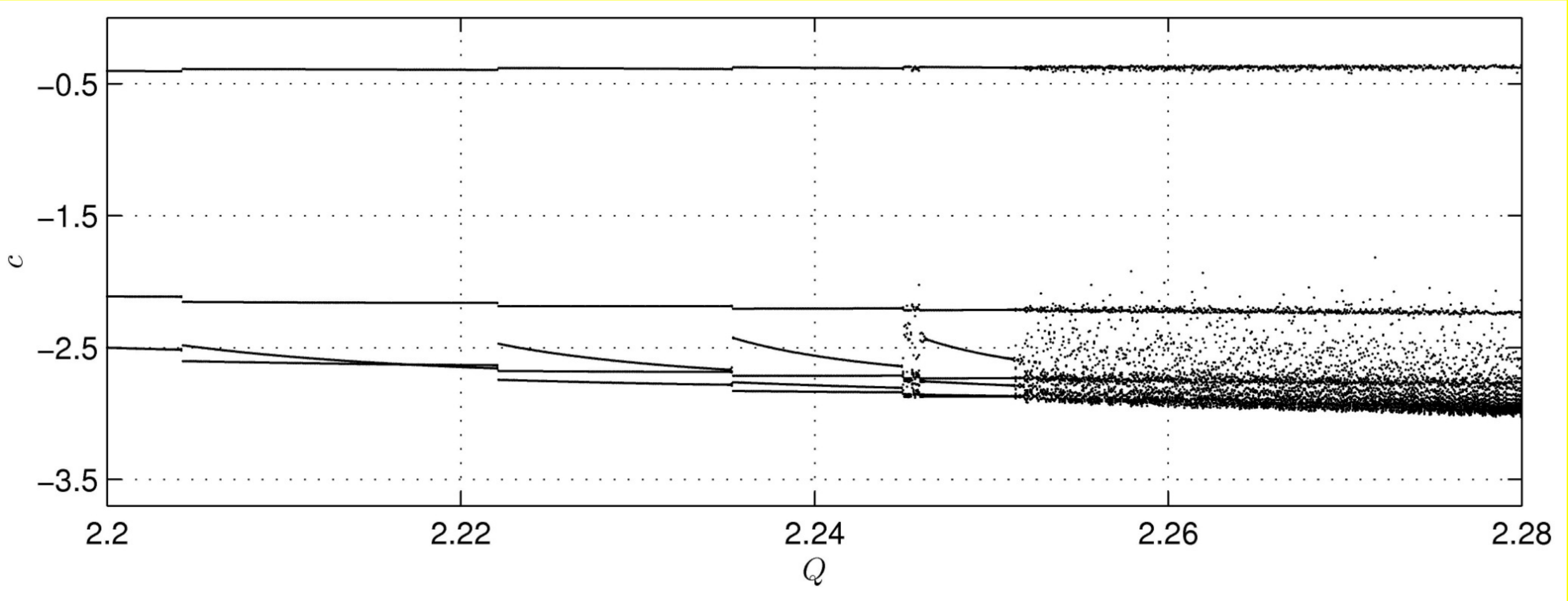
$\alpha=0.75$, $\beta=200$

HB(red square) at $Q_{HB} \approx 2.271$

3. Isolas of Periodic Orbits



Q changes →
Period-Adding
Cascades



Maxima of the C=ln(I) versus Q.

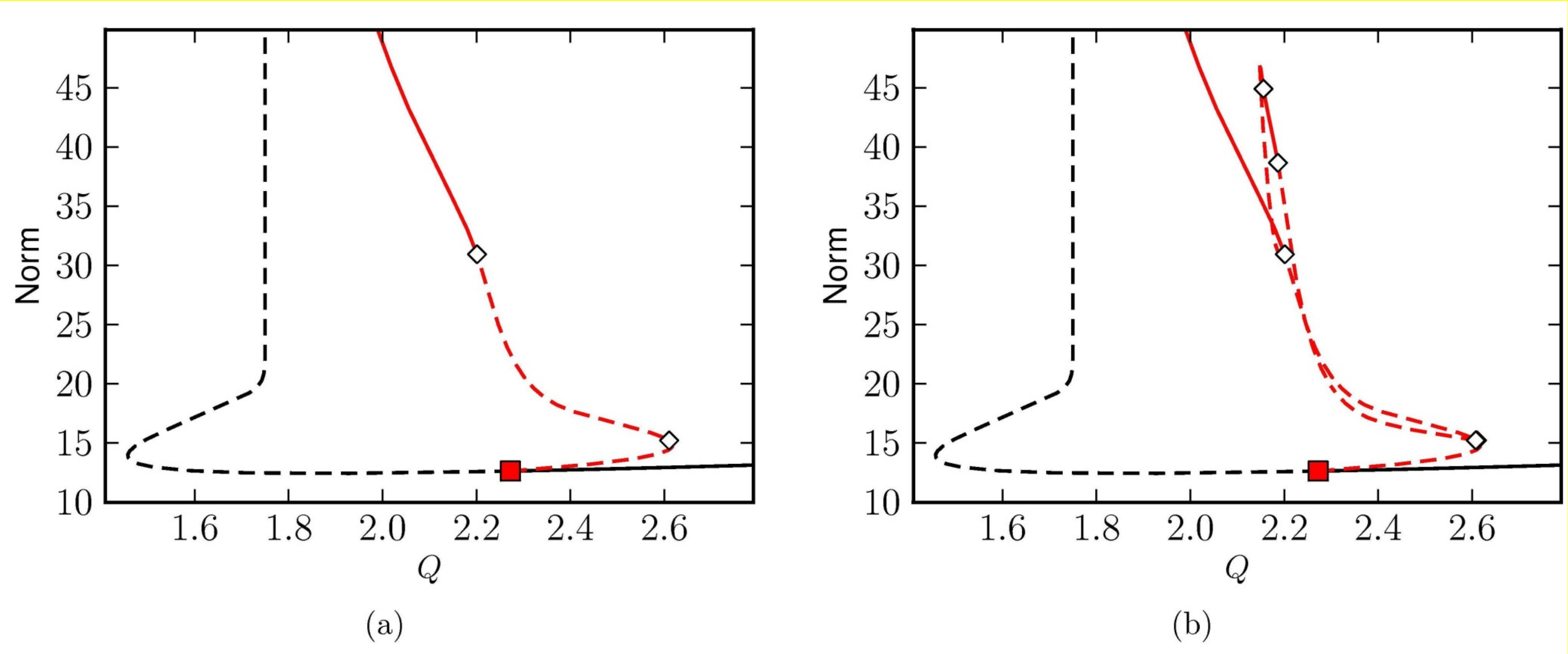
Phase Digram as Q changes → Period-Adding Cascades

The Norm for the Stationary Solutions :

$$\sqrt{c^2 + v^2 + w^2}\ .$$

The Norm for Periodic Solutions :

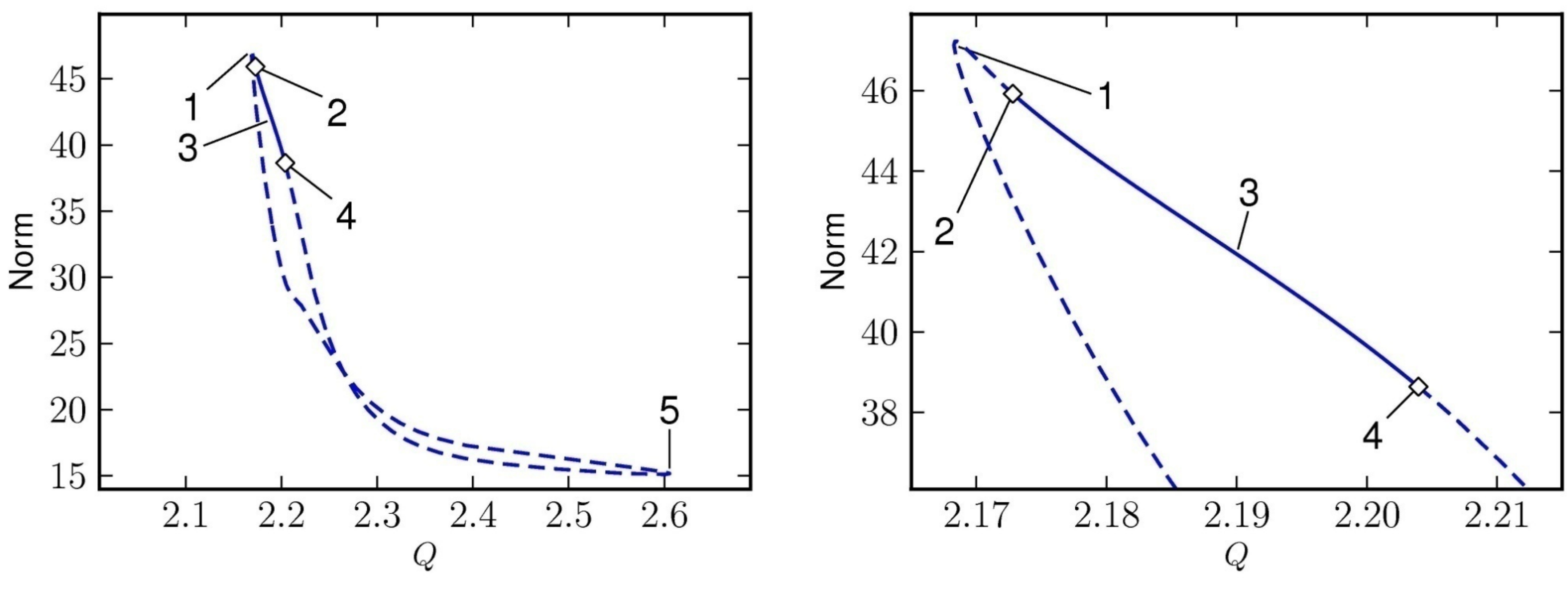
$$\frac{1}{\sqrt{T}} \left[\int_0^T (c(t)^2 + v(t)^2 + w(t)^2)\, dt \right]^{1/2} .$$



Norm → ∞ ► C → −∞ ► I → 0.

Red Square : HB , Diamond : PDB.

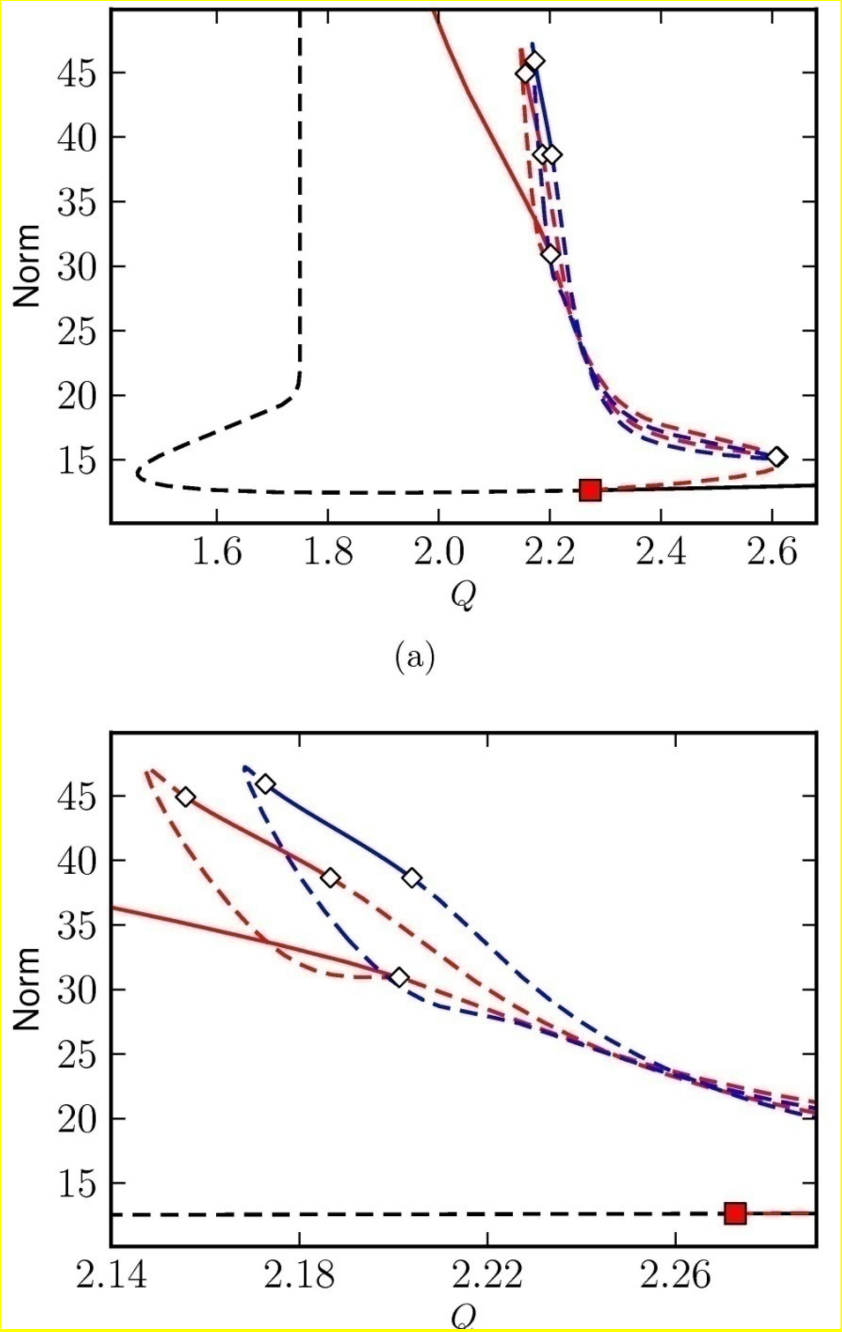
Isola (Closed Branch of Periodic Solutions) as Q is changed.



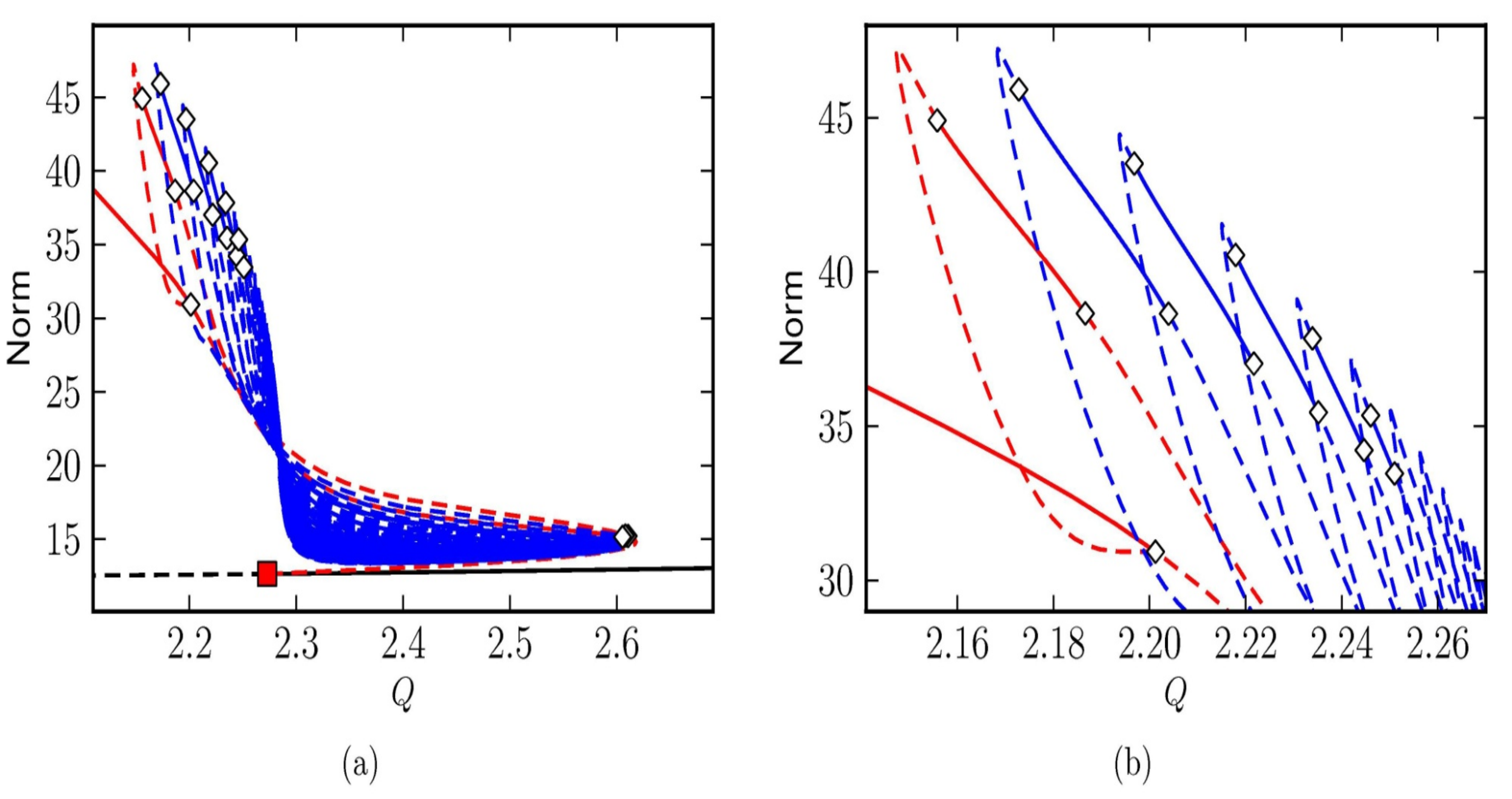
(a) (b)

Isola of periodic orbits (pulsations) with 3 maxima, Π(3).

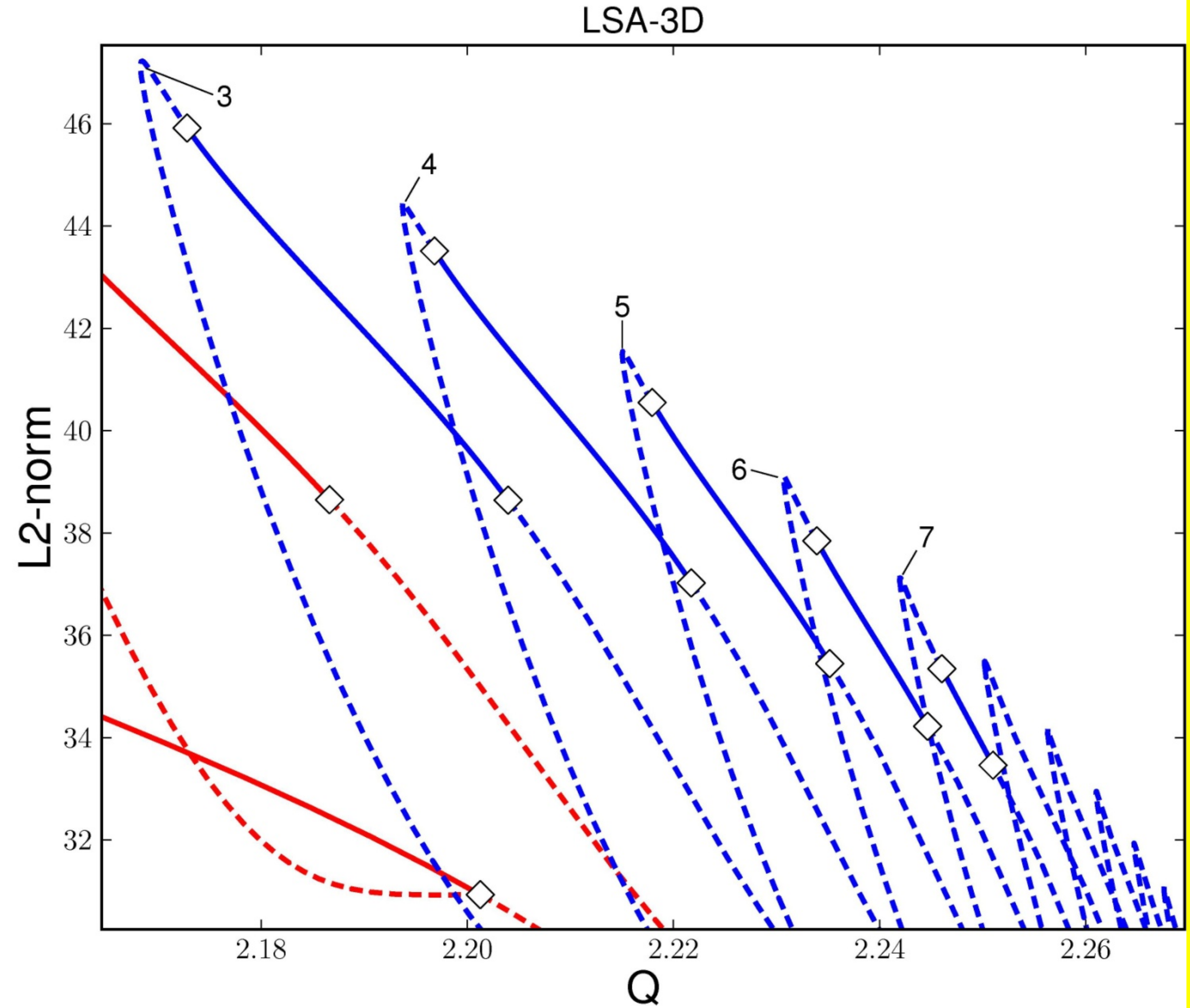
.



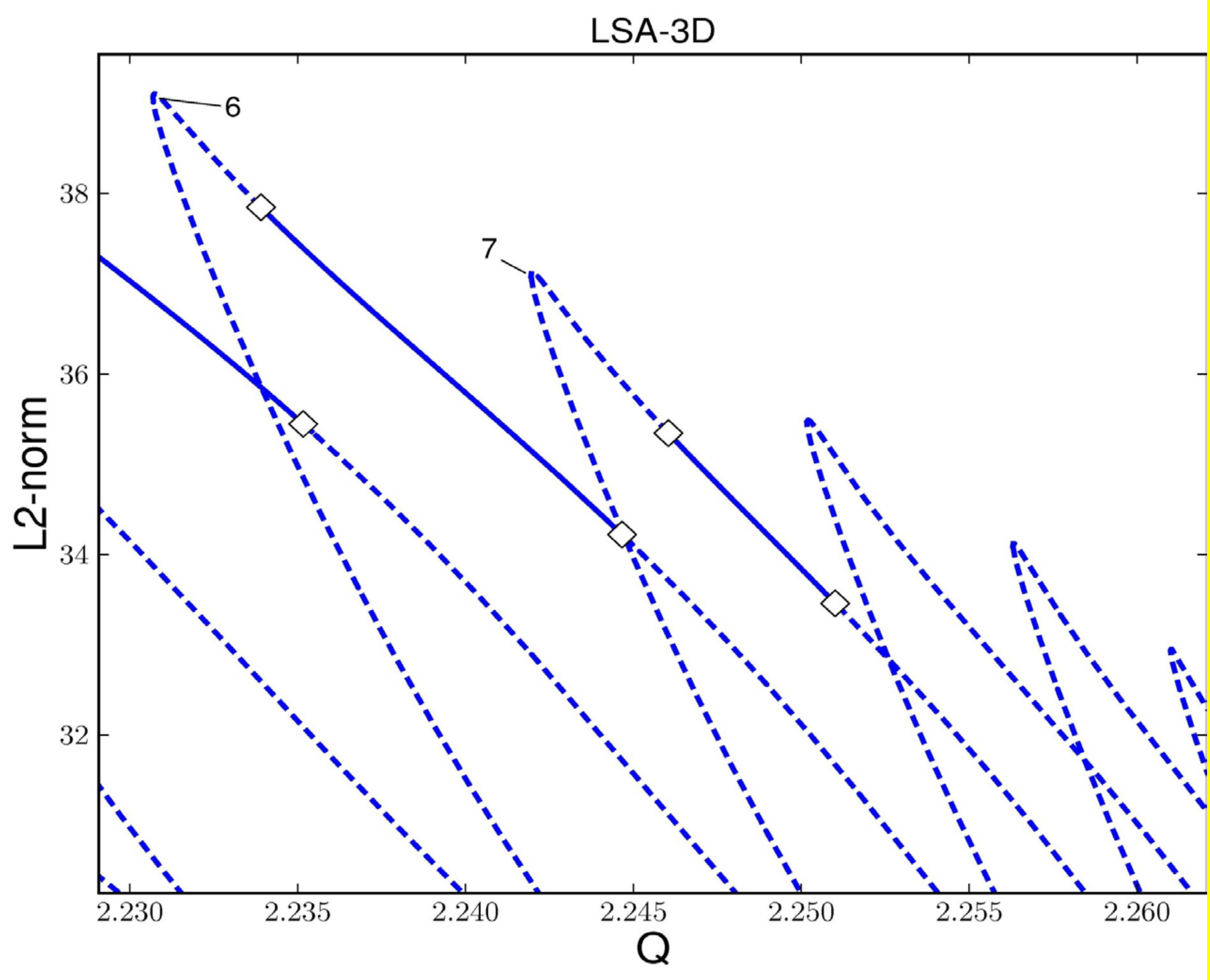
Branches of stationary solutions, main periodic orbits, PD period orbits and periodic orbits with 3 maxima, Π(3).



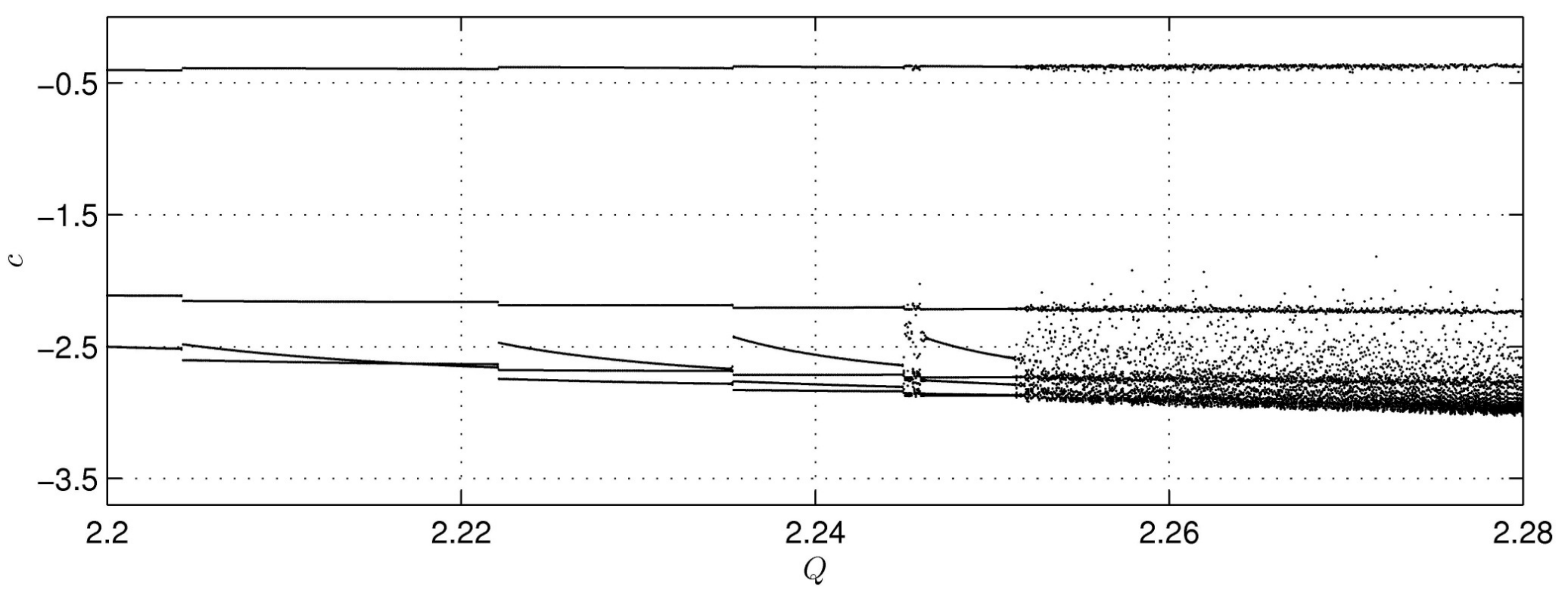
Branches of stationary solutions, main periodic orbits,
PD period orbits and ISOLAS of
periodic orbits with N=3, 4, 5, 6, ……50 maxima, Π(N).



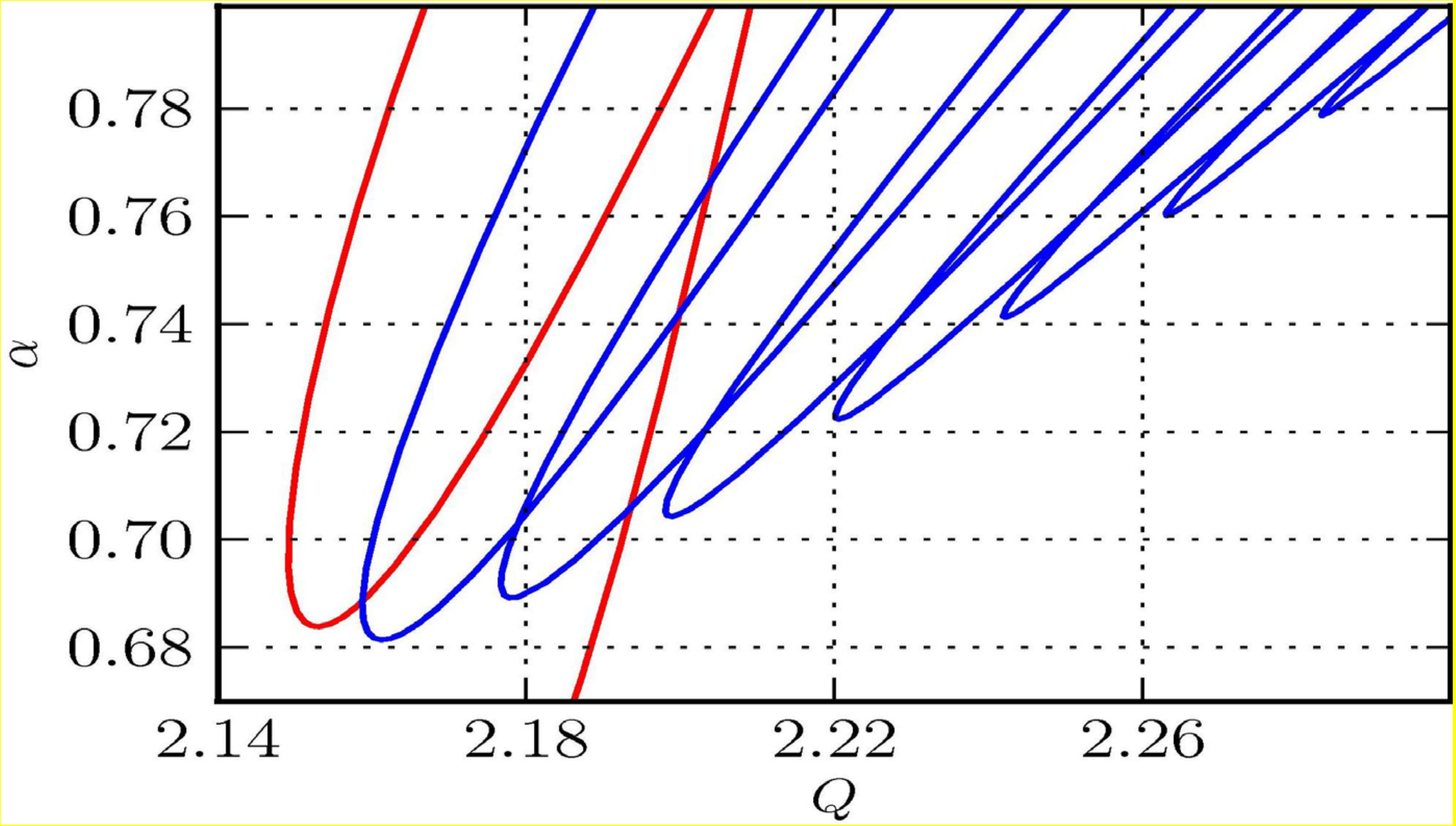
Branches of main periodic orbits,
PD period orbits and ISOLAS of
periodic orbits with N=3, 4, 5, 6, ...50 maxima, Π(N).



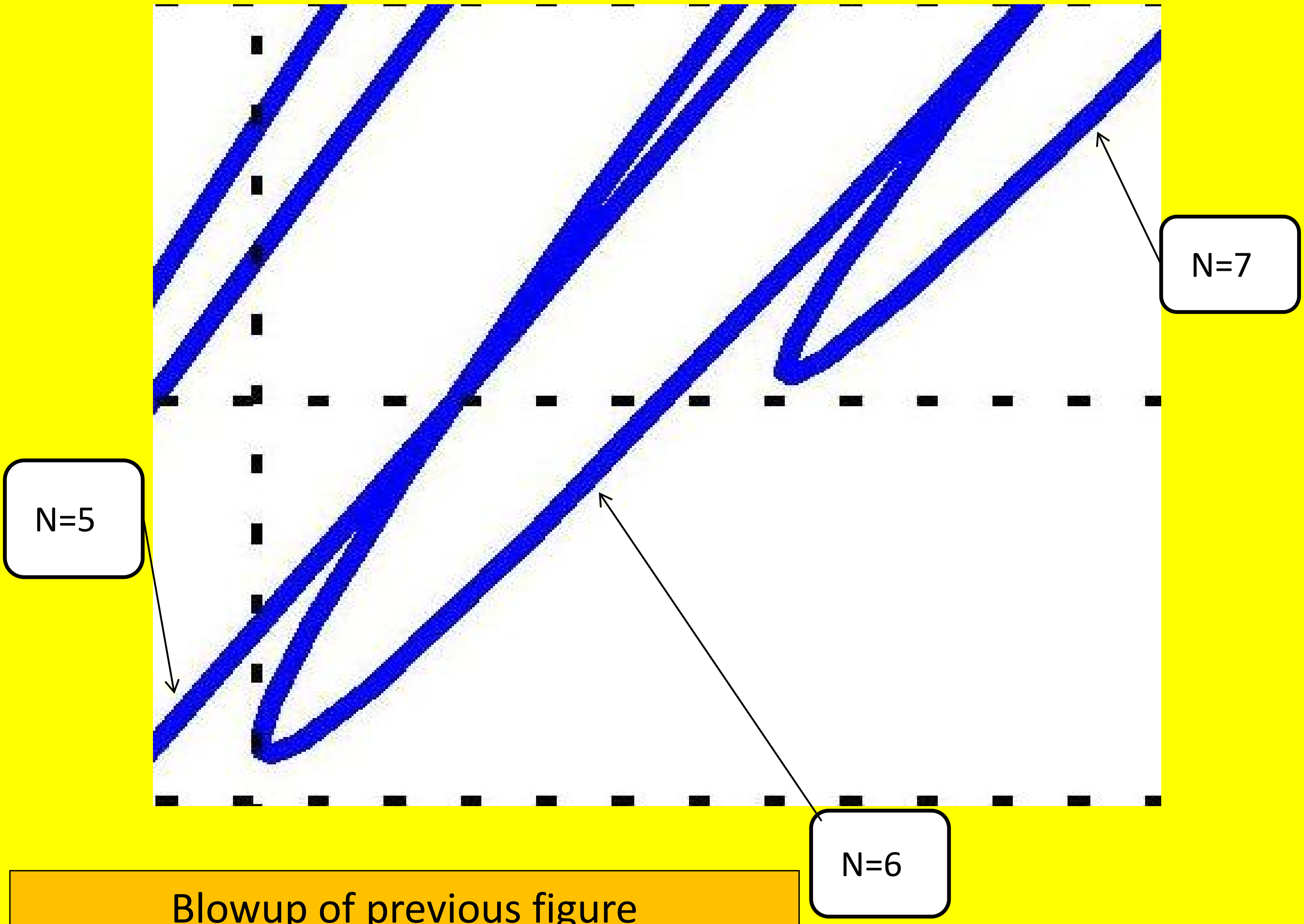
Blowup of previous figure.



Phase Diagram, where “Q” (pump) is swept.



Intervals of stability for the main periodic orbit (red),
The PD period orbit (red) and
periodic orbits with M maxima.

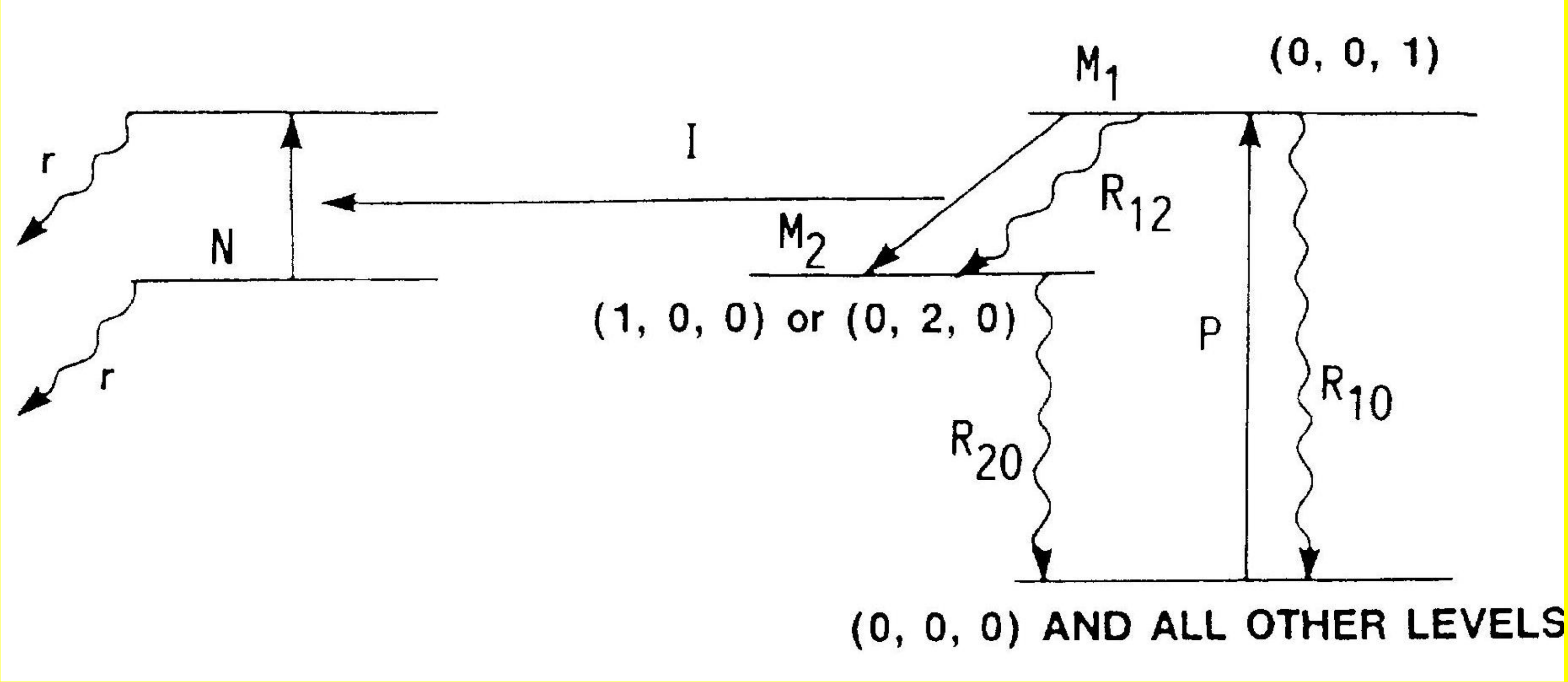


Blowup of previous figure

3:2 Level Model

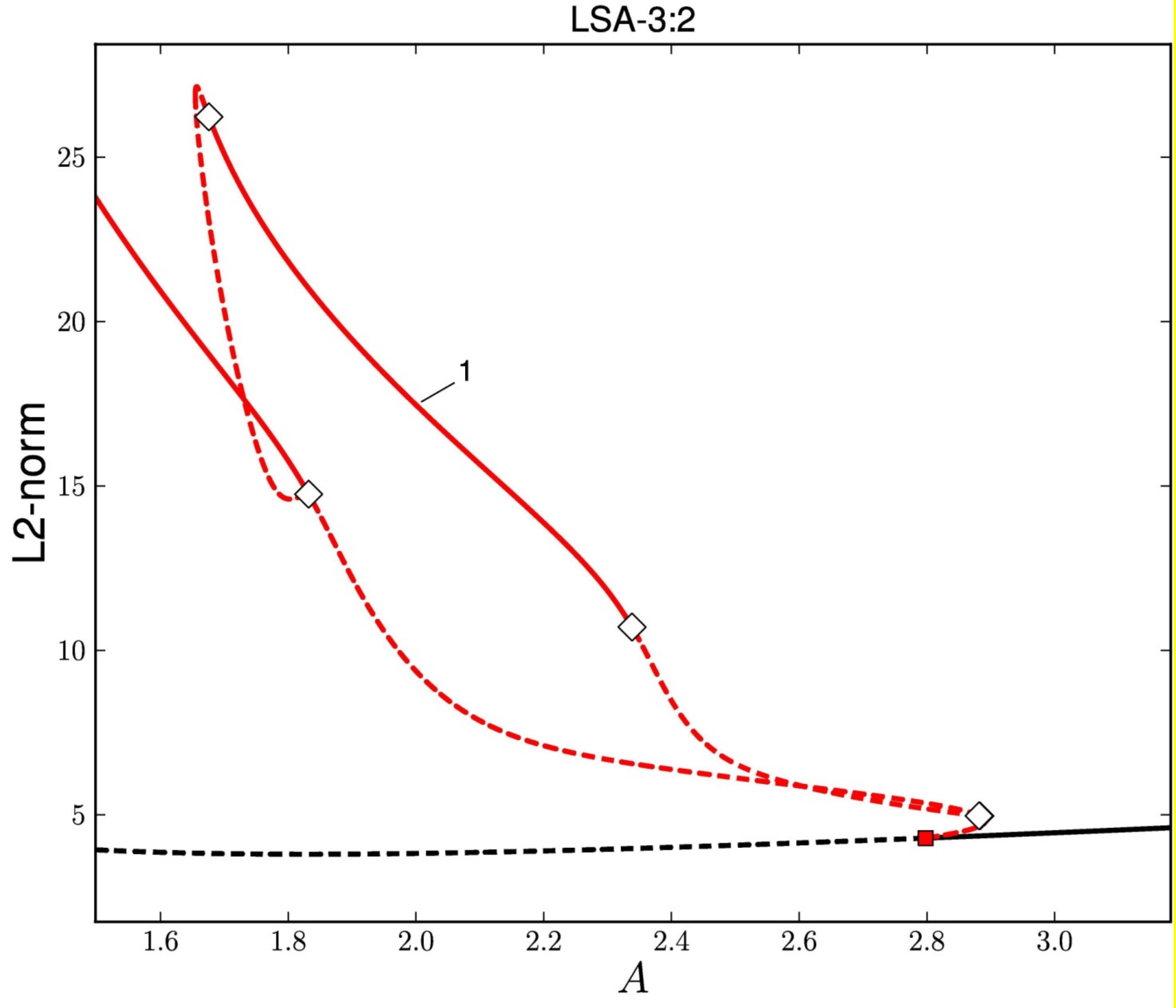
Absorber Medium

Amplifier Medium

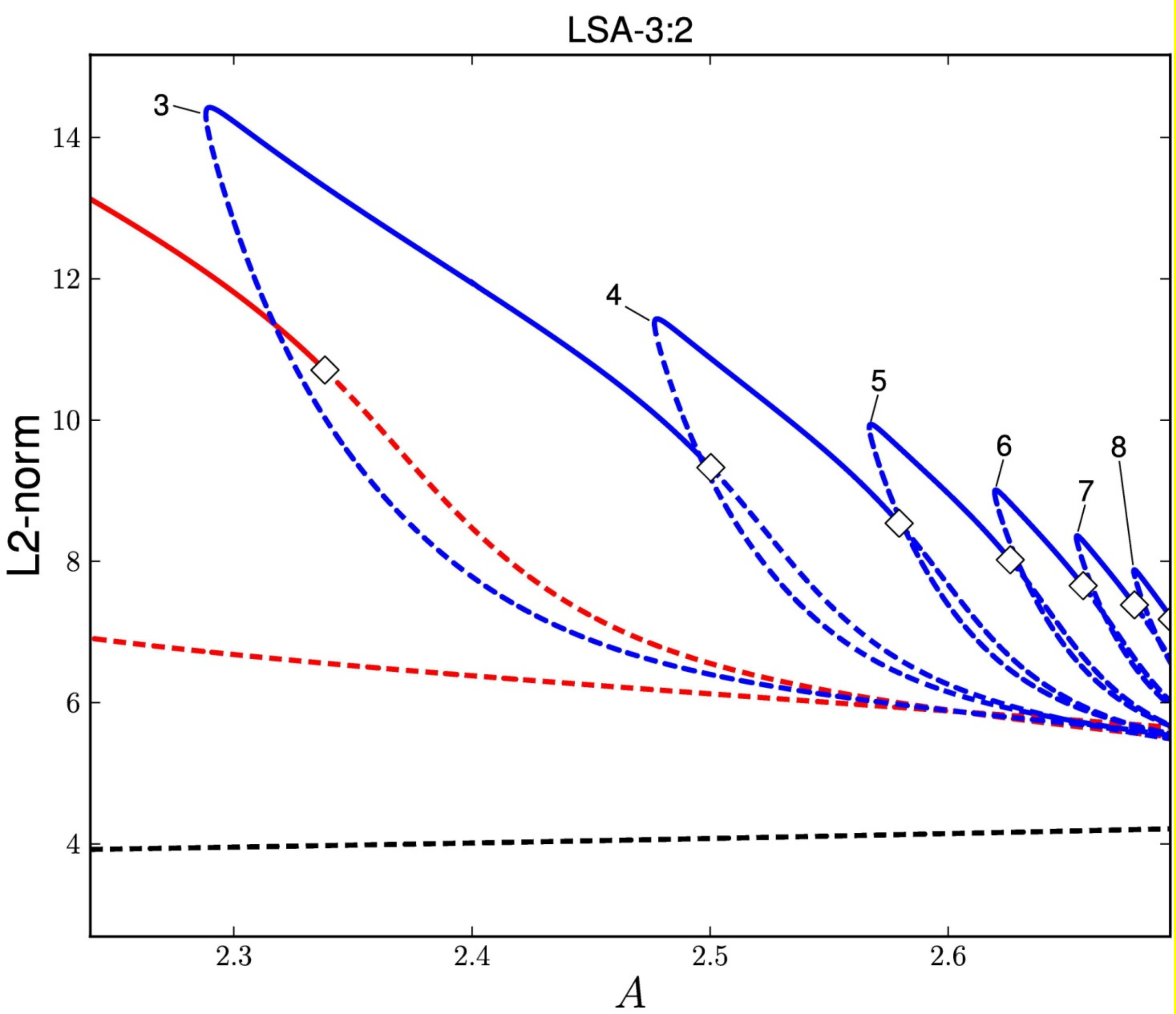


T. Tohei et al, PRA v. 45 p. 5166 (1992)

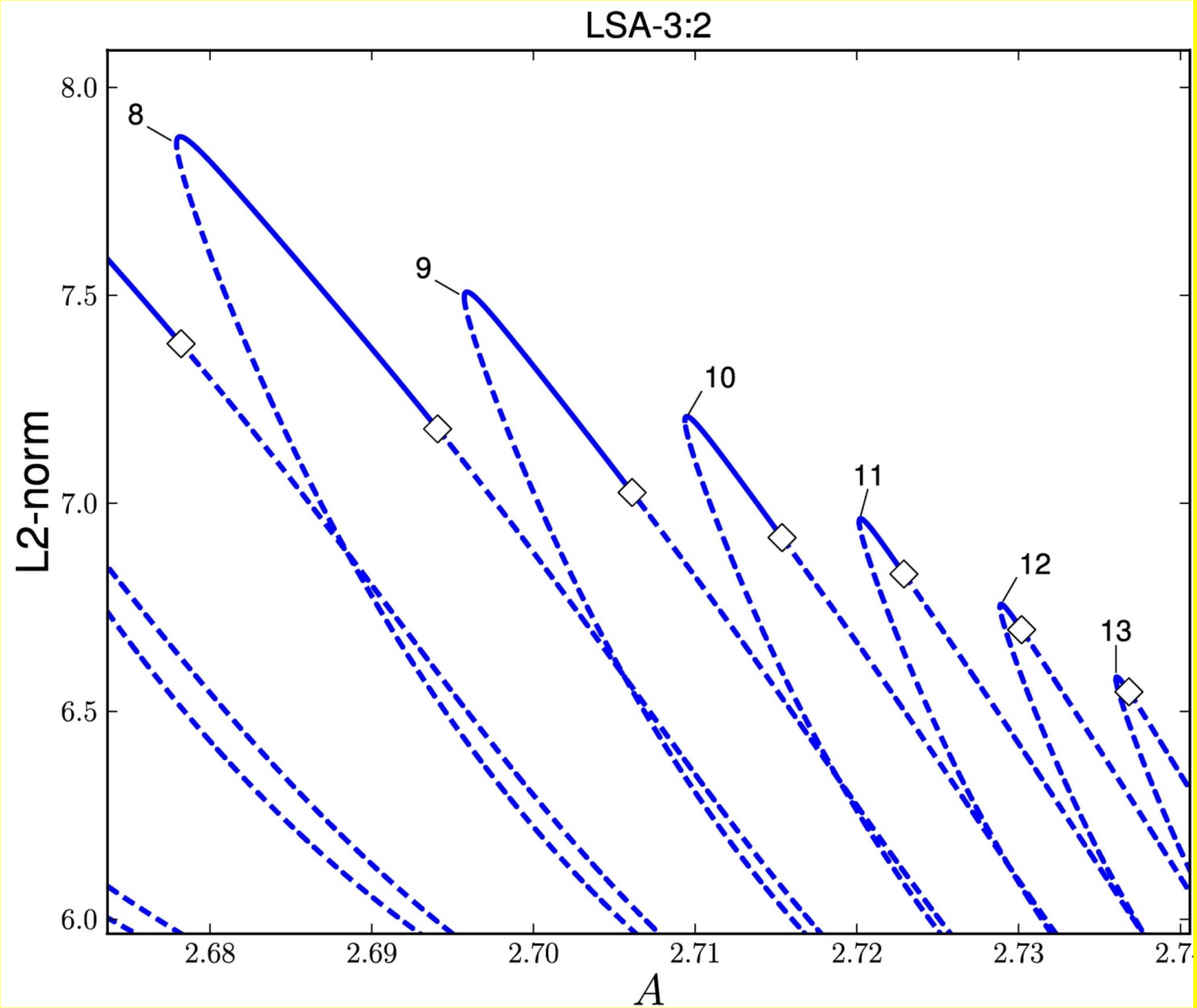
Parameters : SA pressure & discharge current gain medium



Branches of stationary solutions, main periodic orbits, and PD period orbits



Branches of stationary solutions, main periodic orbits,
PD period orbits and ISOLAS of
periodic orbits with N=3, 4, 5, 6, 7, 8 maxima, Π(N).



Blow Up of ISOLAS of
periodic orbits with N=3, 4, 5, 6, 7, 8 maxima, Π(N).

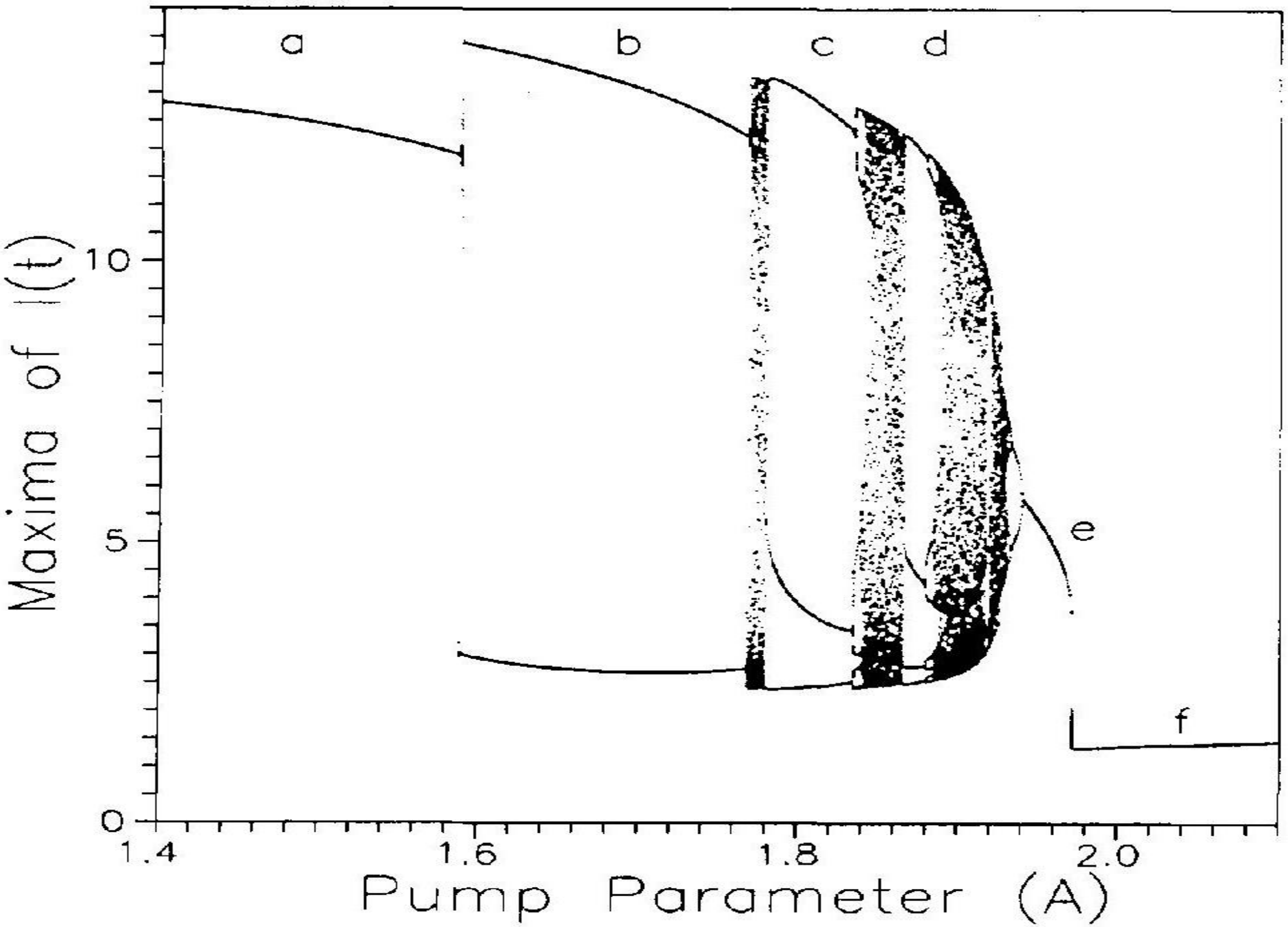


Fig. 5. Bifurcation diagram of the model of the LSA for parameters of Table 1. The letters indicate the following regimes: a, $P^{(0)}$; b, $P^{(1)}$; c, $P^{(2)}$; d, $P^{(3)}$; e, T; f, I_+.

M. Lefranc et al. , JOSA B v. 8 p.239 (1991).

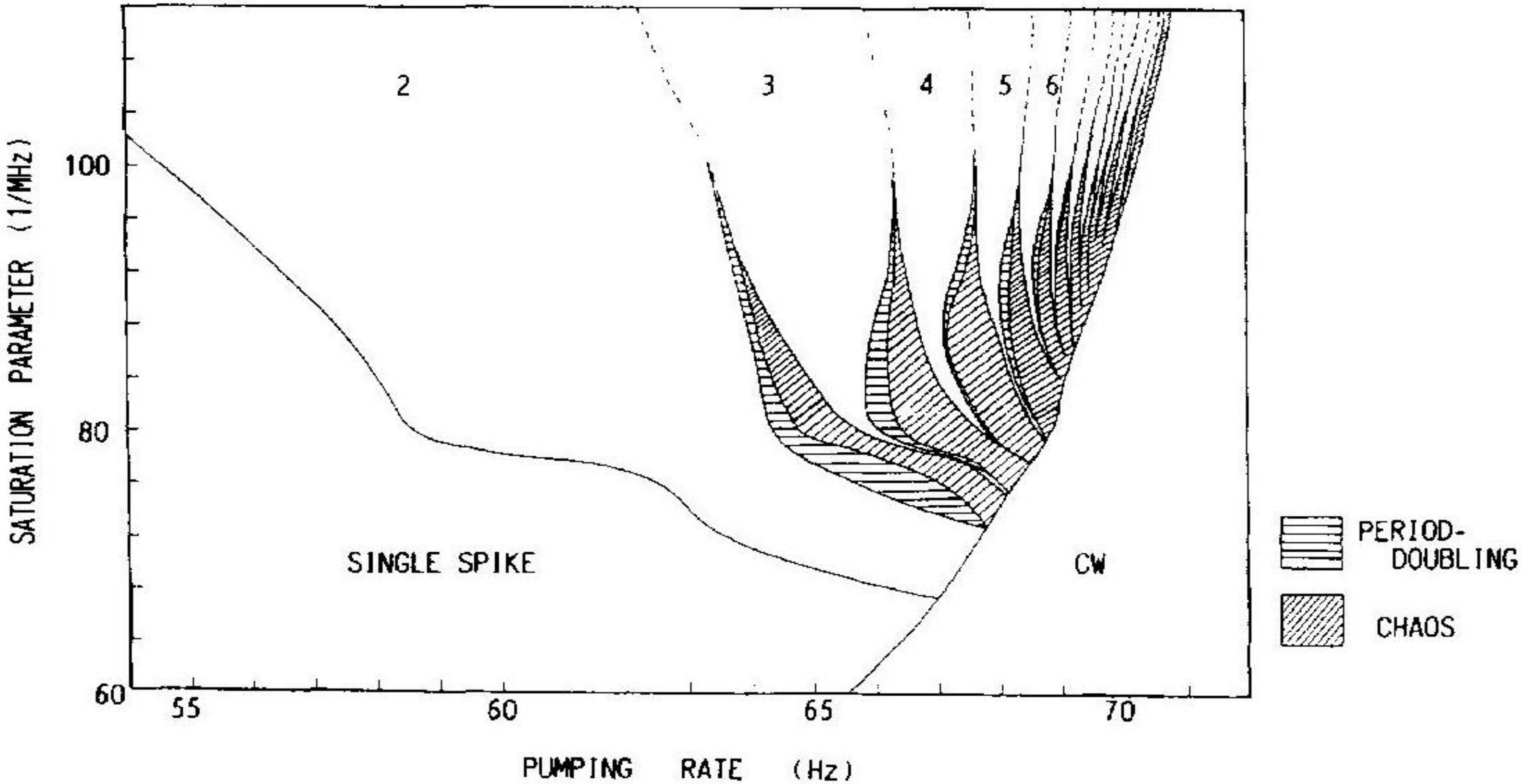
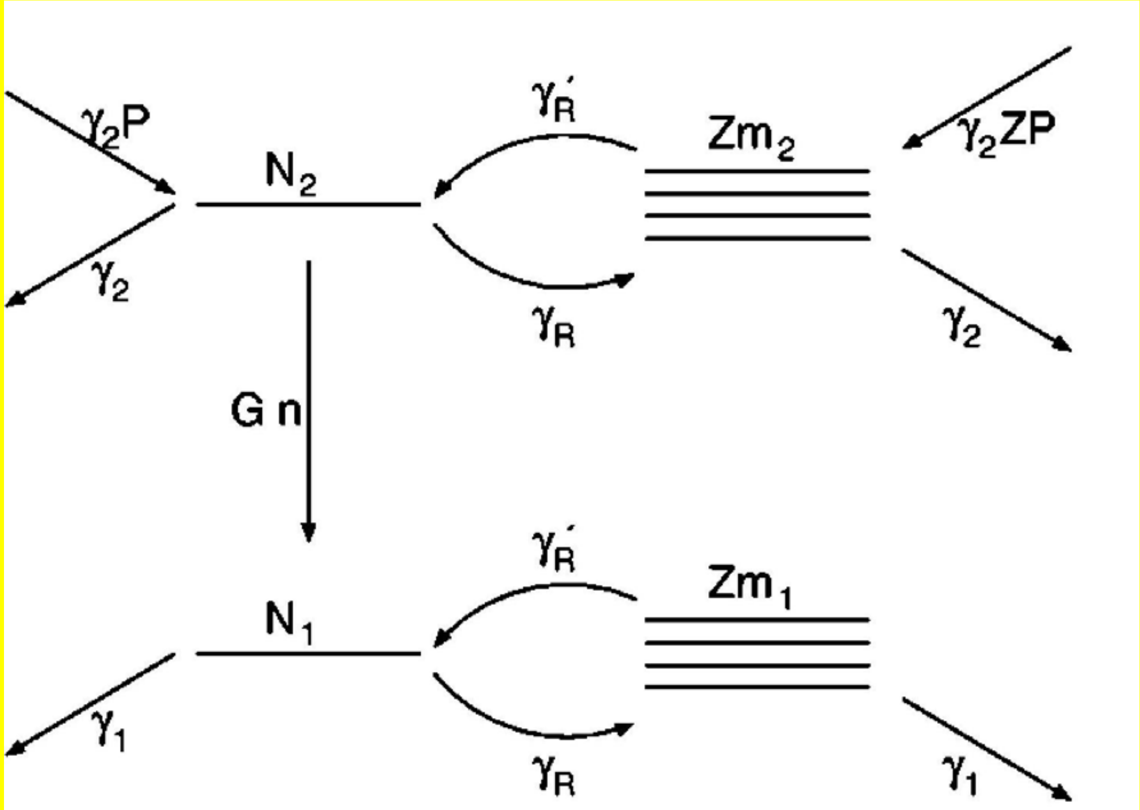


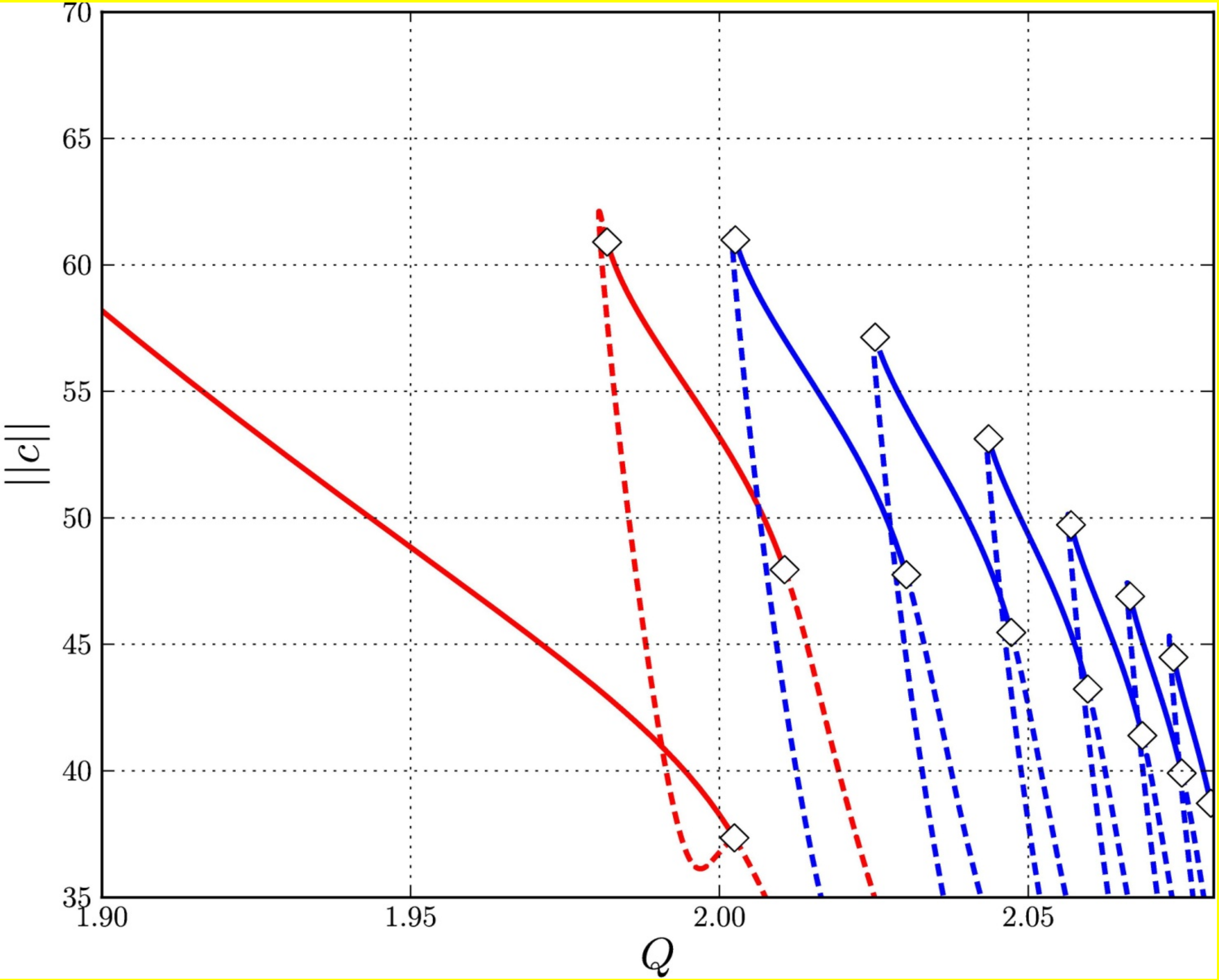
FIG. 2. Theoretically obtained phase diagram for the pumping rate and the saturation parameter where the regions for regular and chaotic PQS are depicted. The figures in the phase diagram indicate the number of peaks involved in a single PQS pulse.

T. Tohei et al. , PRA v. 45 p.5166 (1992).

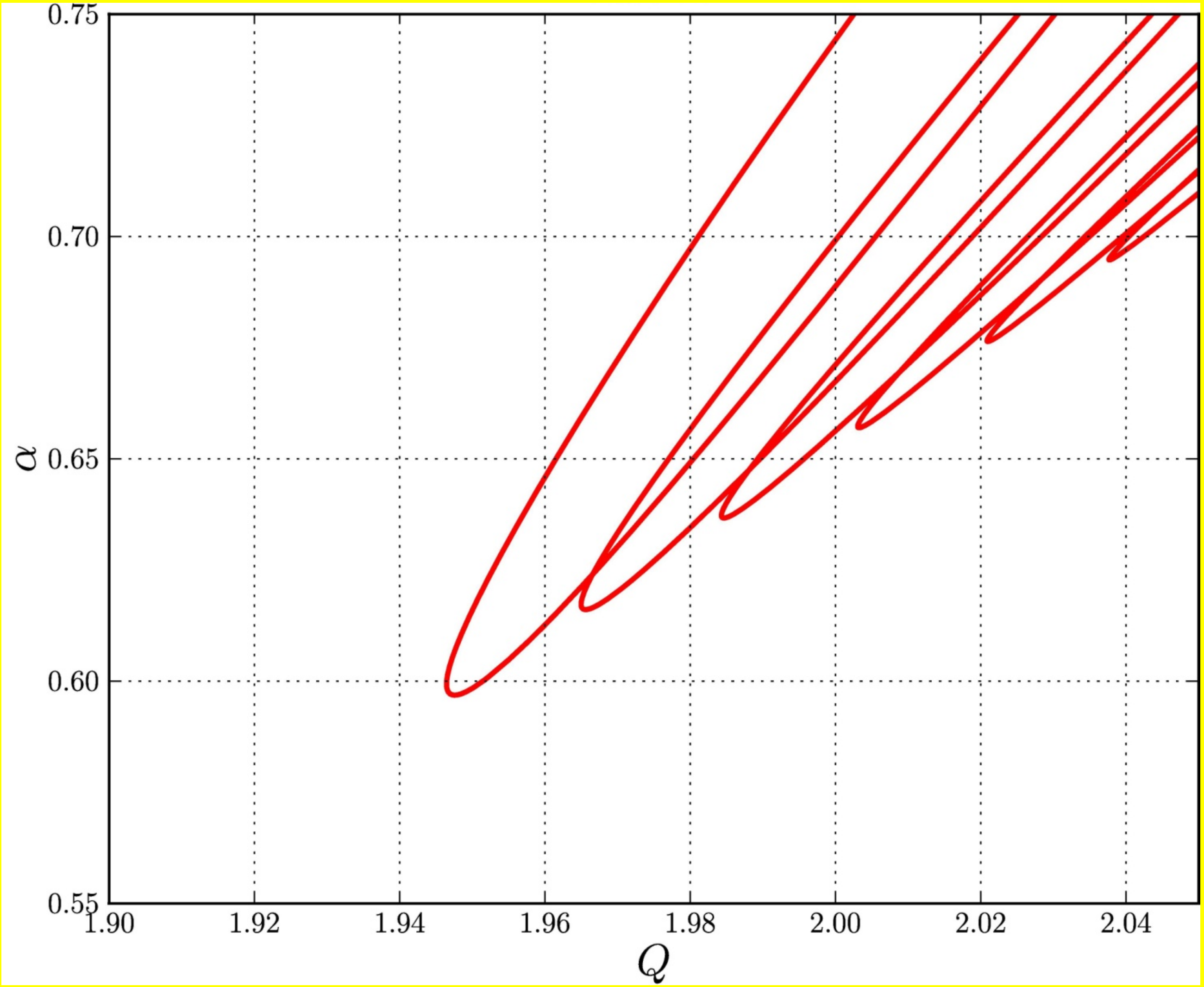
FULL MOLECULAR MODEL : 4LM for Amplifier (CO2) and Absorber (SF6)



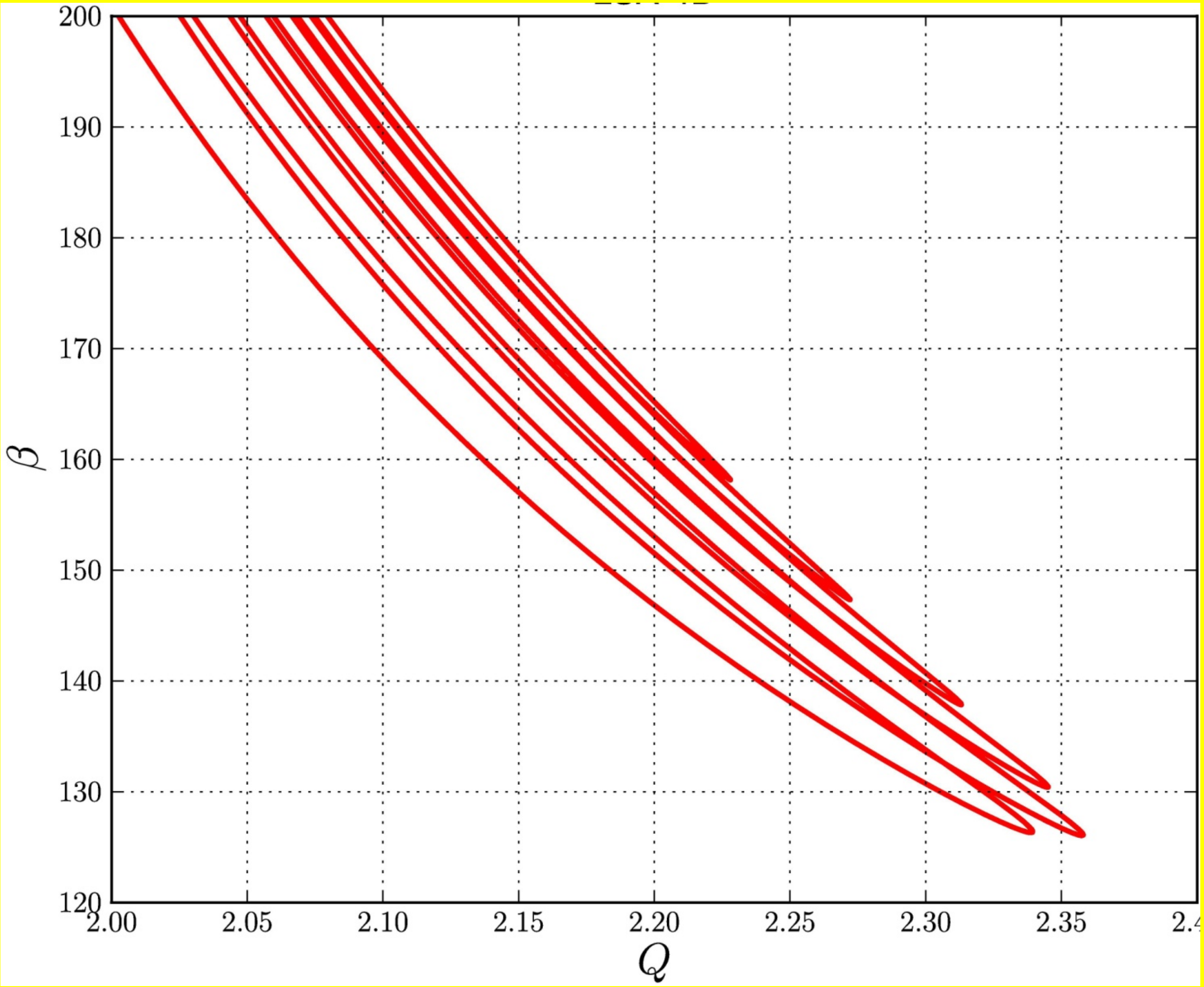
I. Burak et al. , IEEE v. 7 p.73 (1971).
J. Dupré et al., Rev. Phys. Appl. v. 10, p. 285 (1975).
E. Arimondo et al. Applied Physics B v.30 p.57 (1983)



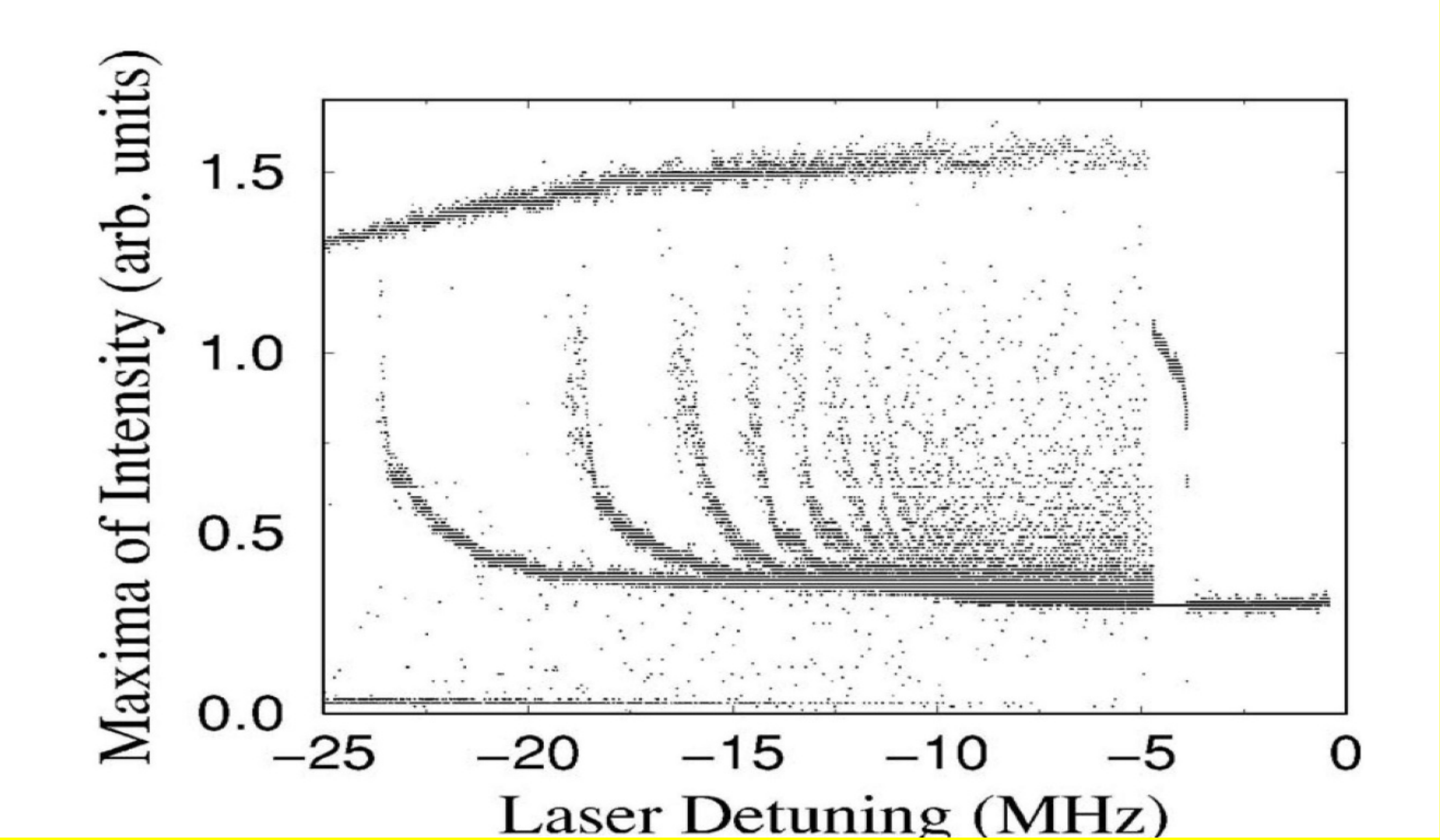
Branches of main periodic orbits, PD period orbits and ISOLAS of periodic orbits with N=3, 4, 5, 6, 7, 8 maxima, Π(N).



Intervals of stability vs. α for the
periodic orbits with N=3 ,4,5,6,7,8 maxima, π(N).

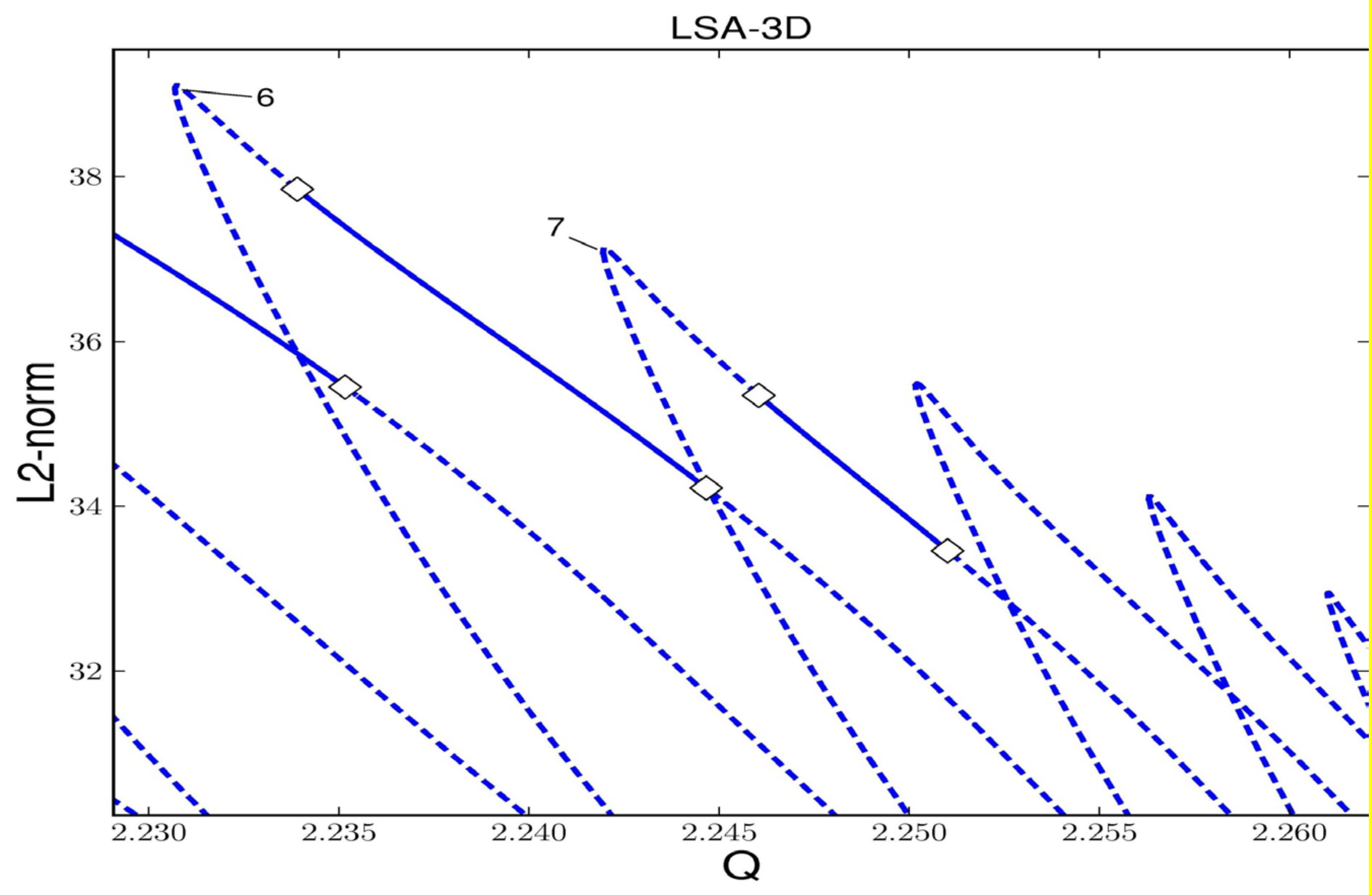


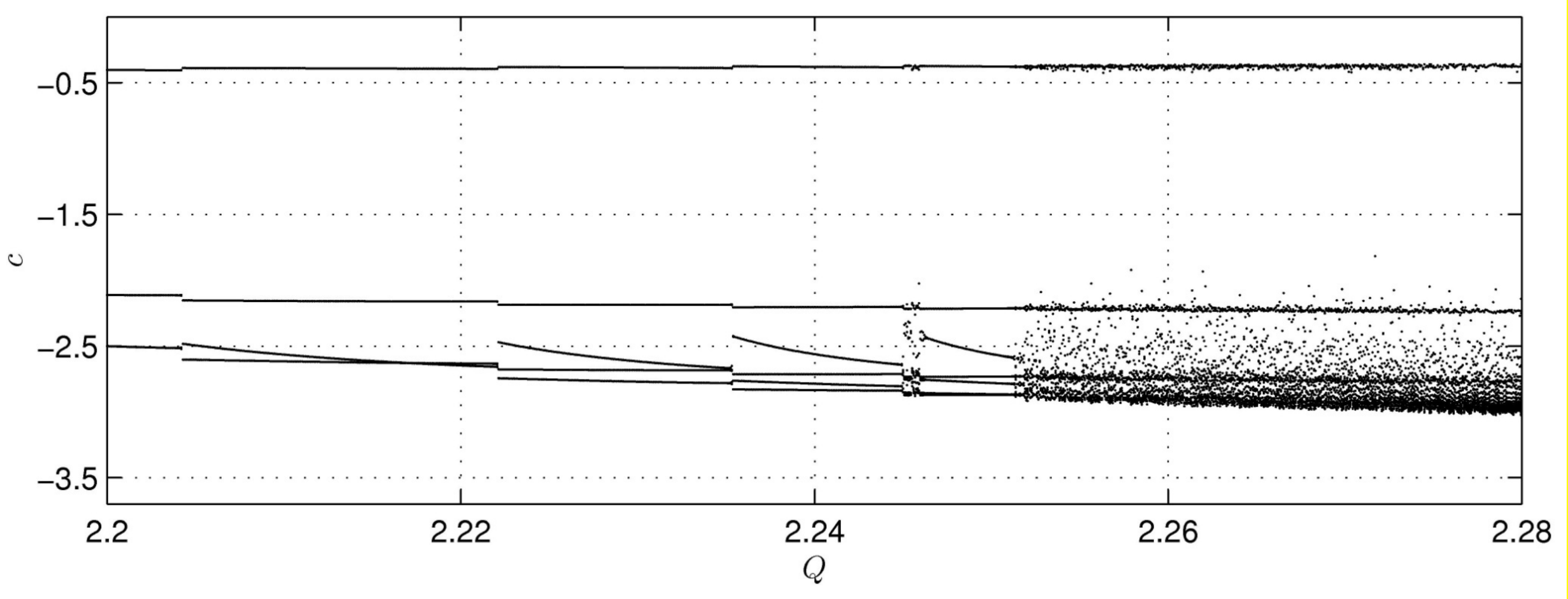
Intervals of stability vs. β for the
periodic orbits with N=3,4,5,6,7,8 maxima, π(N).



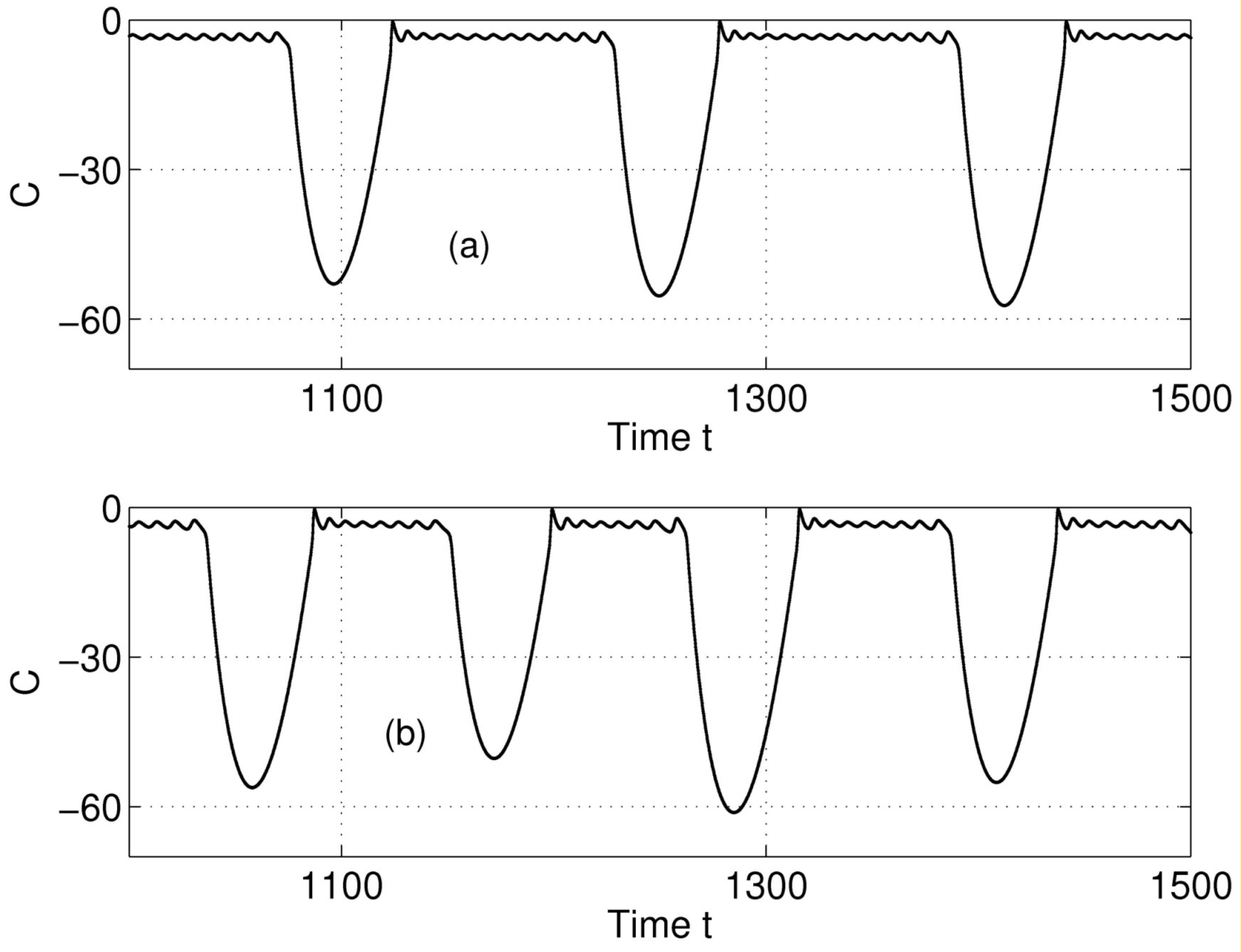
H.L.D. de S. Cavalcante
And J.R. Rios Leite, Chaos
v. 18 , 023107 (2008)

4. LSA Chaotic Dynamics



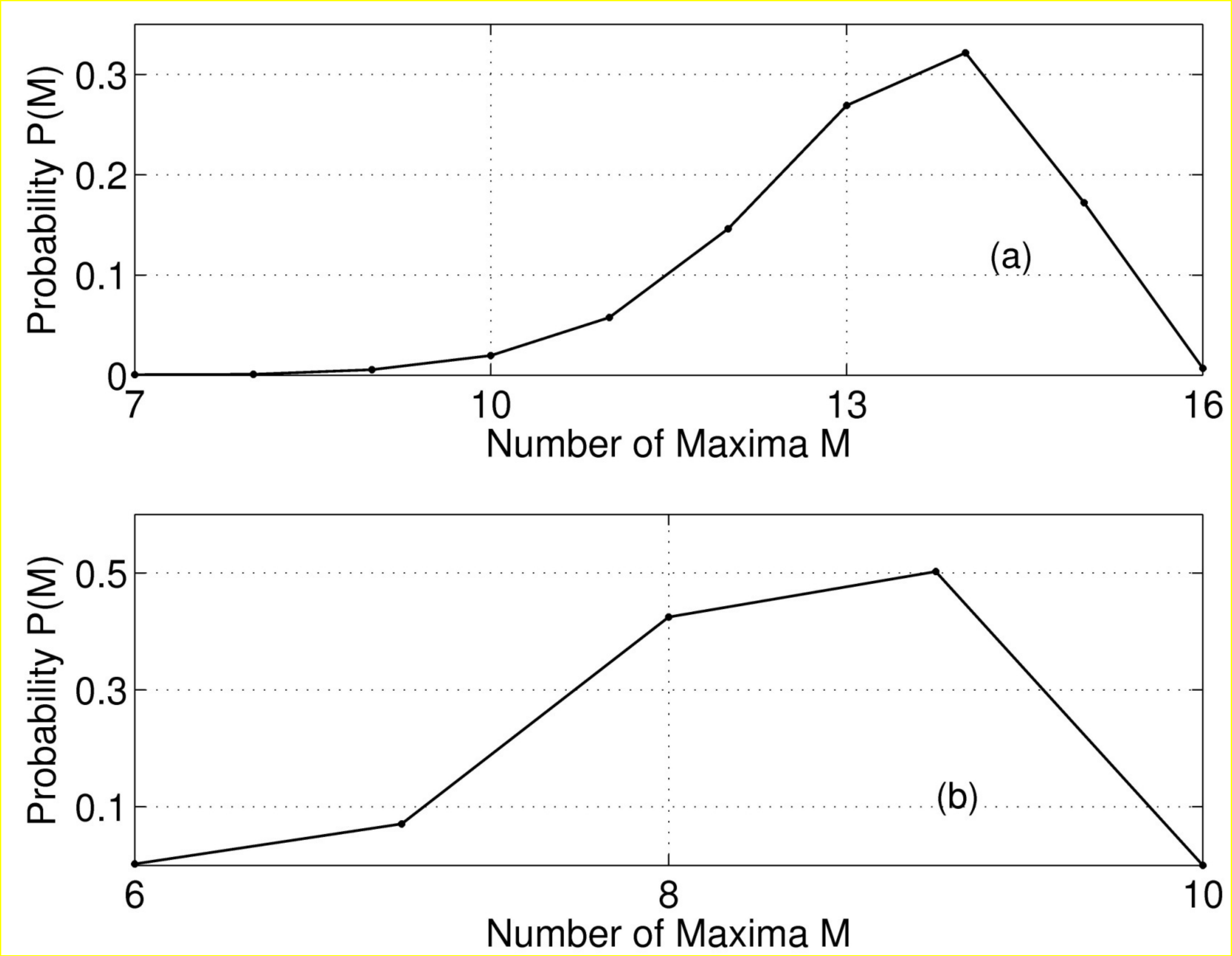


Which is The Mechanisn of the Chaotic Motion ?



Q=2.275

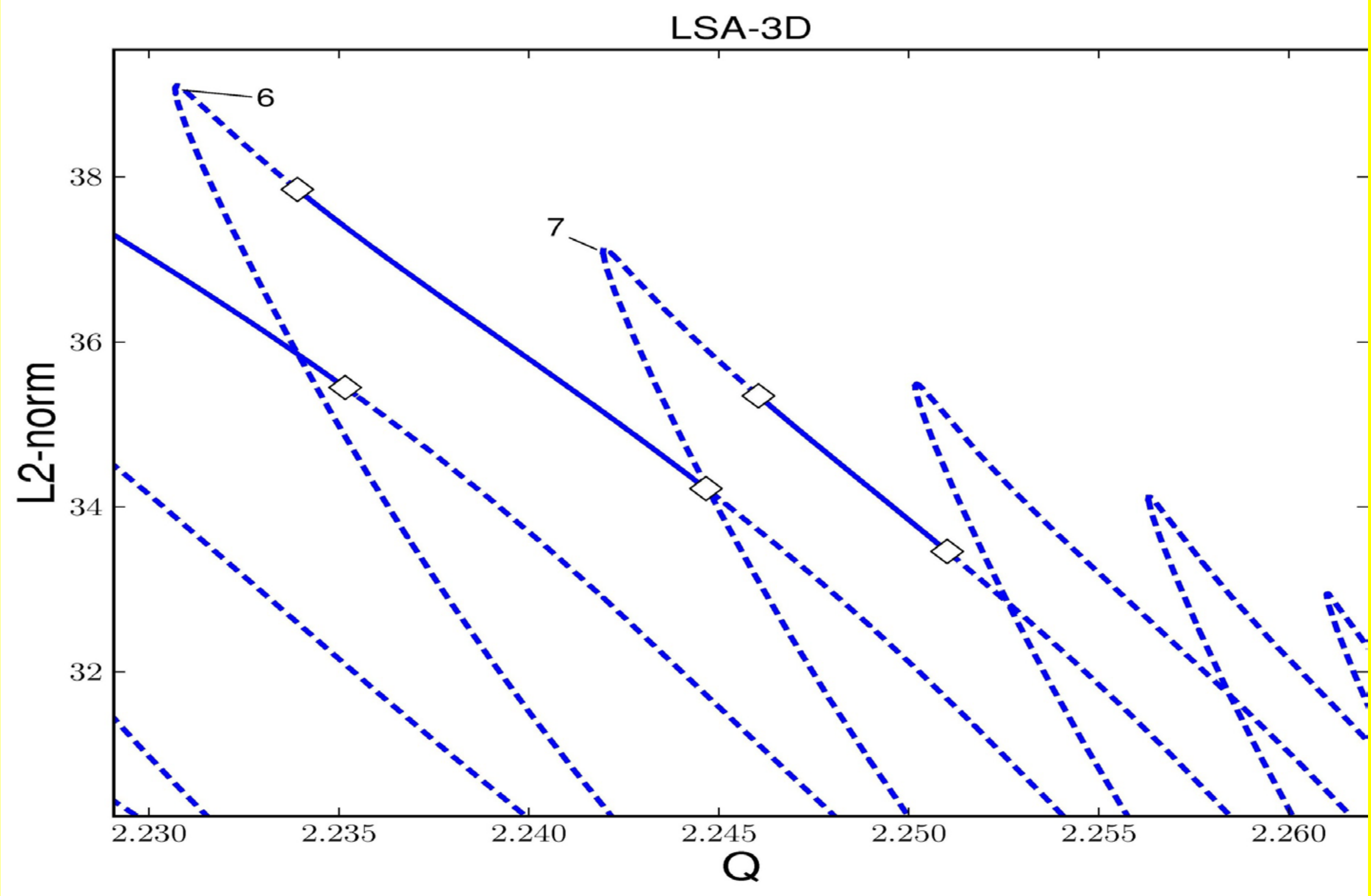
Q=2.26



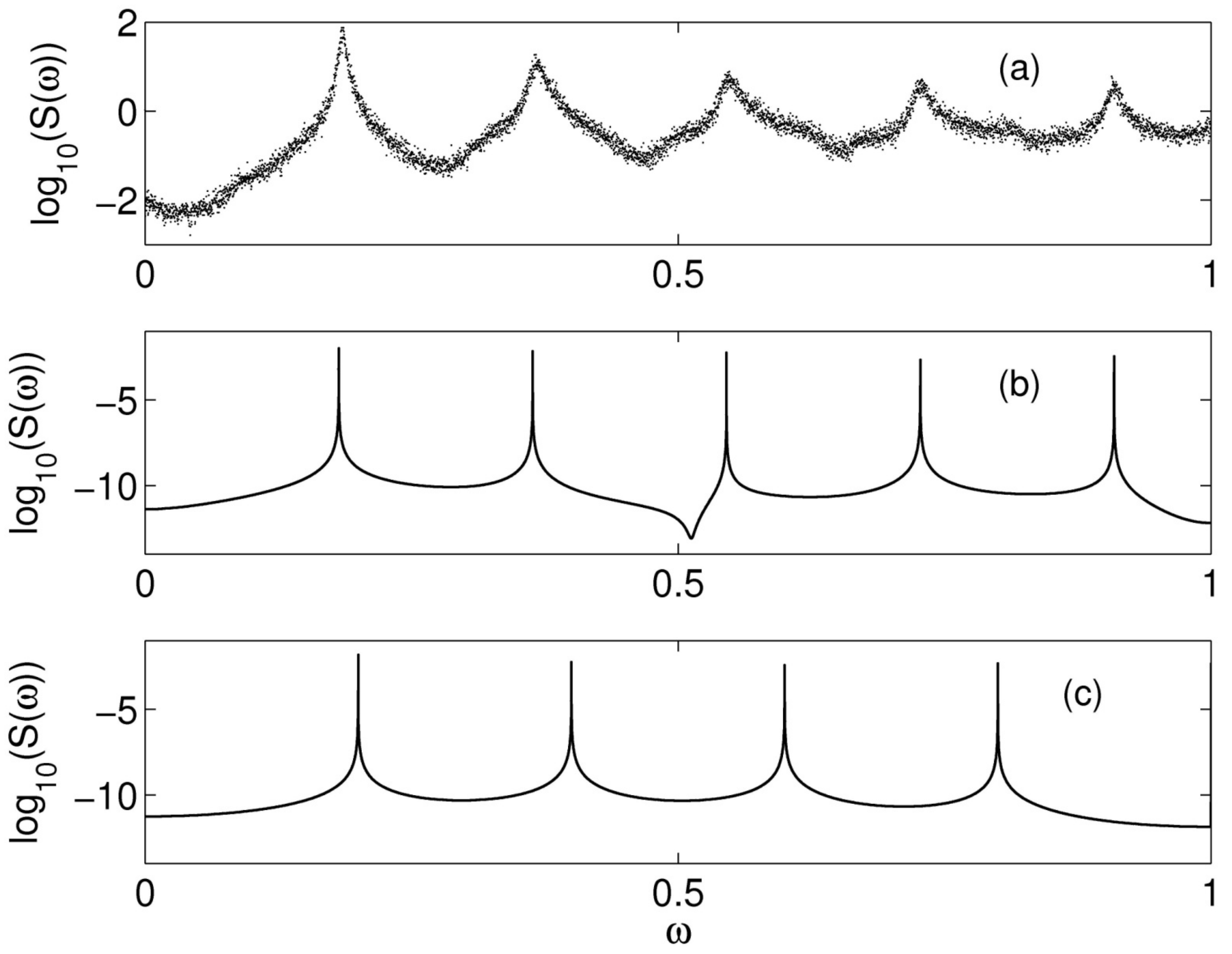
Probability P(M)
0.3
0.2
0.1
0
7
10
13
16
Number of Maxima M
(a)
Probability P(M)
0.5
0.3
0.1
6
8
10
Number of Maxima M
(b)

Q=2.275

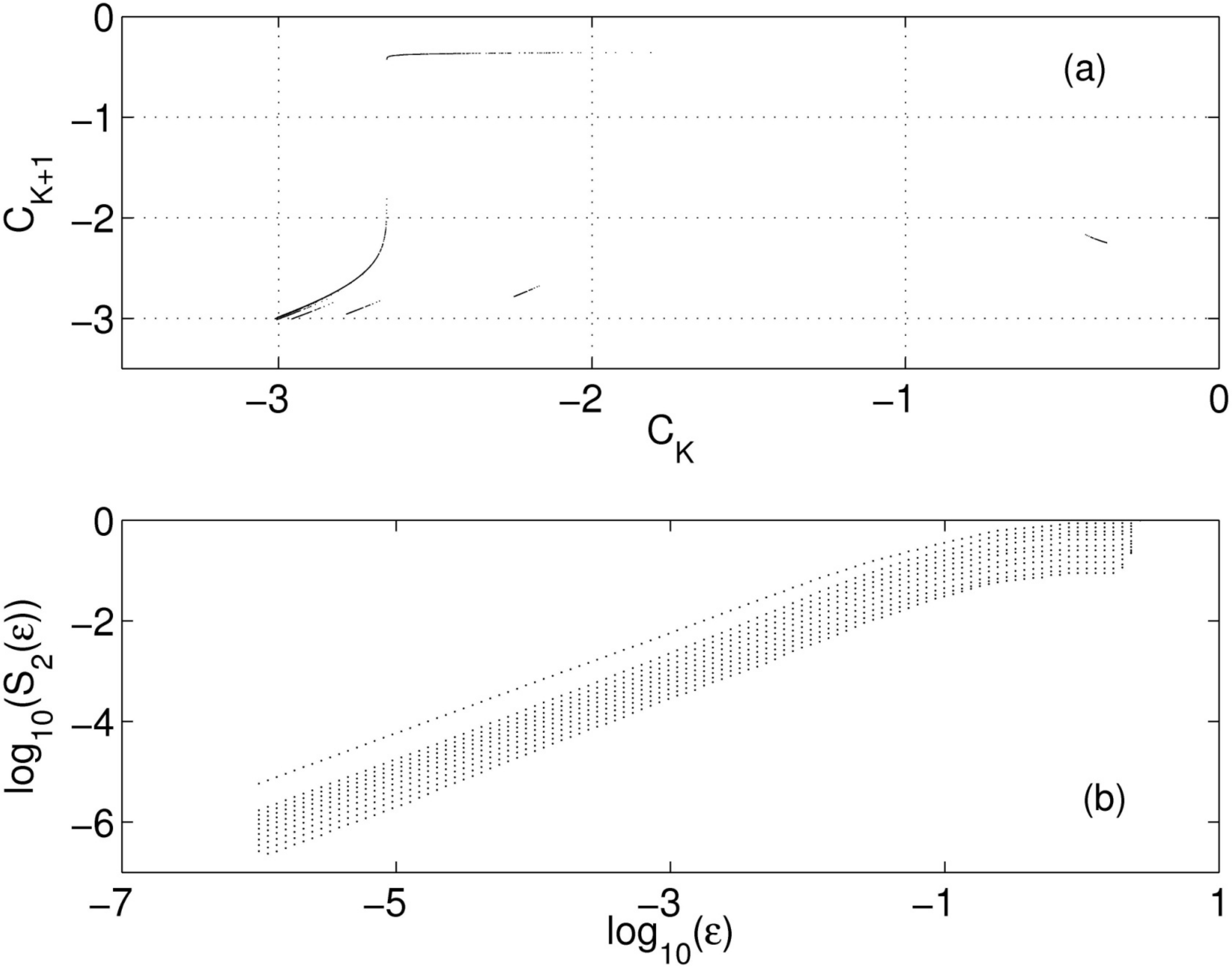
Q=2.26



LSA-3D
6
7
38
36
34
32
L2-norm
2.230
2.235
2.240
2.245
2.250
2.255
2.260
Q



log10(S(ω))
2
0
−2
0
0.5
1
(a)
log10(S(ω))
−5
−10
0
0.5
1
(b)
log10(S(ω))
−5
−10
0
0.5
1
(c)
ω



(a)
C_{K+1}
C_K
0
−1
−2
−3
−3
−2
−1
0
(b)
$\log_{10}(S_2(\varepsilon))$
$\log_{10}(\varepsilon)$
0
−2
−4
−6
−7
−5
−3
−1
1

5. Conclusions

1. All the models are able to display the PQS Pulsations (spikes), Π(N).
Moreover, in all models,
the Π(N) are organized
along Isolas of periodic orbits.

5. Conclusions

with a gaseous saturable absorber in its cavity [2–6]. The two-level model [7] and the four-level model [8,9] for the laser system were not successful in reproducing chaotic PQS, while the three-level–two-level model (the 3-2 model) proposed by Tachikawa *et al*. [10] has given successful interpretations of the observed behaviors of the unstable laser oscillation. In the 3-2 model, the vibrational relaxa-

T. Tohei et al, PRA v. 45 p. 5166 (1992)

Nevertheless, there remained a qualitative discrepancy, since this four-level model is unable to predict the $P^{(n)}$ regimes observed experimentally. A definite step

M. Lefranc et al. , JOSA B v. 8 p.239 (1991).

5. Conclusions

2. The numerical continuation of a
a few periodic orbits explains
to a good extent the phase
diagram (simulations):
the period-adding cascades
and the windows, the observables.

5. Conclusions

3. The subtle differences among the models – such as the number of windows- can help to determine consistently the validity of the models in a given experiment.

C.L. Pando L., PLA v.210 p.391 (1996).

E.J. Doedel, B. Oldeman, C.L. Pando L.,
IJBC v. 21 p.305 (2011),

E.J. Doedel, C.L. Pando L.,
Accepted in PRE, 2011

¡Thank You for your attention!