" Bifurcations and Chaos near relative equilibria in a ring of BEC ".

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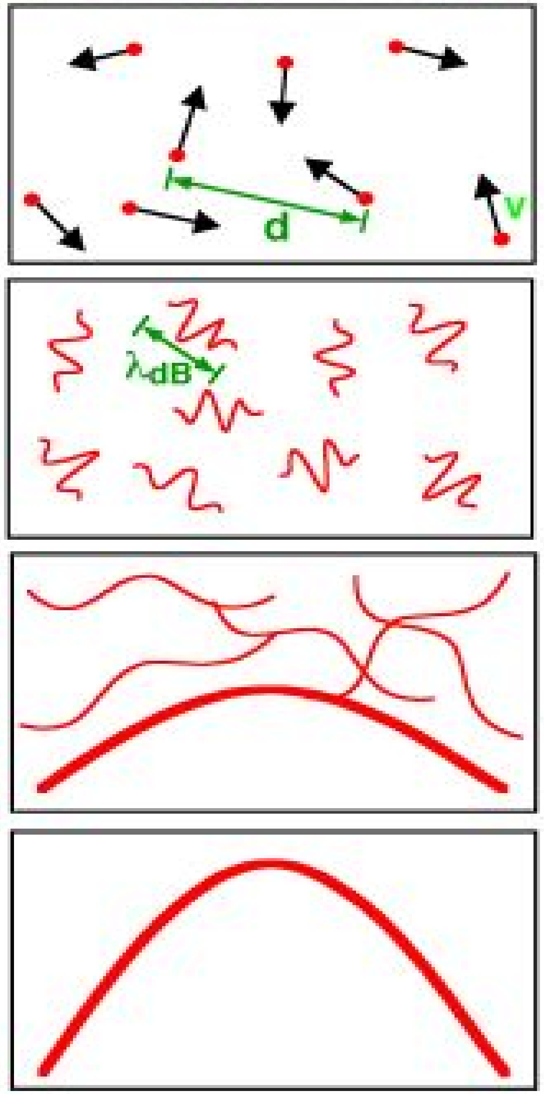
We investigate the bifurcations of a family of relative equilibria and chaos in a ring of seven Bose-Einstein Condensates (BEC). This system is studied using the discrete nonlinear Schrodinger equation (DNLSE) when both the chemical potential ($\sim 1/T_0$) and a local external defect ($= \delta$) act as bifurcation parameters.

What is Bose-Einstein condensation (BEC)?



High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

"Wave packets"

$T=T_{crit}$: Bose-Einstein Condensation

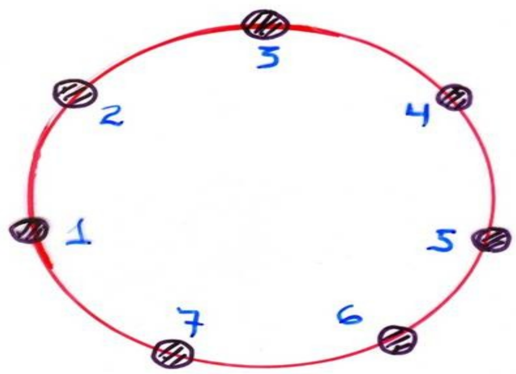
$\lambda_{dB} \approx d$

"Matter wave overlap"

T=0: Pure Bose condensate

"Giant matter wave"

We are considering a ring of seven coupled oscillators.



We modify the chemical potential ($\approx 1/T_0$) or a defect placed at site 3.

The DNLSE equation is given by

$$i\frac{\mathrm{d}\psi_m}{\mathrm{d}\tau} + \delta_m \psi_m + (\psi_{m-1} + \psi_{m+1} - 2\psi_m) + 2|\psi_m|^2 \psi_m = 0,$$

The Hamiltonian is given by

$$H = \sum_{m=1}^{M} (|\psi_m - \psi_{m+1}|^2 - |\psi_m|^4 - \delta_m |\psi_m|^2).$$

The second constant is the norm, which is given by

$$P = \sum_{m=1}^{M} |\psi_m|^2.$$

$$\psi_m = \sqrt{N_m}\exp(-i\theta_m)$$

$$\begin{aligned}\frac{\mathrm{d}N_m}{\mathrm{d}\tau} &= 2\sqrt{N_m N_{m-1}}\sin(\theta_{m-1}-\theta_m)\\ &\quad + 2\sqrt{N_m N_{m+1}}\sin(\theta_{m+1}-\theta_m),\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\theta_m}{\mathrm{d}\tau} &= 2-\delta_m-\sqrt{\frac{N_{m-1}}{N_m}}\cos(\theta_{m-1}-\theta_m)\\ &\quad -\sqrt{\frac{N_{m+1}}{N_m}}\cos(\theta_{m+1}-\theta_m)-2N_m.\end{aligned}$$

N_m : Number of Atoms at Site m ; θ_m: Phase at site m

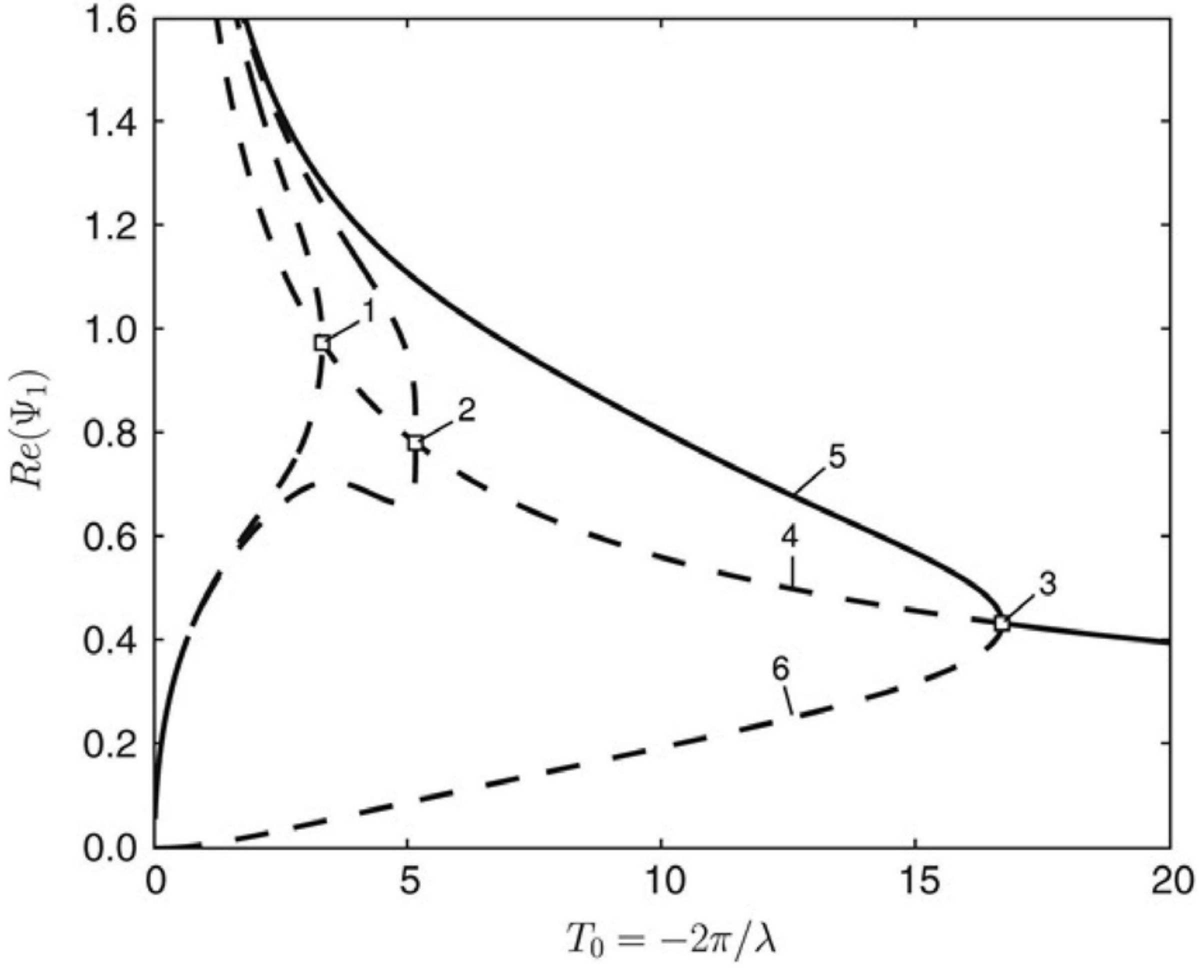
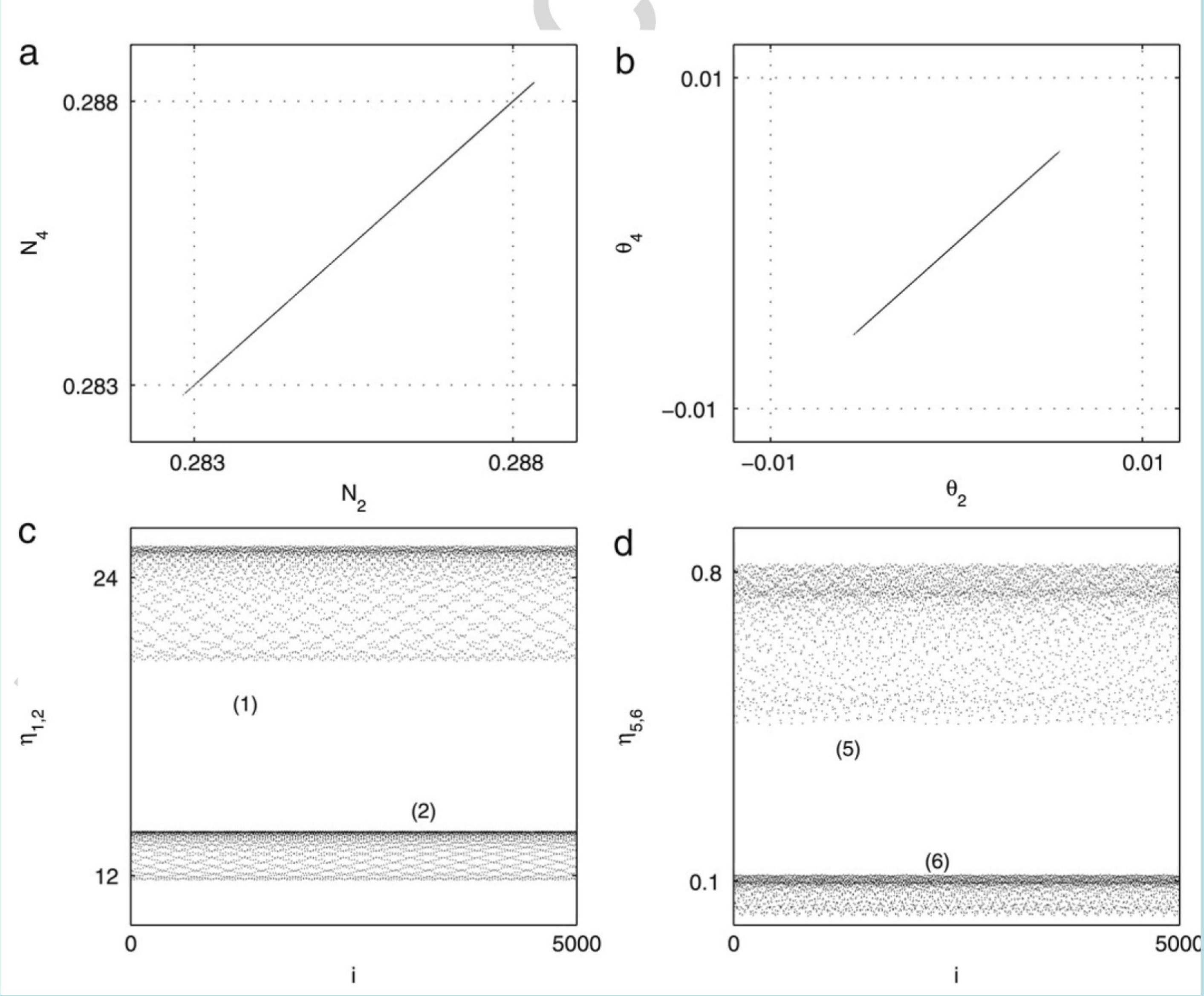


Fig. 1. Plot of $r_1 = \sqrt{N_1} = Re(\psi_1)$ versus the period of the relative equilibrium $T_0 = \frac{-2\pi}{\lambda}$. Here $\delta = 0$. Solid curves denote spectrally stable solutions; dashed curves denote unstable solutions. $Re(\psi_1)$ stands for the real part of ψ_1.

For point “5”, the conditions of the KAM theorem hold : no-resonance relations and nondegeneracy condition.

Averaging Principle : Symmetric Atomic Populations



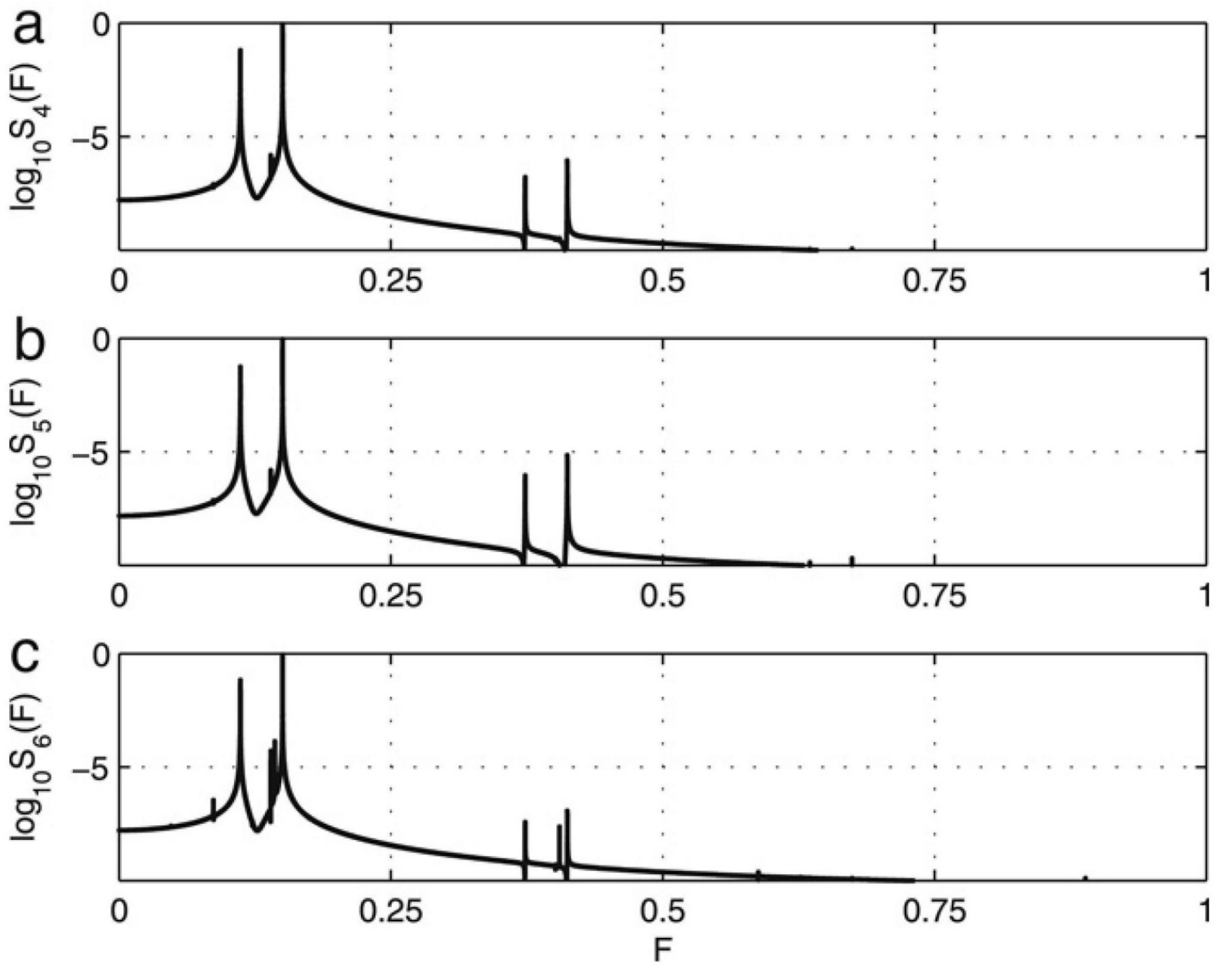


Fig. 9. (a) Power spectral density (PSD), $S_4(F)$, versus frequency F for the continuous time sampling of the slow action $I_4 = \frac{N_2 - N_4}{2}$. (b) The same as (a), but for $I_5 = \frac{N_1 - N_5}{2}$. (c) The same as (a), but for $I_6 = \frac{N_7 - N_6}{2}$. The parameters are the same as those of the previous figures.

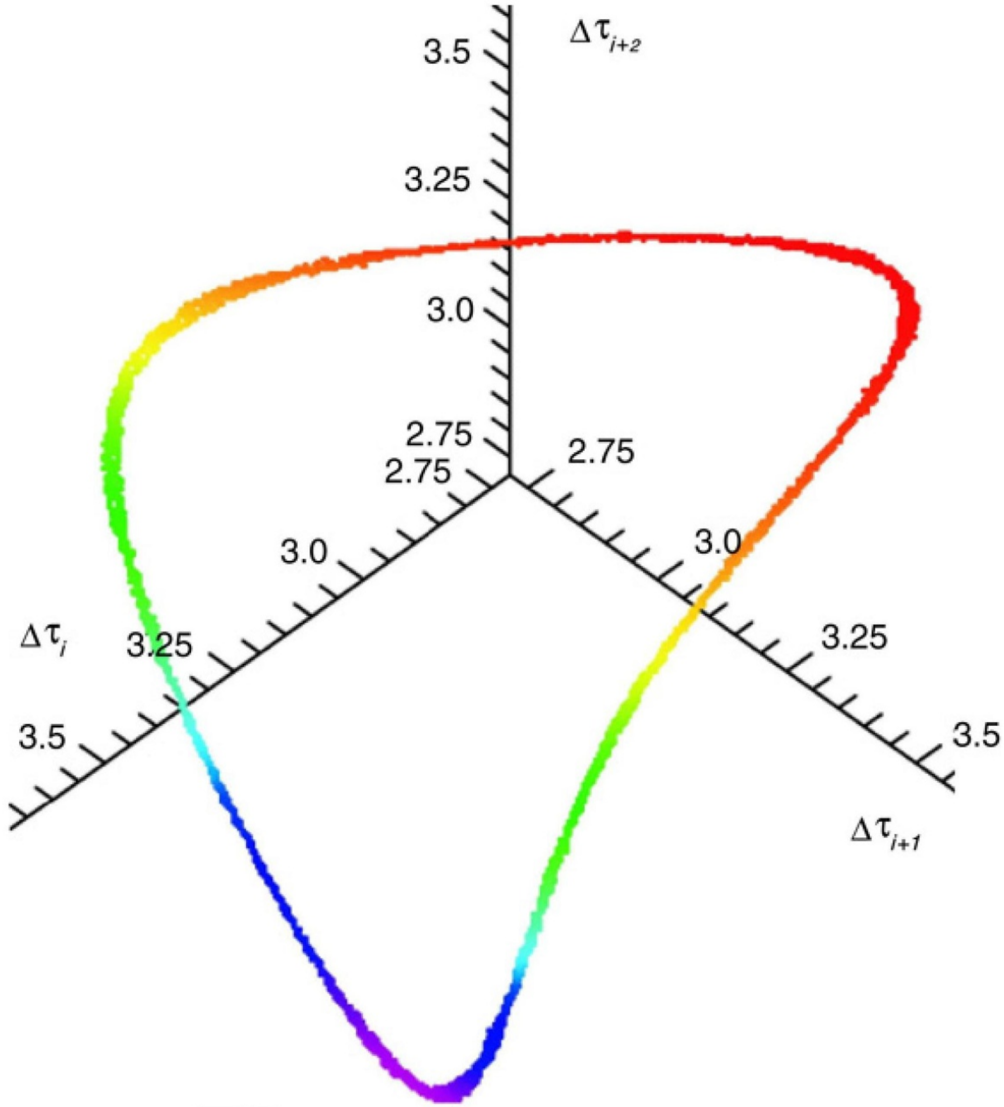


Fig. 11. Three-dimensional embedding of the return times ($\tau_{i+1} - \tau_i$, $\tau_{i+2} - \tau_{i+1}$, $\tau_{i+3} - \tau_{i+2}$). The data come from the simulations carried out for Fig. 4.

• *Conclusions*

- ***A neighbourhood of stable Breather-like relative equilibria shows KAM Quasiperiodic or Chaotic dynamics (convex). The time series of the recurrence times is useful in the study of the dynamics of this system.***

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