Modos Dominantes, Difusión de Arnold y Sincronización Caótica Simbólica en la Vecindad de Equilibrios Relativos en la Ecuación Discreta No Lineal de Schroedinger (DNLSE) y sus extensiones.

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Esquema de la presentación.

(1) The DNLSE, extensions and physical examples.

(2) A family of Relative Equilibria, stability.

(4) The Poincaré section: diffusion of slow actions.

(5) Quasiperiodic dynamics.

(6) Chaotic dynamics (Arnold diffusion, diffusion of slow actions).

The 1-D Discrete Nonlinear Schroedinger Equation (DNLSE)

$$i\frac{\partial \Psi_m}{\partial t} + \Delta_m \Psi_m + K(\Psi_{m-1} + \Psi_{m+1}) + \rho|\Psi_m|^2 \Psi_m = 0,$$

ψ_m : Wave function or Electric Field

$$i\frac{\partial \psi_m}{\partial \tau} + \delta_m \psi_m + (\psi_{m-1} + \psi_{m+1} - 2\psi_m) + 2|\psi_m|^2 \psi_m = 0,$$

$$H = \Sigma_{m=1}^{M} (\mid \psi_m - \psi_{m+1} \mid^2 - \mid \psi_m \mid^4 - \delta_m \mid \psi_m \mid^2) \,.$$

Hamiltonian

$$P = \Sigma_{m=1}^{M} \mid \psi_m \mid^2 \quad .$$

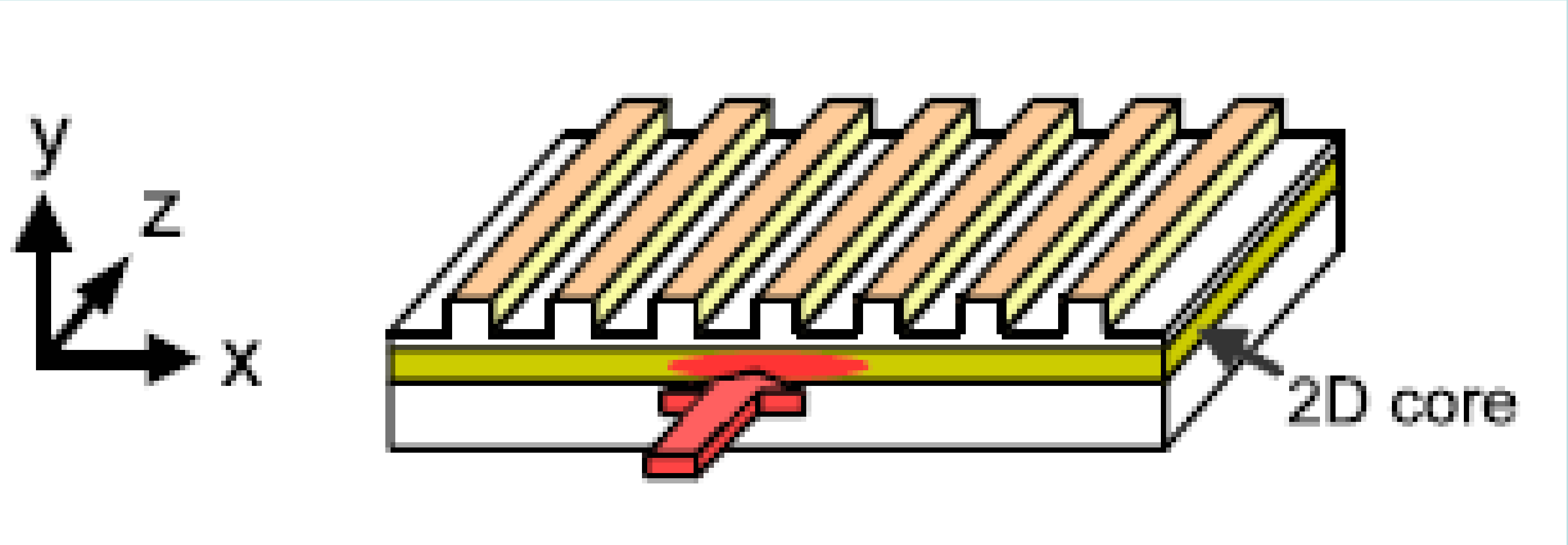
Angular Momentum

DNLSE describes many physical systems:

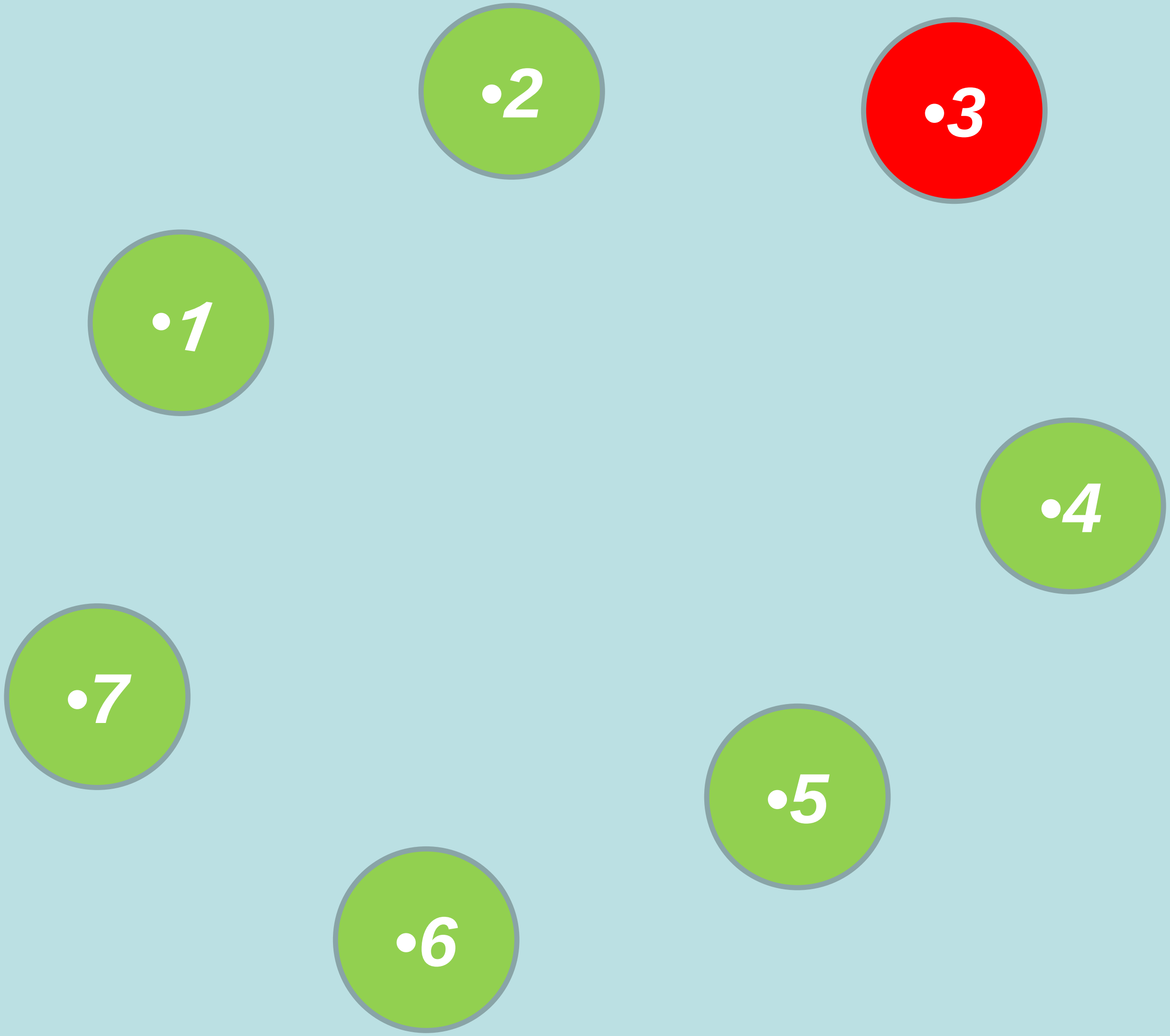
- ***Arrays of optical fibers (1-d, 2-d).***
- ***Small molecules (Benzene)***
- ***BEC trapped in a multiwell periodic potential (1-d, 2-d, 3-d)***

BREATHERS : spatially localized, time periodic (quasiperiodic)
and (very) stable solutions of DNLSE, but in infinite one-dimensional lattices.

- ***Fixed Boundary Conditions***



- ***One-dimensional waveguide lattice: Kerr Medium (cubic medium).***
- ***Geometry for a waveguide lattice: Quadratic Medium (2 waves/fiber).***



• BEC in SEVEN Deep and Coupled Wells

•Periodic Boundary Conditions (Ring)

•Attractive BEC

•C.L. Pando L, EJ. Doedel, PRE v. 71 056201 (2005)

Populations at Each Site M (1,2,..7)
of the Hyperfine Sublevels “+1”,”0” & “-1”.
Each site has 3 Hyperfine Sublevels.

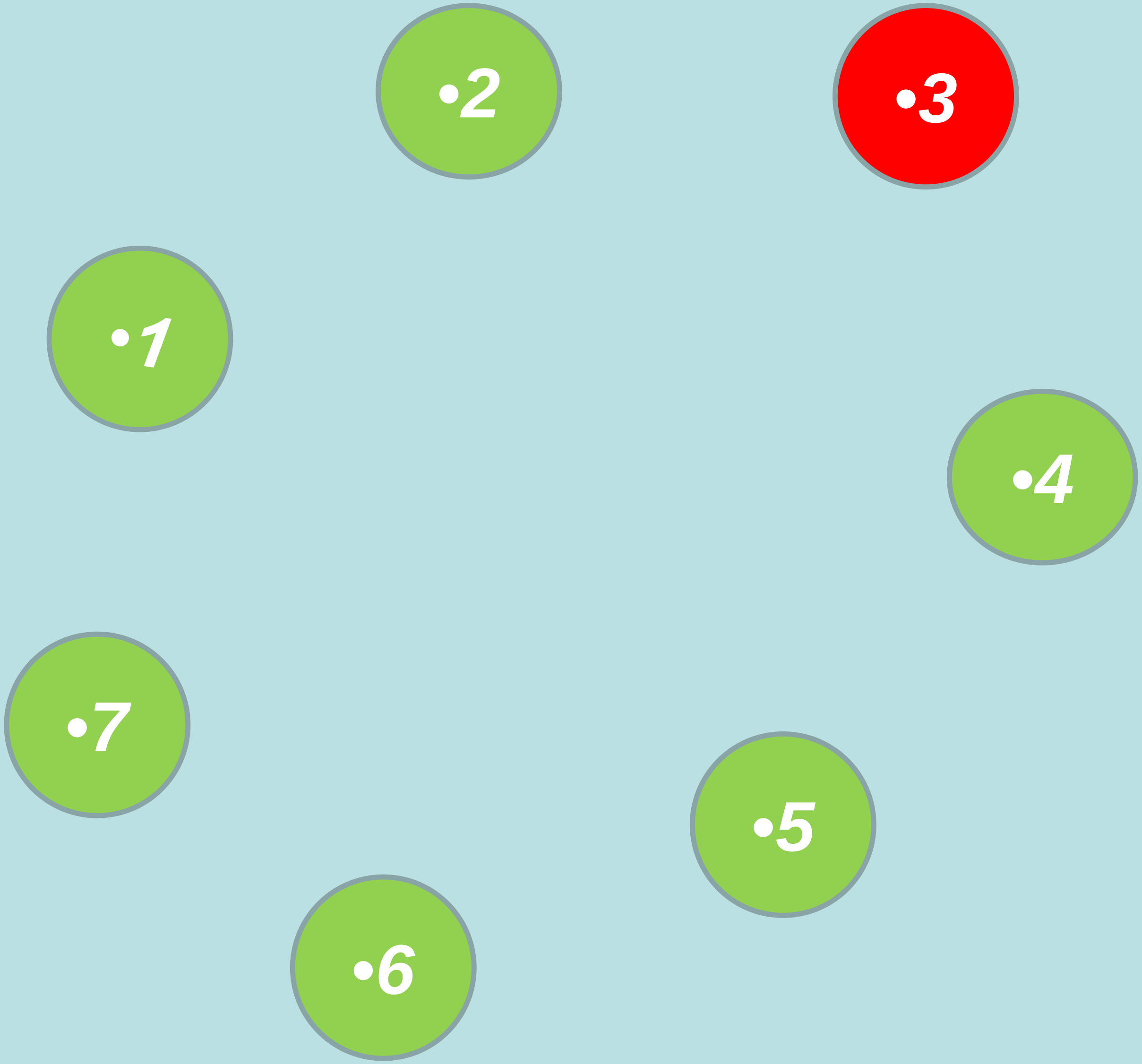
$$i\frac{\partial \psi_m}{\partial \tau} + \delta_m \psi_m + (\psi_{m-1} + \psi_{m+1} - 2\psi_m) + 2|\psi_m|^2 \psi_m = 0,$$

$$H = \Sigma_{m=1}^{M}(\mid \psi_m - \psi_{m+1} \mid^2 - \mid \psi_m \mid^4 - \delta_m \mid \psi_m \mid^2) \ .$$

Hamiltonian

$$P = \Sigma_{m=1}^{M} \mid \psi_m \mid^2 \ .$$

Angular Momentum



•RING with SEVEN Oscillators

•Attractive Interaction

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$$\psi_m(\tau) = \sqrt{N_m}\exp(-i\lambda\tau + i\theta_0)$$

$$\begin{aligned}\frac{dN_m}{d\tau} &= 2\sqrt{N_m N_{m-1}}\sin(\theta_{m-1} - \theta_m) \\ &\quad + 2\sqrt{N_m N_{m+1}}\sin(\theta_{m+1} - \theta_m),\end{aligned}$$

$$\begin{aligned}\frac{d\theta_m}{d\tau} &= 2 - \delta_m - \sqrt{\frac{N_{m-1}}{N_m}}\cos(\theta_{m-1} - \theta_m) \\ &\quad - \sqrt{\frac{N_{m+1}}{N_m}}\cos(\theta_{m+1} - \theta_m) - 2N_m.\end{aligned}$$

- N_m : action ; θ_m: angle

The family of relative equilibria that we study is obtained by setting $\frac{dN_m}{d\tau} = 0$, and $\theta_n = \theta_m$, for any $n \neq m$ in Eq. (4). Moreover, we can define the frequency of the resulting periodic orbit, *i.e.*, relative equilibrium, by setting $\frac{d\theta_m}{d\tau} = \lambda$, where λ is a constant.

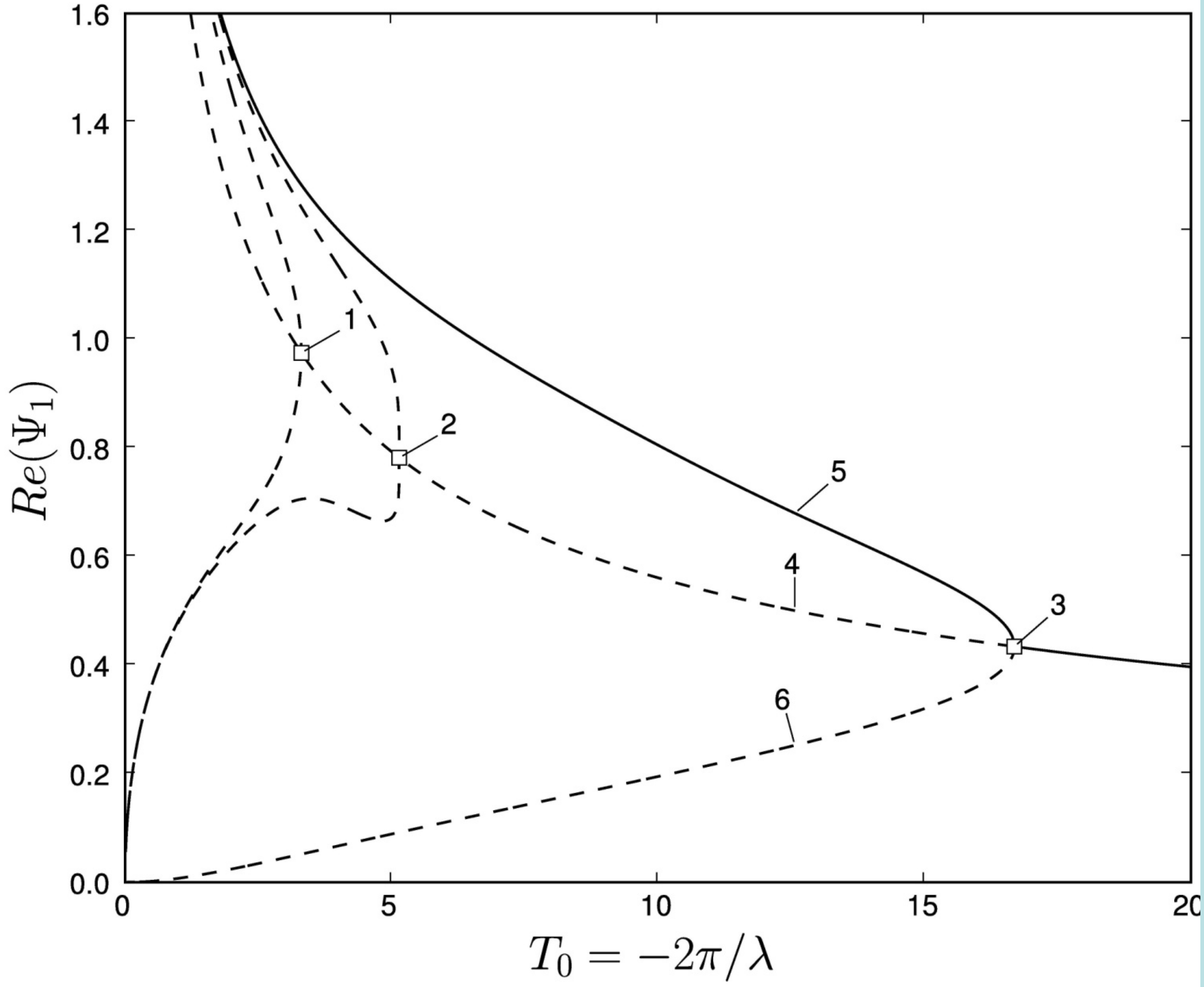


FIG. 1. Plot of $r_1 = \sqrt{N_1} = Re(\psi_1)$ versus the period of the relative equilibrium $T_0 = \frac{-2\pi}{\lambda}$. Here $\delta = 0$. Solid curves denote spectrally stable solutions; dashed curves denote unstable solutions. $Re(\psi_1)$ stands for the real part of ψ_1.

•C.L. Pando L, EJ.Doedel, Physica-D v. 238 p. 687 (2009)

- ***The Hessian of H_0 (integrable part) of the Reduced Hamiltonian H ($H= H_0 +H_1$) determines the stability of the equilibria (in the family of relative equilibria having the same phase).***

- ***A neighbourhood of stable Breather-like relative equilibria ("5") shows QP dynamics (KAM conditions fulfilled).***

•C.L. Pando L, EJ.Doedel, Physica-D v. 238 p. 687 (2009)

When the system is perturbed with a small term (that with parameter δ) , QP dynamics, and

chaotic dynamics (convex and non-convex) may occur near “5”.

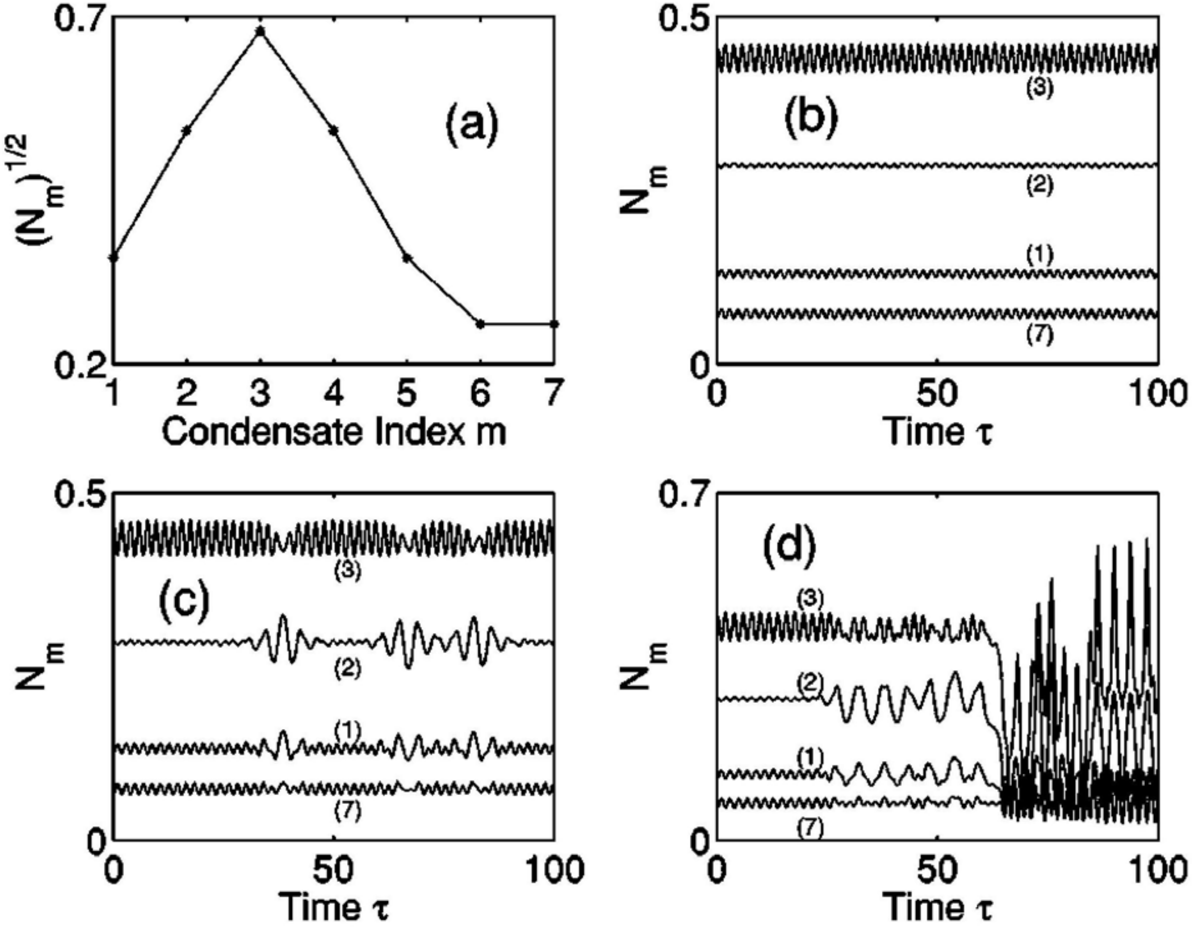


FIG. 1. (a) Plot of stationary solution amplitude $\sqrt{N_m}$ versus condensate index m where $\Gamma=2.5$. Plot of N_m versus time τ for δ_3 = (b) -0.005, (c) -0.0065, and (d) -0.007. The labels 1, 2, 3, and 7 are the condensate indices. The variable τ has been further rescaled by dividing by 4π.

•C.L. Pando L, EJ. Doedel, PRE v. 71 056201 (2005)

Poincaré Section:

$$(N_1 - N_5) + (N_2 - N_4) + (N_7 - N_6) = 0$$

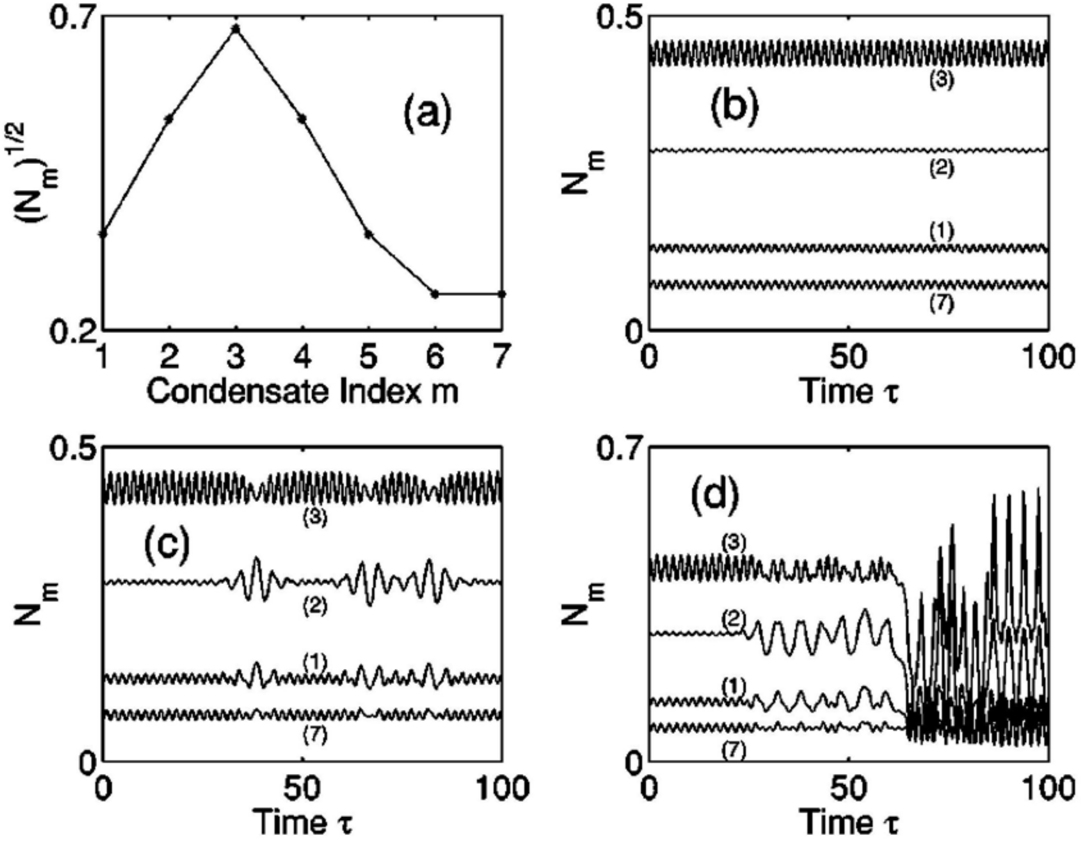
At t=0 and P.S. , $N_1 - N_5 = 0$

At t=0 and P.S. , $N_2 - N_4 = 0$

At t=0 and P.S. , $N_7 - N_6 = 0$

- $N_1 - N_5$,….*are slow actions*

• *Quasiperiodic Motion*



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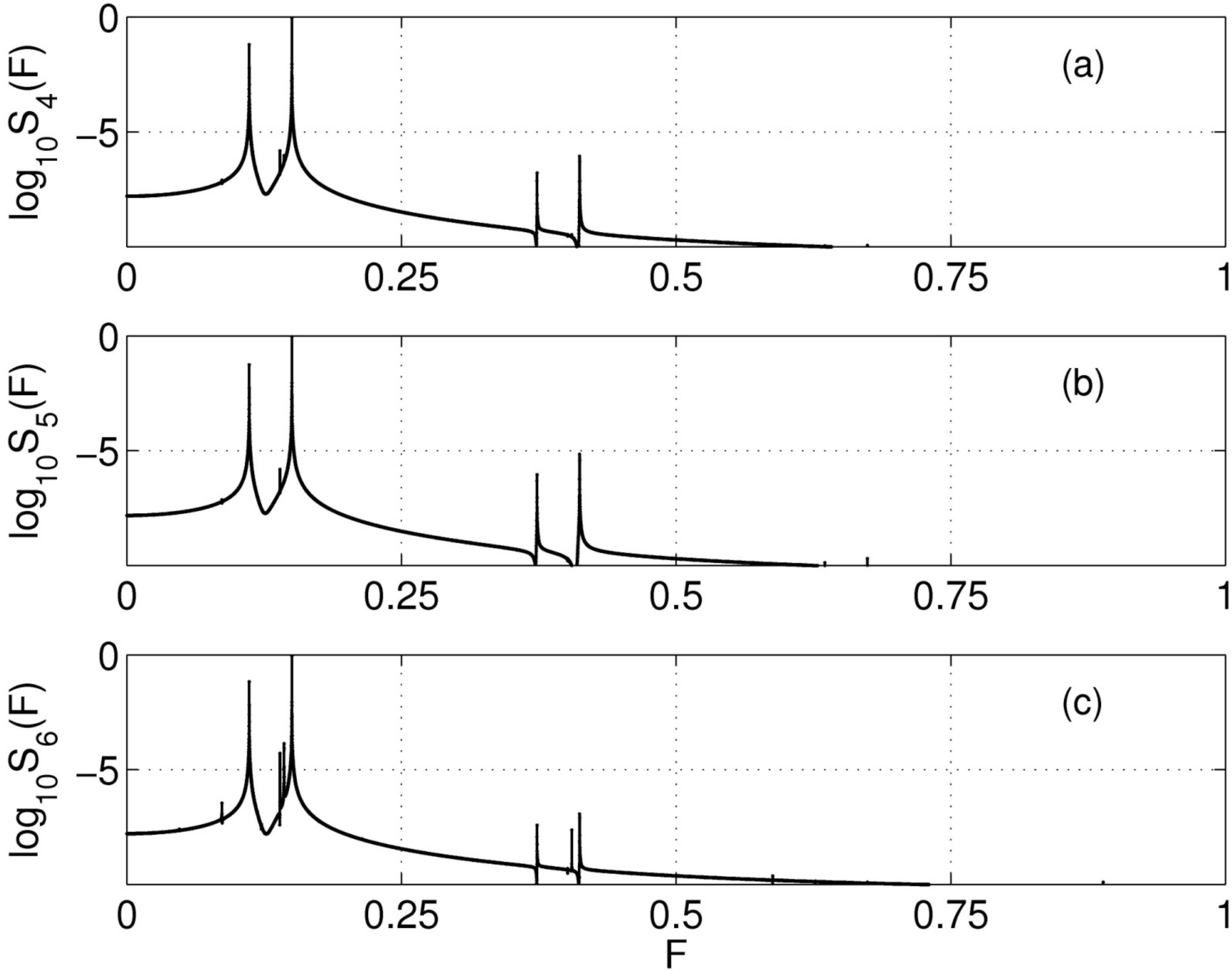


FIG. 9. (a) Power spectral density (PSD), $S_4(F)$, versus frequency F for the continuous time sampling of the slow action $I_4 = \frac{N_2 - N_4}{2}$. (b) The same as (a), but for $I_5 = \frac{N_1 - N_5}{2}$. (c) The same as (a), but for $I_6 = \frac{N_7 - N_6}{2}$. The parameters are the same as those of the previous figures.

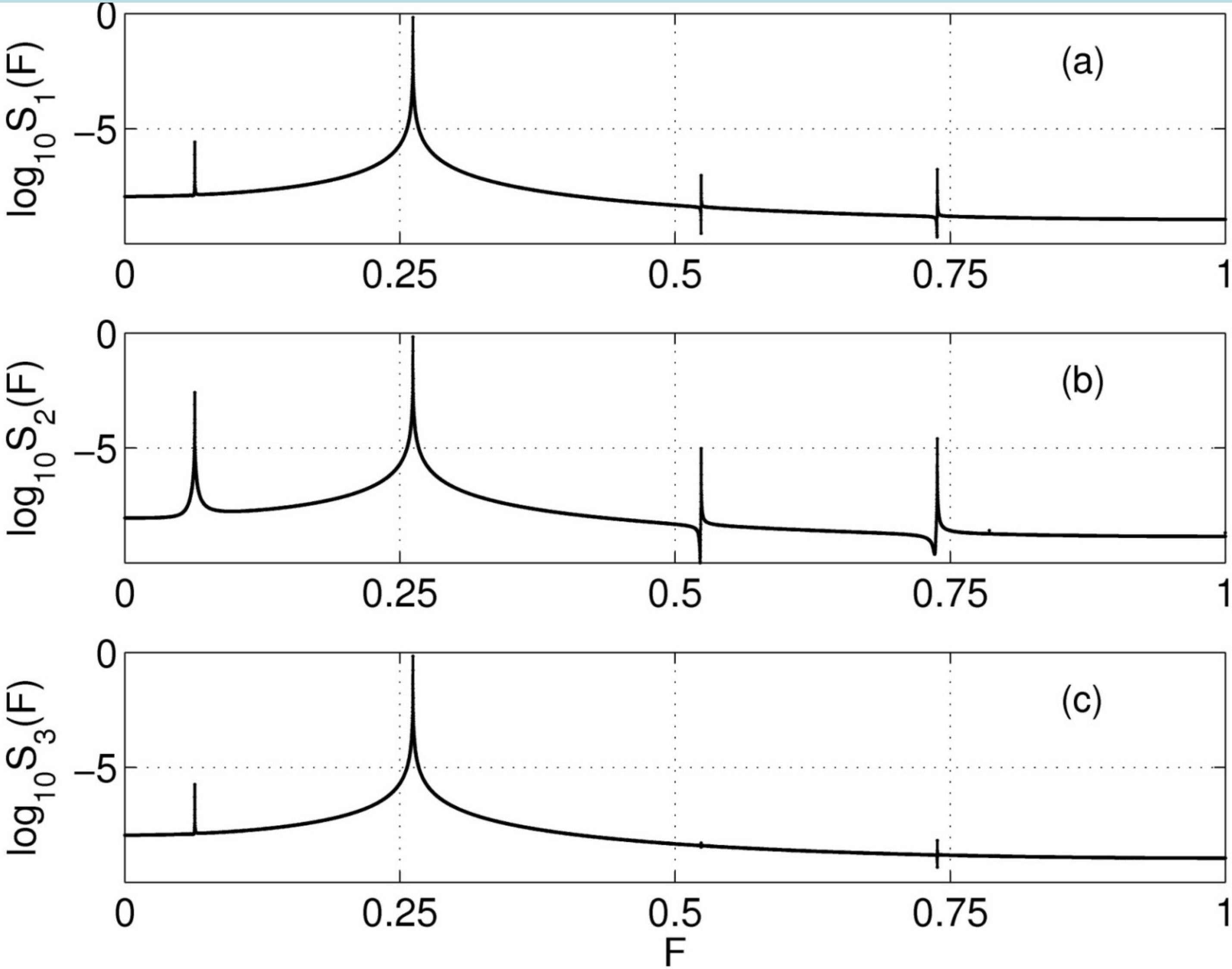
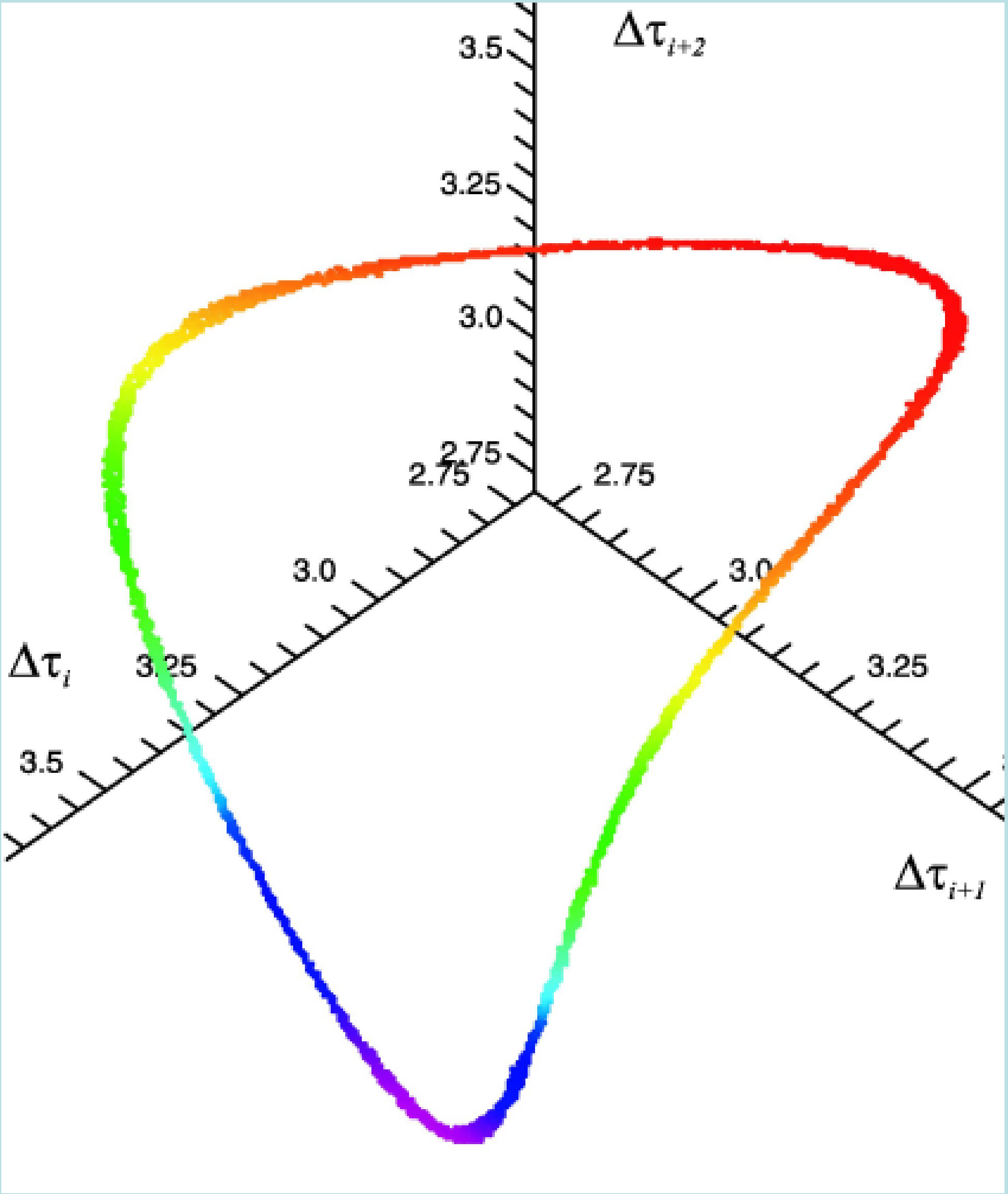


FIG. 10. (a) Power spectral density (PSD), $S_1(F)$, versus frequency F for the continuous time sampling of the fast action $I_1 = \frac{N_2+N_4}{2}$. (b) The same as (a), but for $I_2 = \frac{N_1+N_5}{2}$. (c) The same as (a), but for $I_3 = \frac{N_7+N_6}{2}$. The parameters are the same as those of the previous figures.

•C.L. Pando L, EJ.Doedel, Physica-D v. 238 p. 687 (2009)



- ***Delayed Coordinates using the Recurrence Times to the Poincaré section.***

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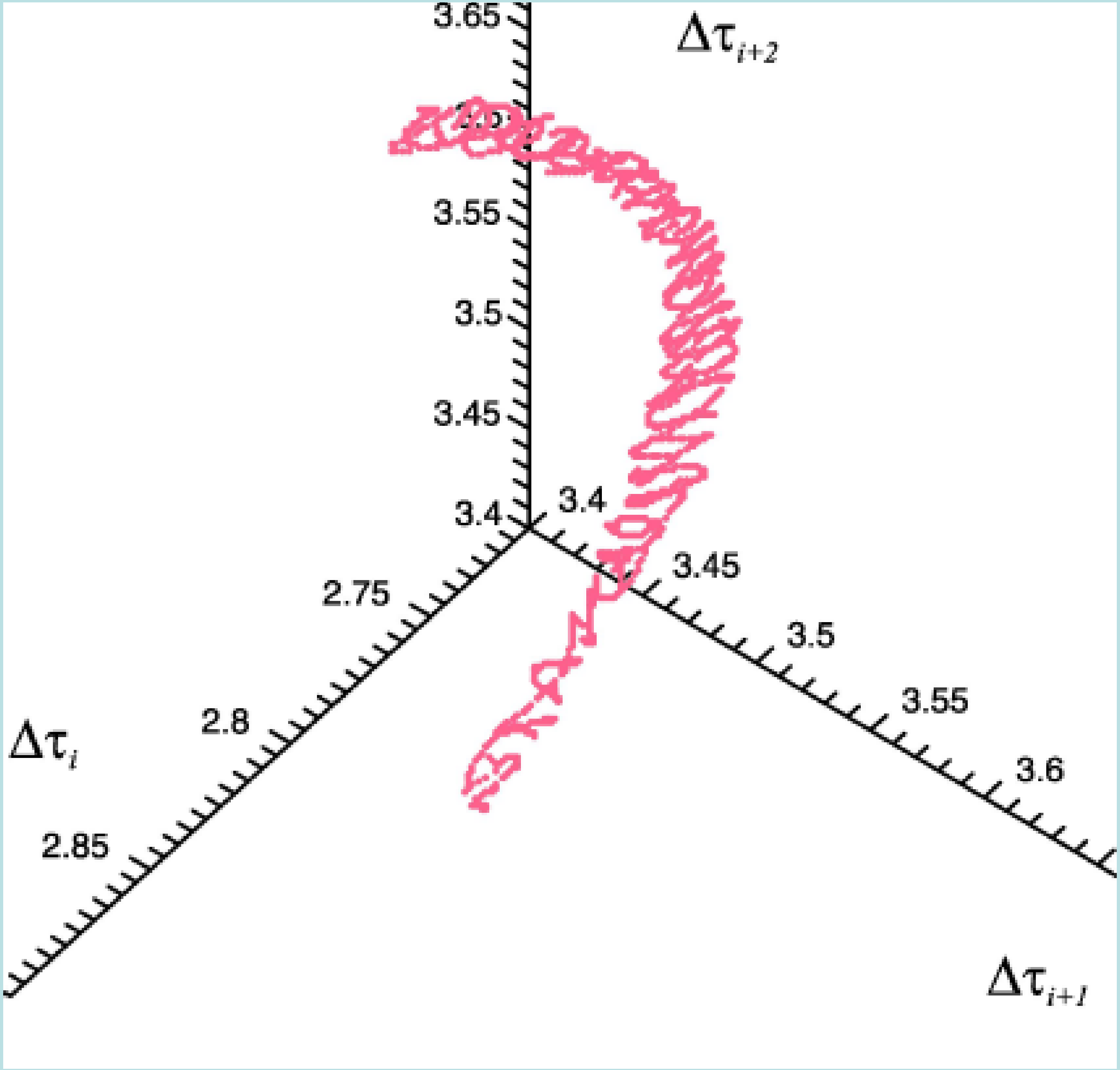
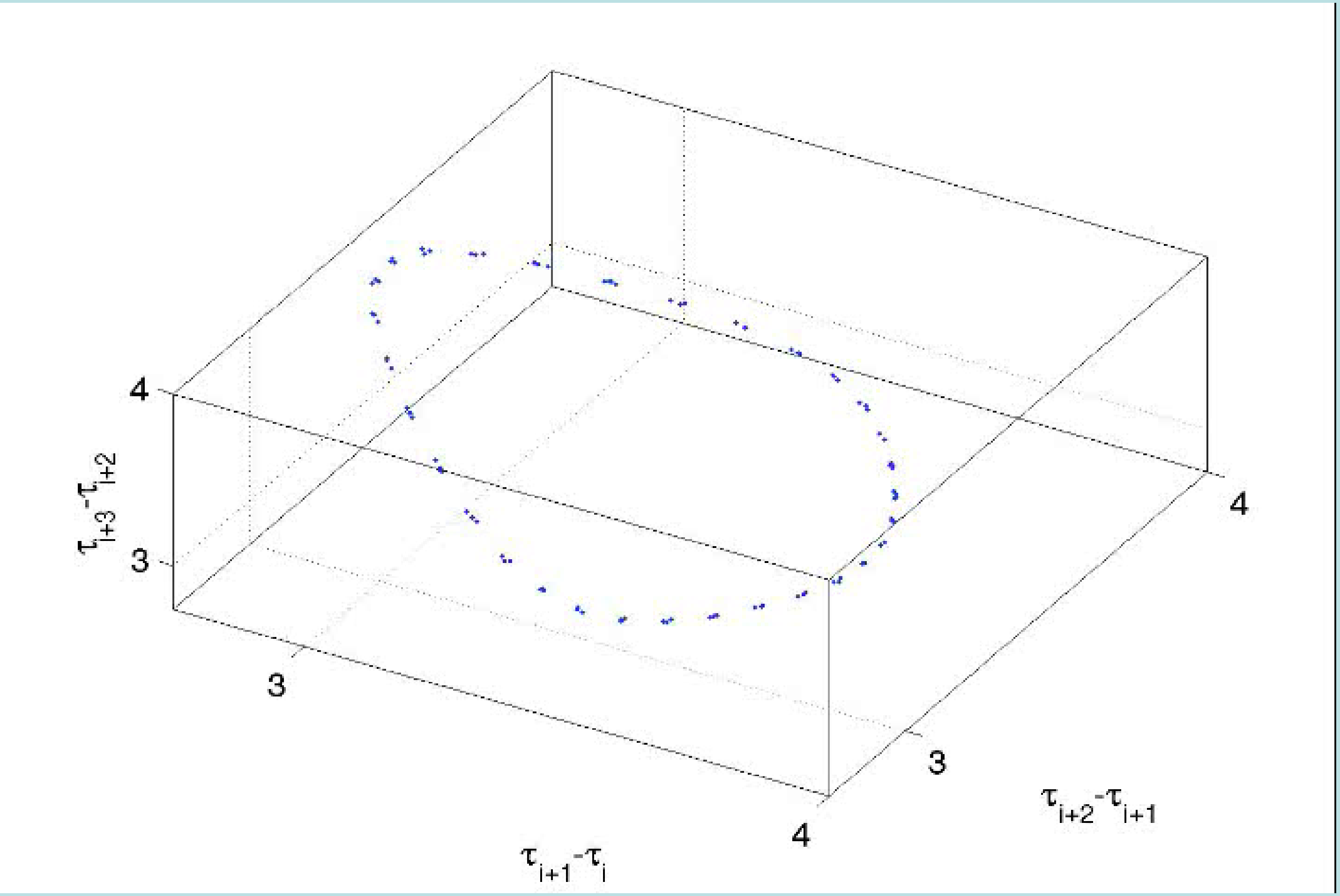
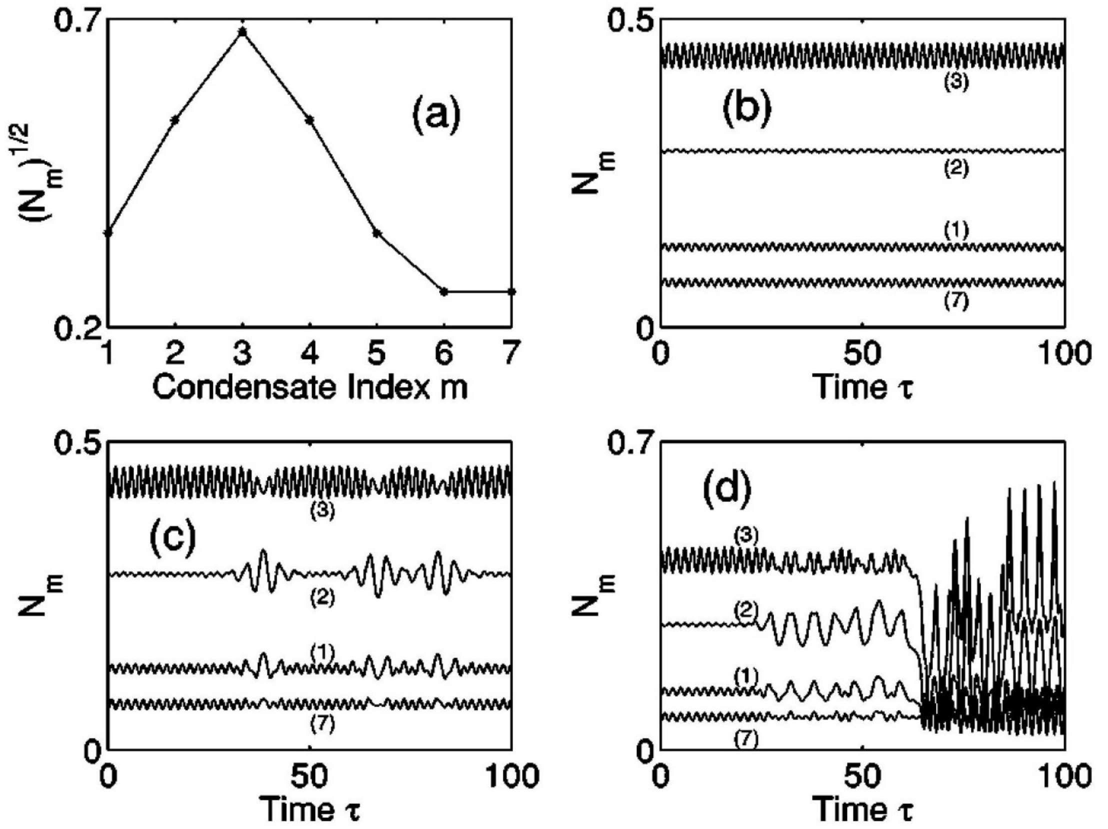


3.65
$\Delta\tau_{i+2}$
3.55
3.5
3.45
3.4
3.4
3.45
2.75
3.5
3.55
2.8
$\Delta\tau_i$
3.6
2.85
$\Delta\tau_{i+1}$

Dynamics for Delta=-0.005, QP



- ***Localized Chaotic Motion (Arnold Diffusion)***



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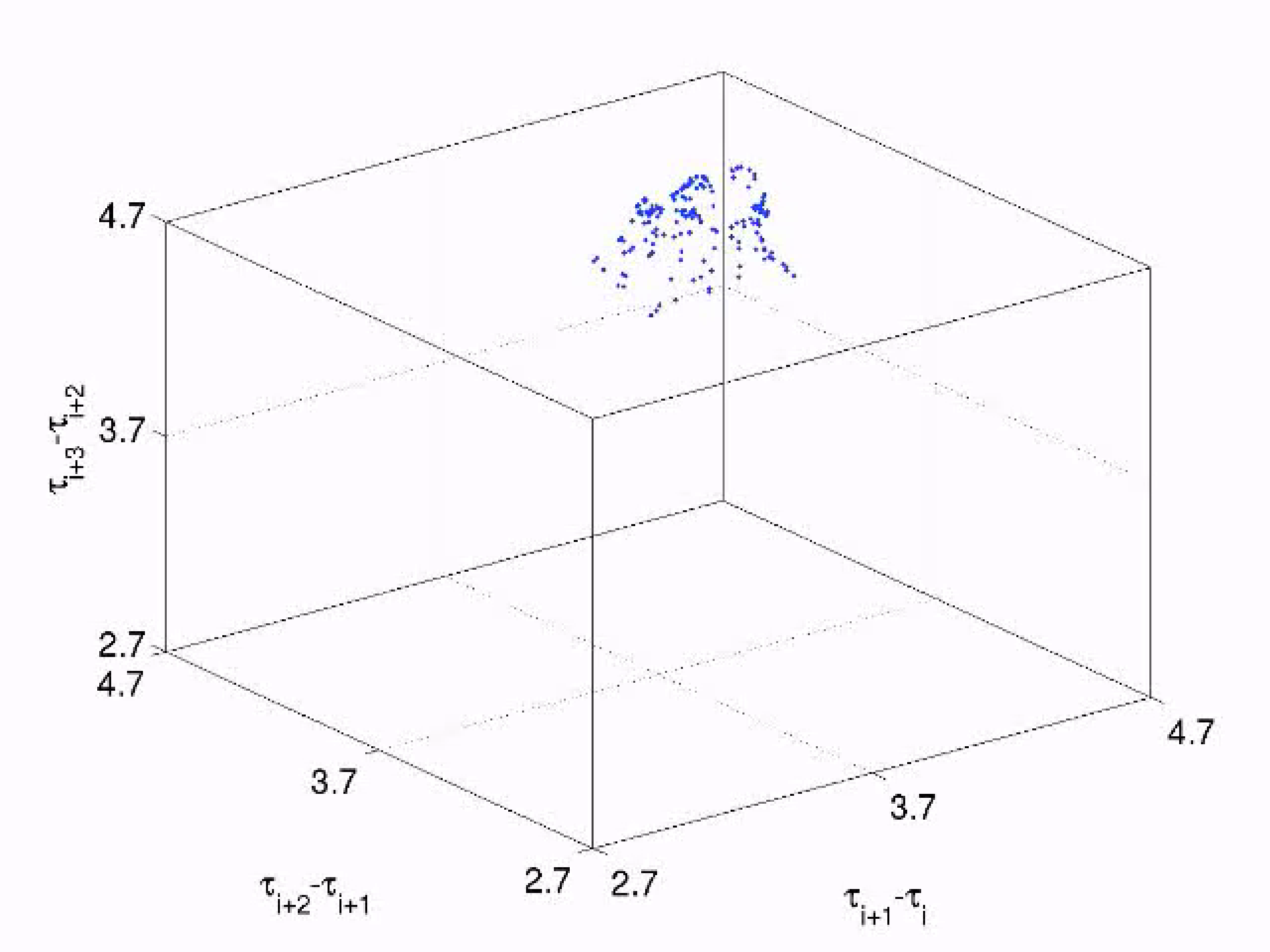
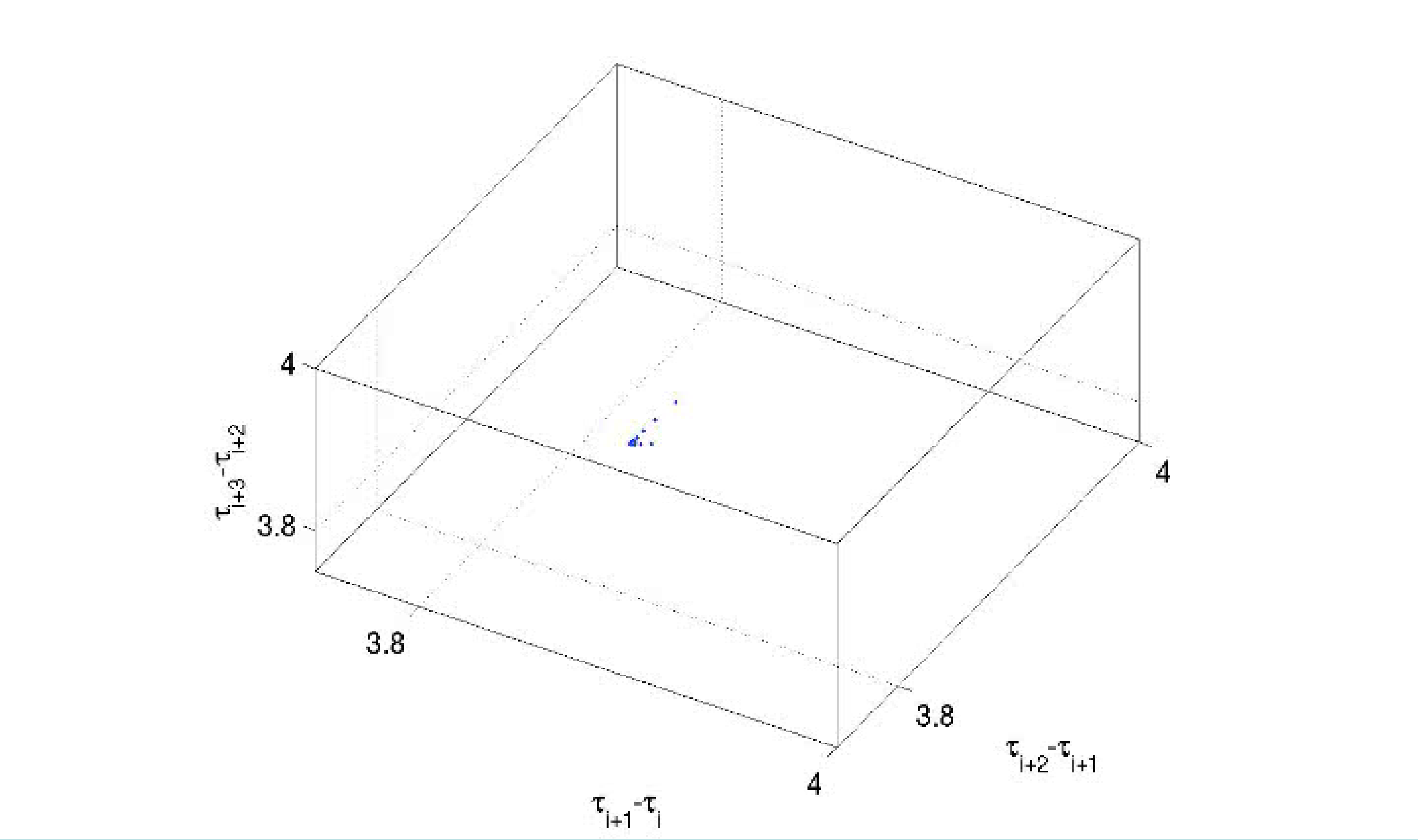
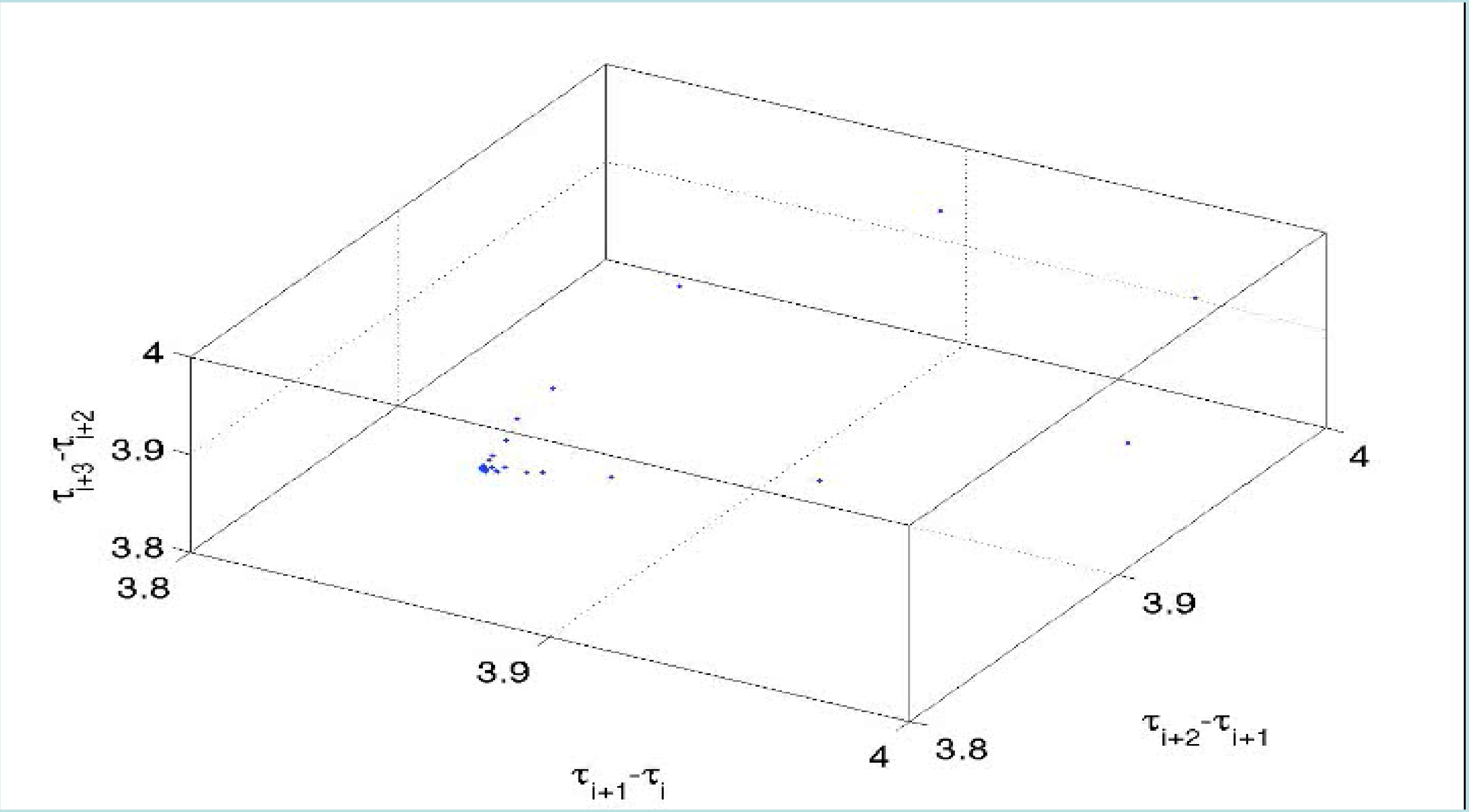
- ***Dynamics Sample 1, -0.00625***

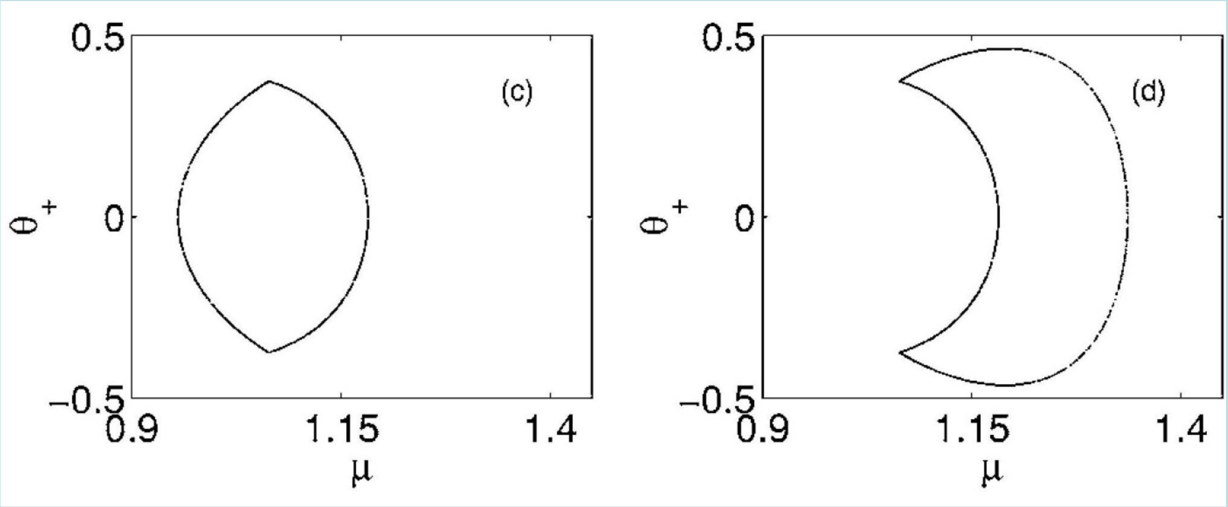


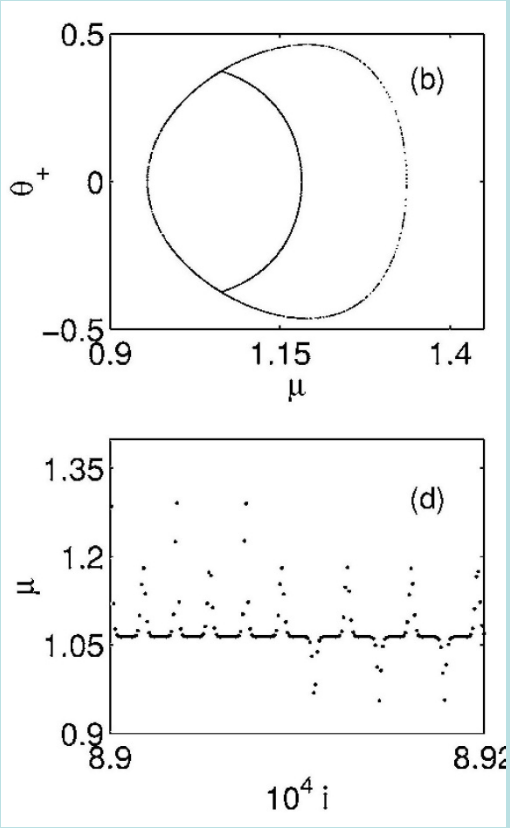
Dynamics for delta=-0.0055, Detail.



Dynamics for delta=-0.0056, Detail.

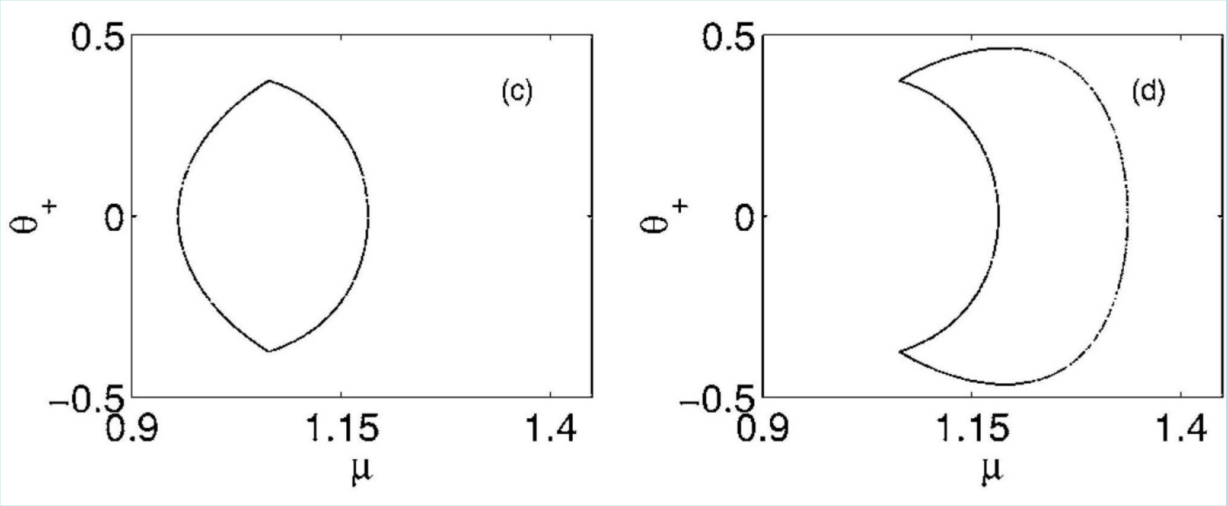


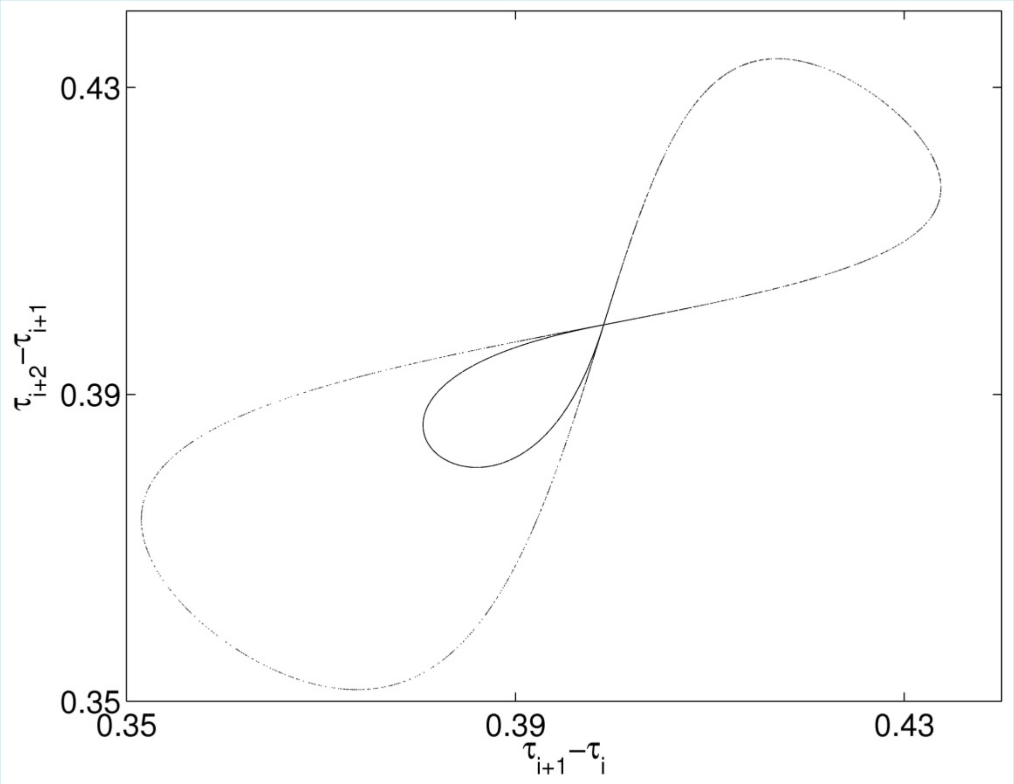




- ***RING with N=3 sites; DNLSE***

•C.L. Pando L, EJ. Doedel, PRE v.75 016213 (2007)





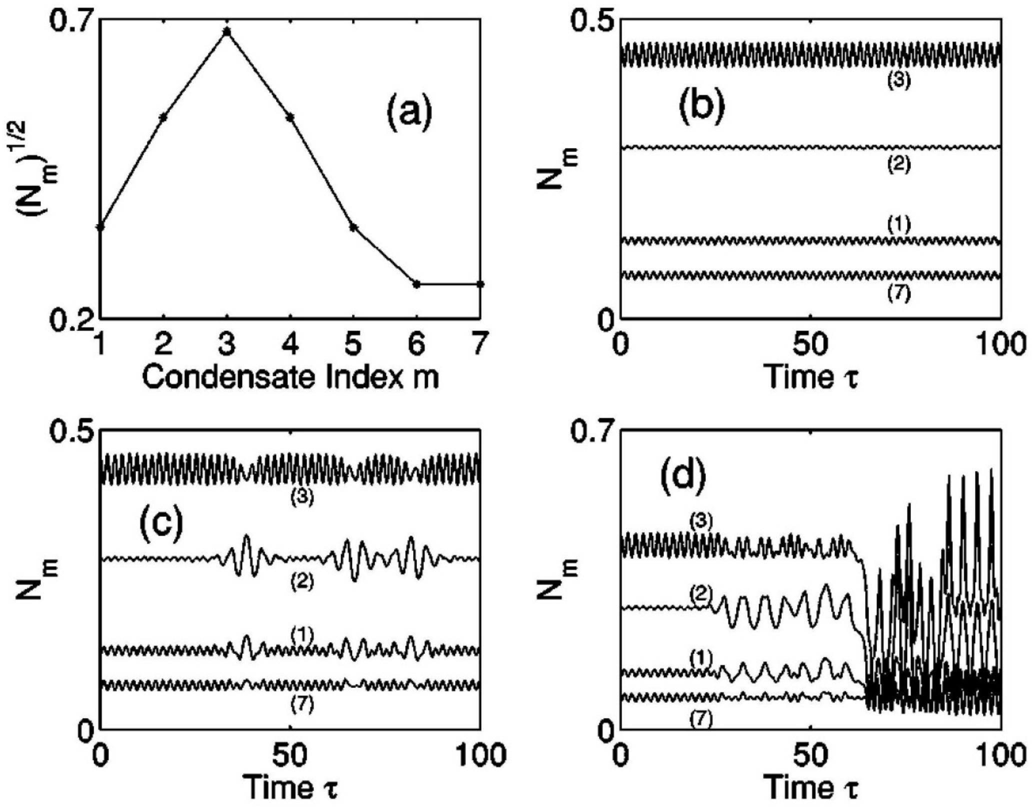
- ***RING with N=3 sites; DNLSE***

•C.L. Pando L, EJ. Doedel, PRE v.75 016213 (2007)

Convexity of H_0 implies convexity of RH
For the "single-phase solutions".

Convexity of RH implies existence of six
Distinct families of periodic orbits : Nonlinear
Normal modes.
(Liapunov Theorem, Moser-Weinstein).

• *Global Chaotic Motion (Action Diffusion)*



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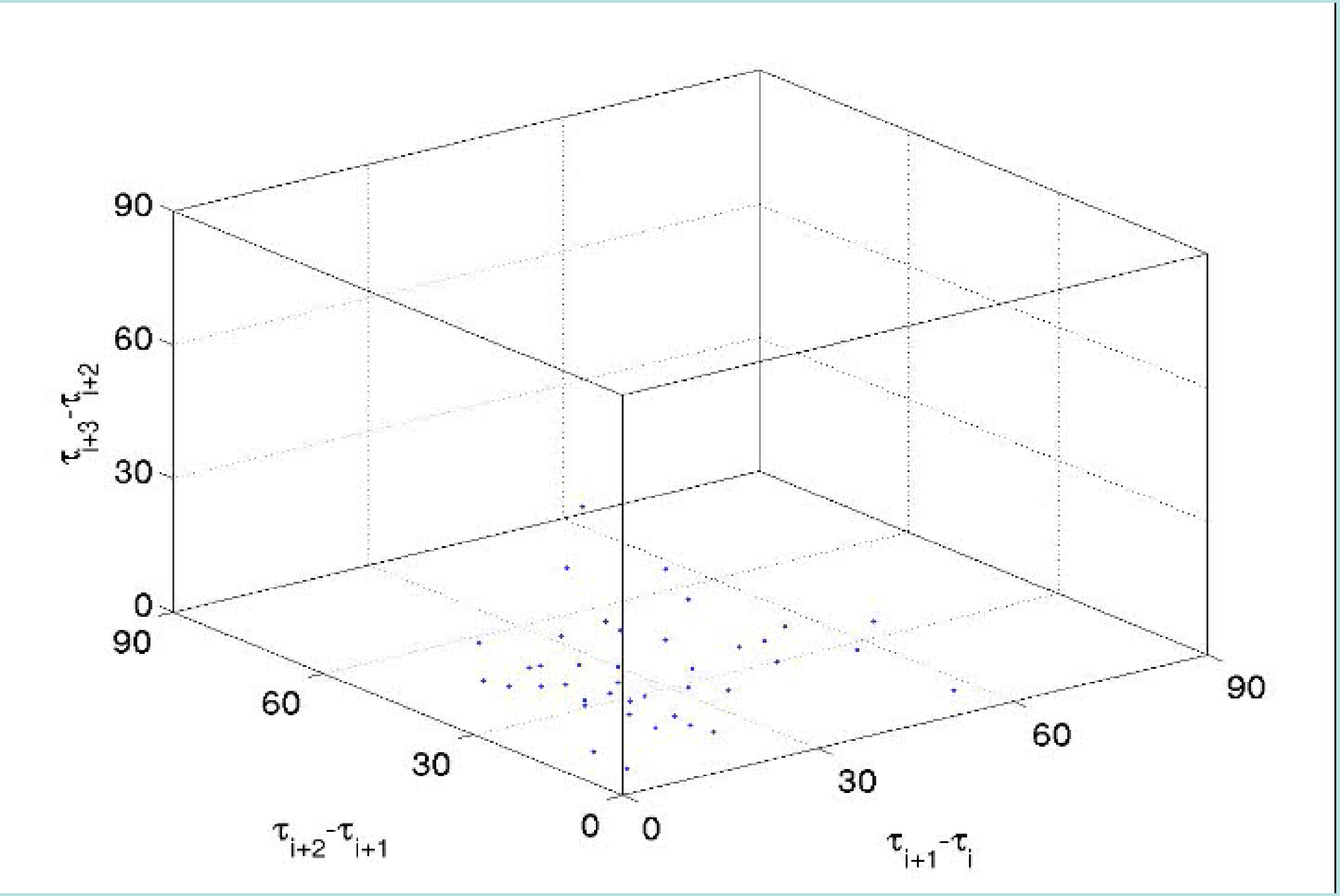
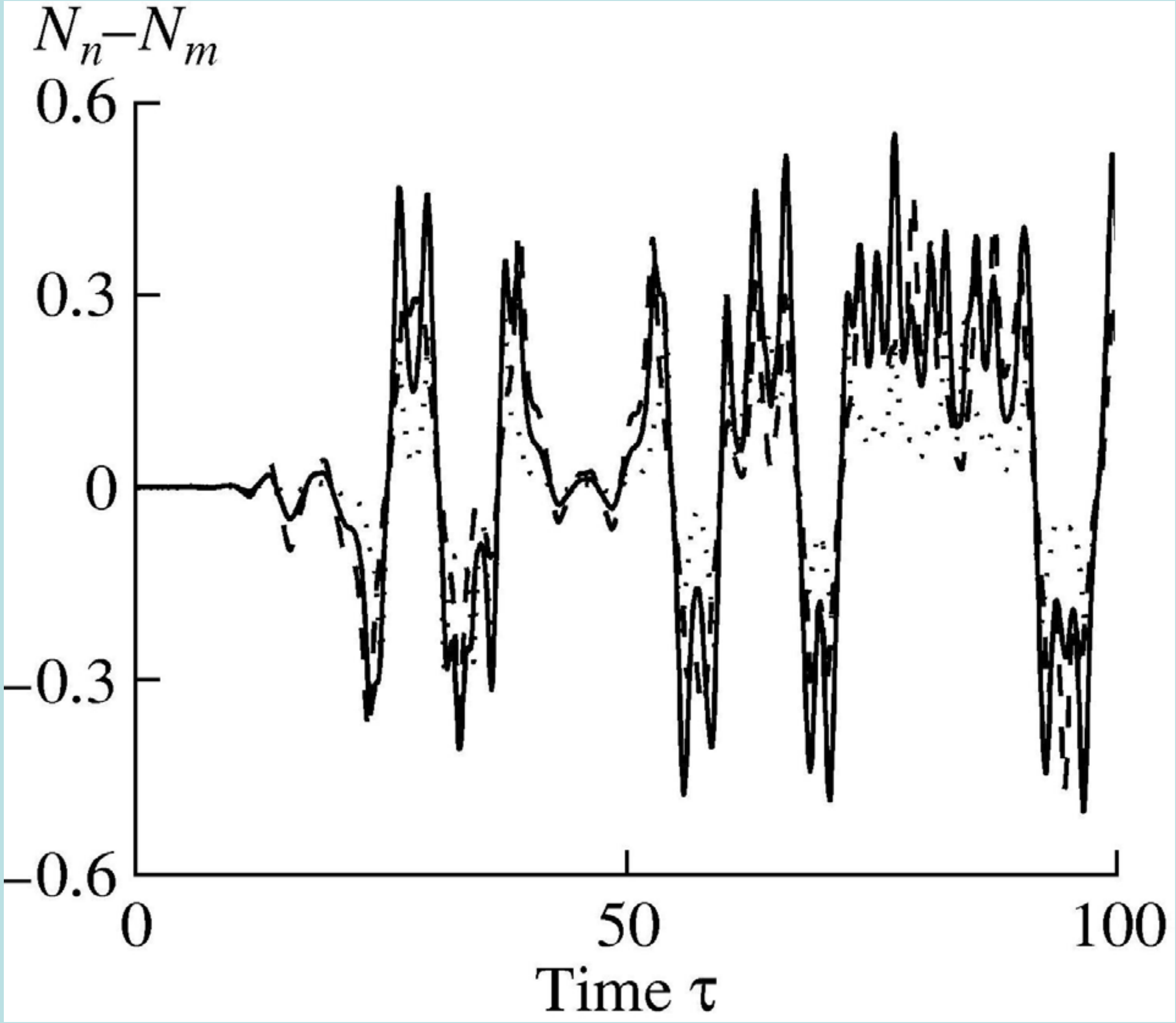
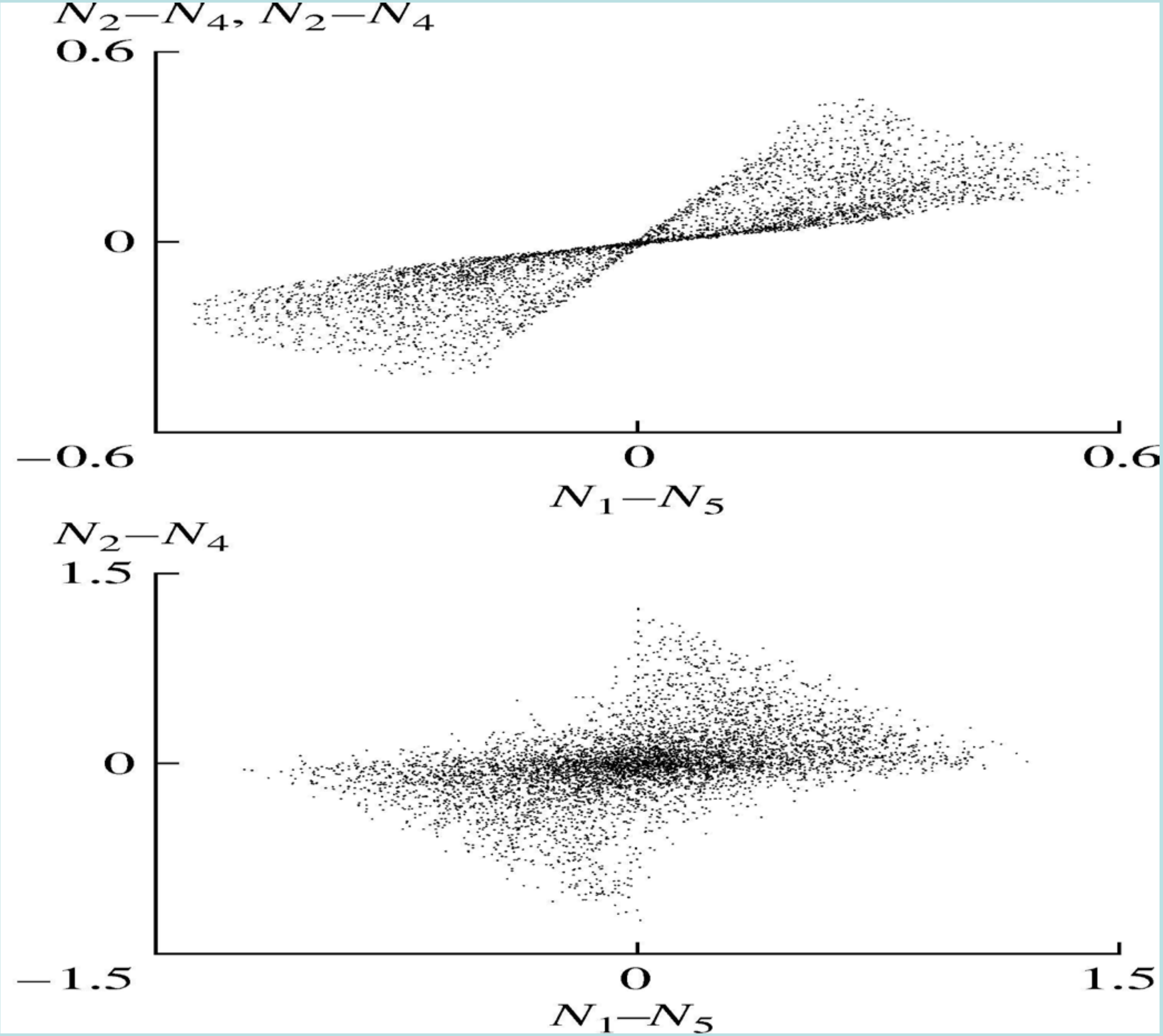
•C.L. Pando L, EJ. Doedel, PRE v. 71 056201 (2005)

Dynamics for delta=-0.007, global chaos.

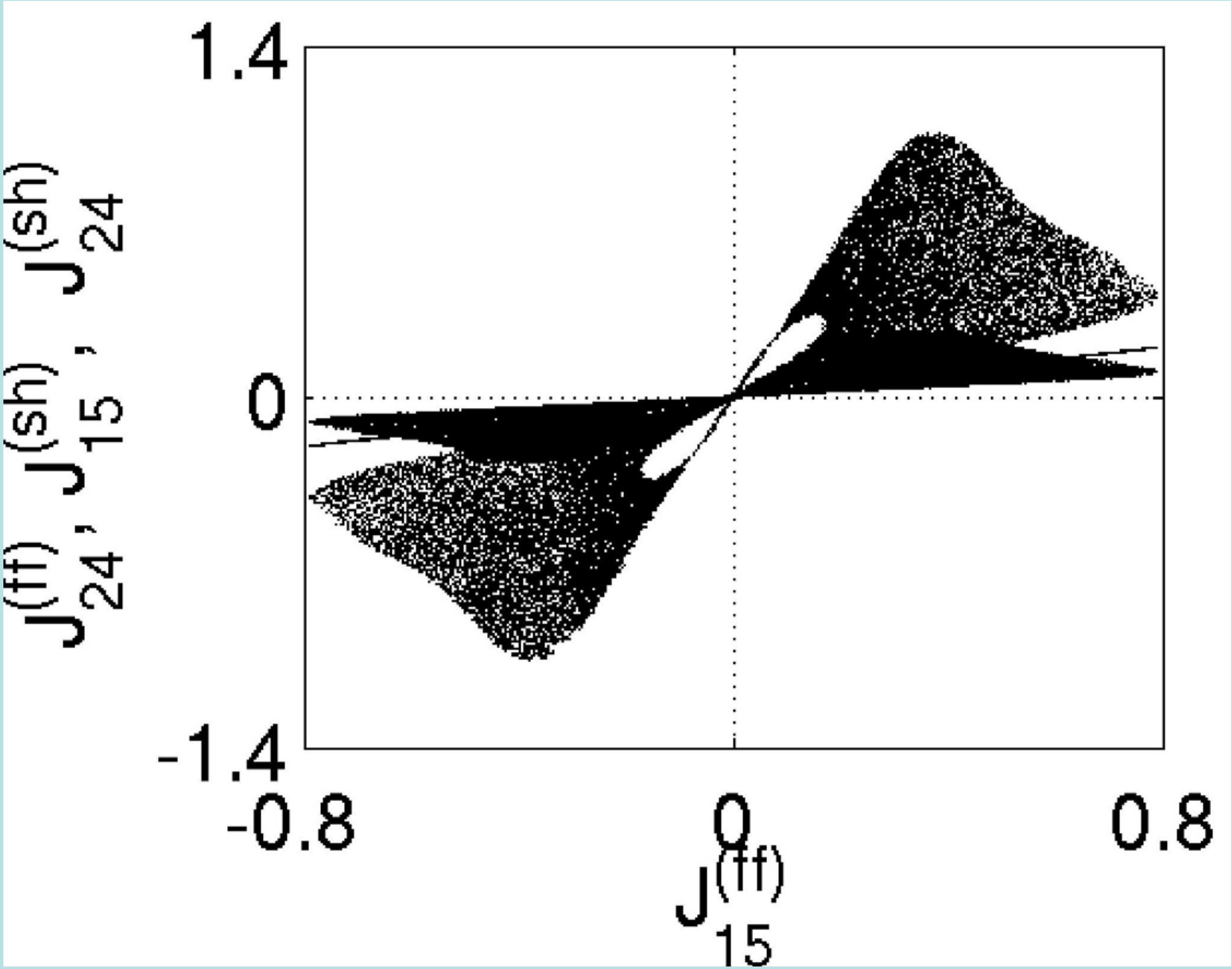




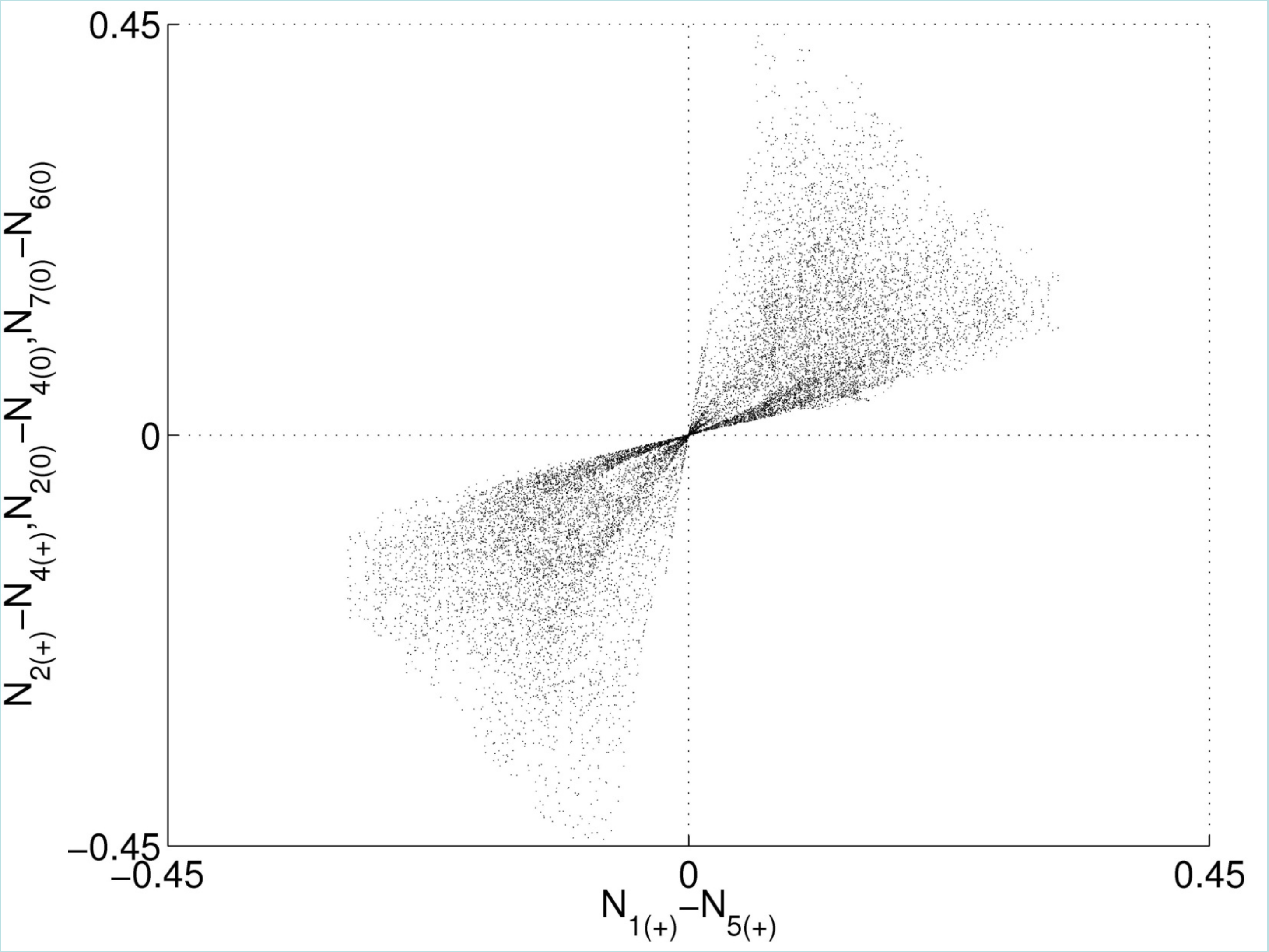
- ***RING with N=7 sites; DNLSE***



- ***RING with N=7 sites; DNLSE***



- ***RING with N=5 sites; Quadratic Nonlinear Media***



- ***RING with N=7 sites; SPINOR-1 DNLSE***

- ***Conclusiones.***

- ***The Hessian of H_0 of a Reduced Hamiltonian H determines the stability in that family of relative equilibria having the "same phase".***

- ***A neighbourhood of stable Breather-like relative equilibria show KAM dynamics and chaotic dynamics (convex and non-convex).***

- ***The time series of the recurrence times is useful in the study of the dynamics of the system, in contrast to following , in the continuous time, actions etc.***

- ***¡¡ Mil Gracias por su atención !!***