Rare Event Mitigation via Optical Feedback in Coupled Lasers

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Outline

(1) Introduction : Extreme Rare Events in a GeneralContext.

(2) Laser Models : Coherently & Mutually (& Unidirectionally) Coupled CO_2 Lasers with Saturable Absorber (LSA).

(3) Weak Mutual Coupling:

A. Onset of Rogue Waves (Extreme Rare Events)

B. Rare Event Mitigation & Suppression via Optical Feedback.

(4) Conclusions.

1. INTRODUCTION.

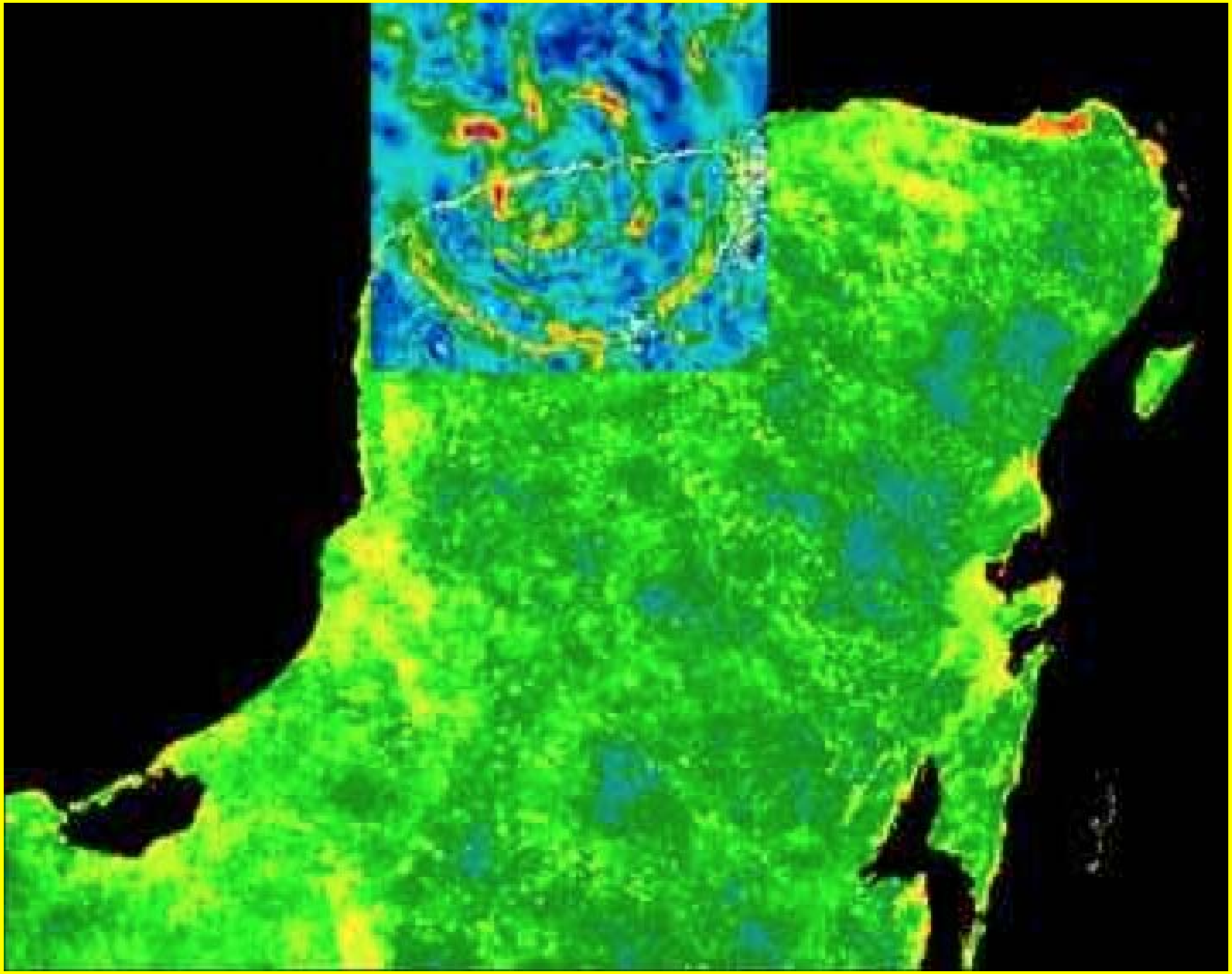
Rare Events are Recurrent Outliers with a "Qualitatively Different" Nature :

1) Rare Events of Extremely large amplitude: Climatology(droughts, hurricanes (cat. 5), tornadoes), earthquakes (mag. > 7, or in Lasers, review: J. Opt. v.18, N. 6, 2016.

2) Flow Cytometry: 1 tumor cell out of 10^(7) cells in peripherial blood (2007, A. Donnenberg et al.)

3) Meteoroid impacts (asteroid crashes):
Diameter (D) versus Frequencies:

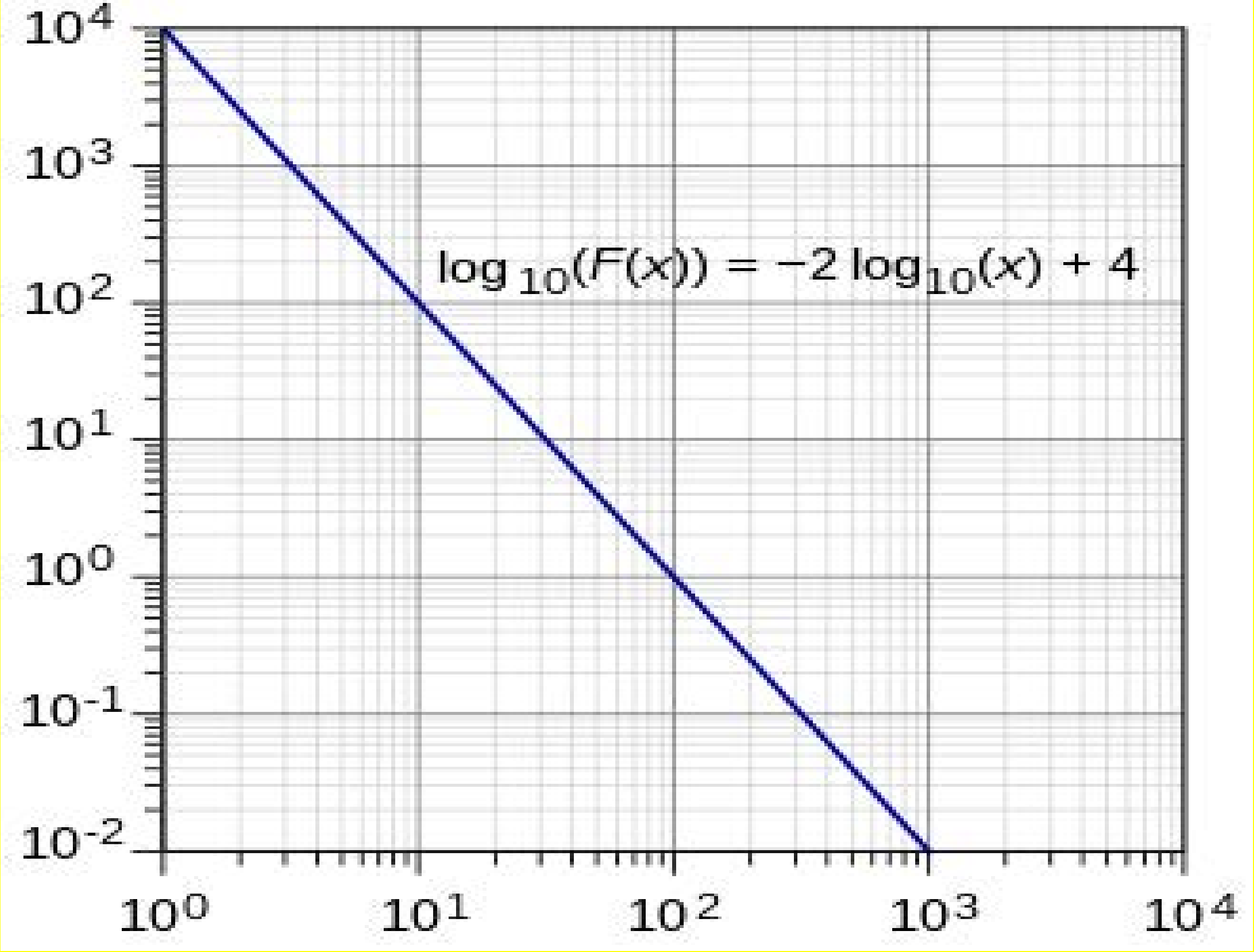
1) D ~ 4 meters – around 1/year
2) D ~ 7 meters – around 1/5 years
3) D ~ 1 km – around 1/500,000 years
4) D ~ 5 km – around 1/20 million years
5) Cretaceous-Paleogene extinction event, 66 million years ago (Mexico's Yucatan Peninsula): D ~ 10 km diameter , Extinction ~ 75 % Species.
6) Tunguska event in 1908 (Siberia, Rusia): D ~ 100 meters ; exploded 10 km from earth; massive forest fires in several miles (F ~ 10^3 y).



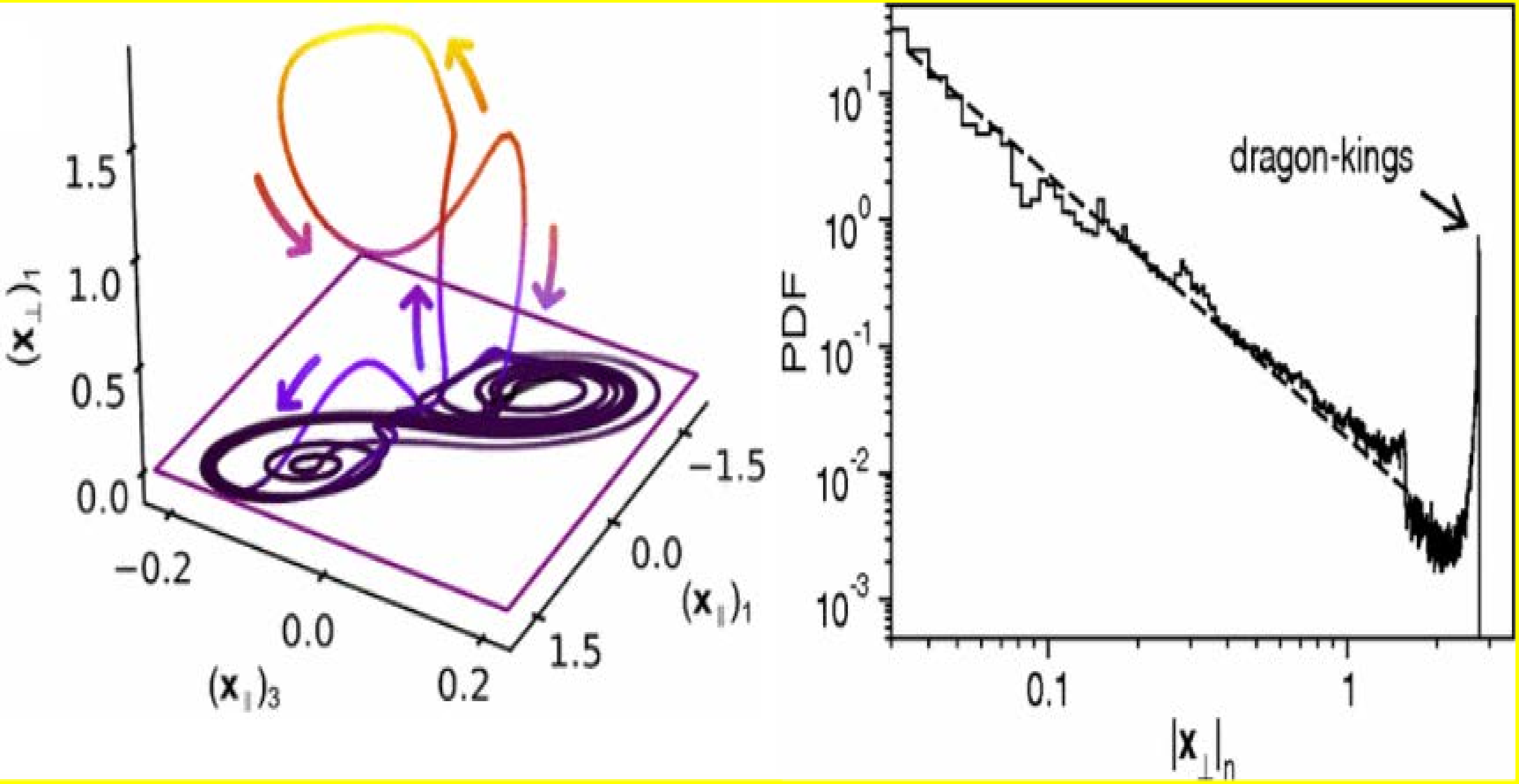
Map of the Chicxulub Crater in Mexico's Yucatan Peninsula, the asteroid caused the extinction of 75 percent of Earth's species including the dinosaurs 65 million years ago (NASA).

NASA organized the Planetary Defense Coordination Office (PDCO) in 2016. The PDCO funds efforts to identify Near-Earth Objects (NEOs) that may become a threat to the planet, and warns in case of possible hazards.

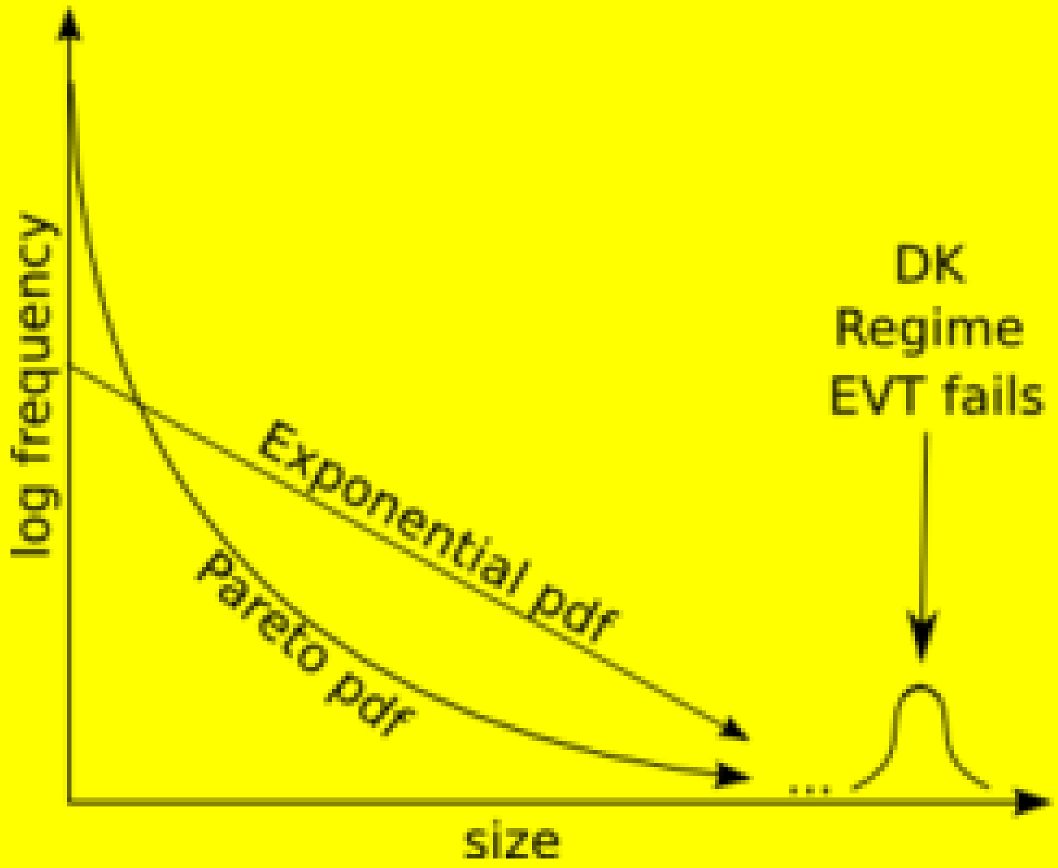
How to assess the frequency of such events? If there is a Power-Law relationship, we can estimate the power law from low-intensity events, such as minor earthquakes or small meteoroid impacts, and then extrapolate to rare events.



Power-Law Distribution (log-log plot):
Extreme Rare Event Estimation via Extrapolation of the Power-Law,



Master-Slave Electric Circuit: Dragon-Kings
H.D.L. Calvacante, D. Sornette , PRL 111.19 (2013): 198701



Generic PDF cores displaying Dragon-Kings (DK)
(EVT : Extreme value theory)

- Characteristic times:

Lasers → micro, nano-seconds…

Chemistry/Biology → milliseconds, days.

- Large amounts of data sets in lasers: Good Scenario to study Rare Events…

(Optical Extreme Events: J. Optics, v.18, N. 6, 2016; Akhmediev et al.)

4) Recurrent Outliers (outside the regular events PDF) : In particular,

in Dynamical Systems Rare Events

- Show up as (Rare) New Points in a New Region on the Poincaré Section.
- Impact on chaotic trajectories: sensitivity to small perturbations…

2. LASER MODELS WITH SATURABLE ABSORBER

2 A. Uncoupled CO_2 Lasers (LSA)

These systems display typically Mixed Mode Oscillations (MMO):

- At least two different time scales for the fast variable(s):

Quiescent & Bursting time intervals

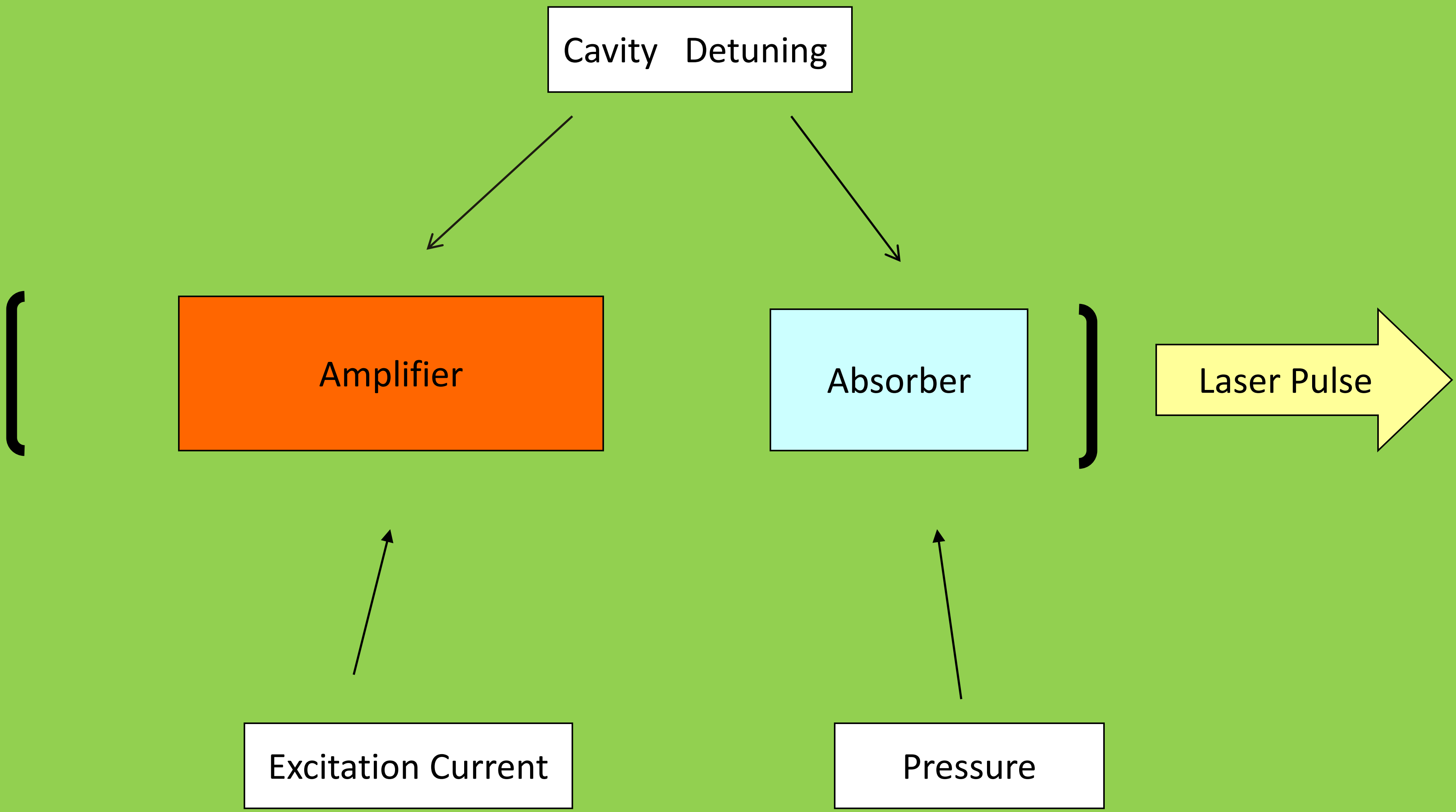


FIG.1. Laser with a Saturable Absorber (LSA): Optical "Neuron" (Spikes & Bursts)

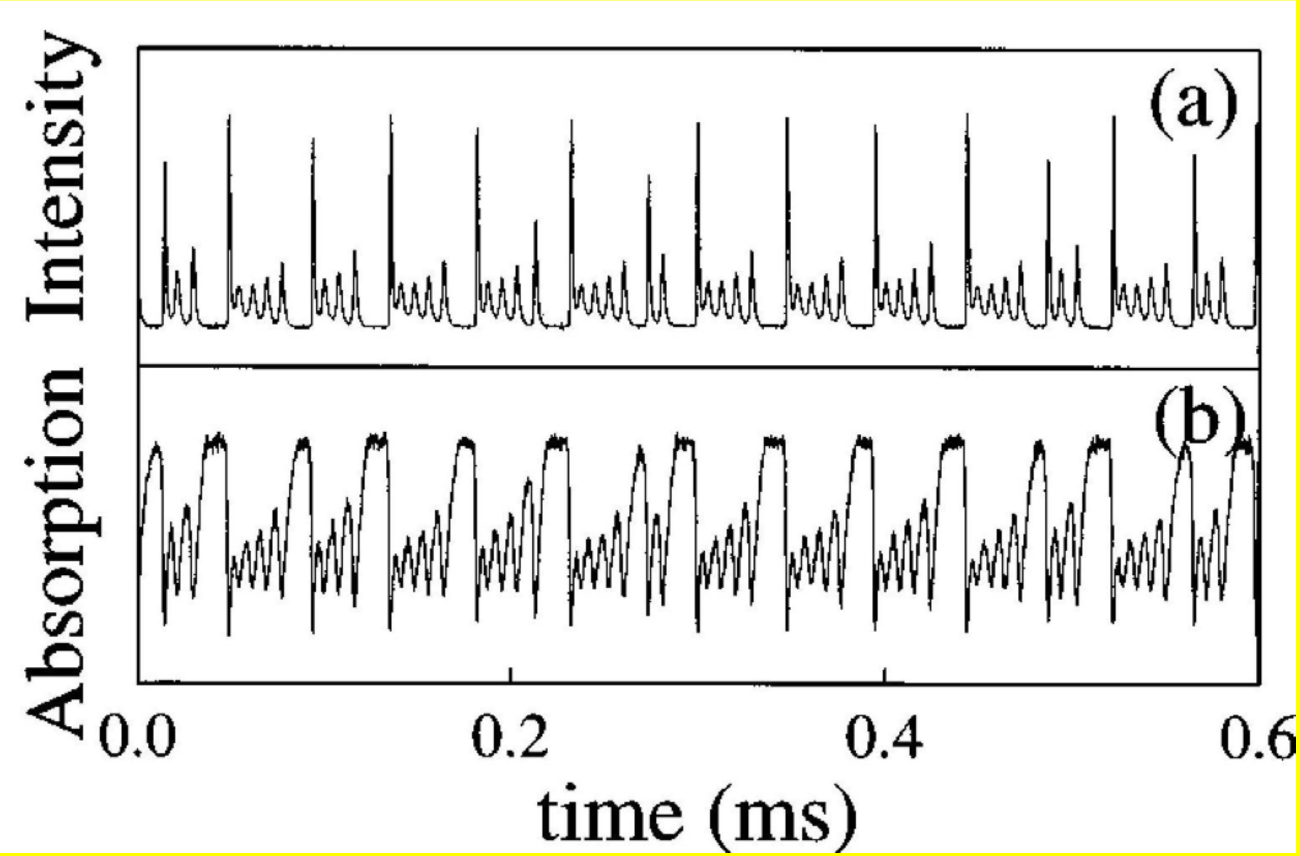


FIG.2 Experiment, P.C. de Oliveira et al, PRA , v. 55 p. 2463 (1997)
Parameters : Cavity detuning & discharge current gain medium.

2 B. Unidirectionally Coupled CO_2 Lasers

Consider two LSA as in the Fig. 1,
(lasers with intracavity saturable absorber):

But,

…. with Inhibitory (optical) coupling

like in Neurons….

FIG.3 Two Unidirectionally Coupled CO2 LSA; Experiment.

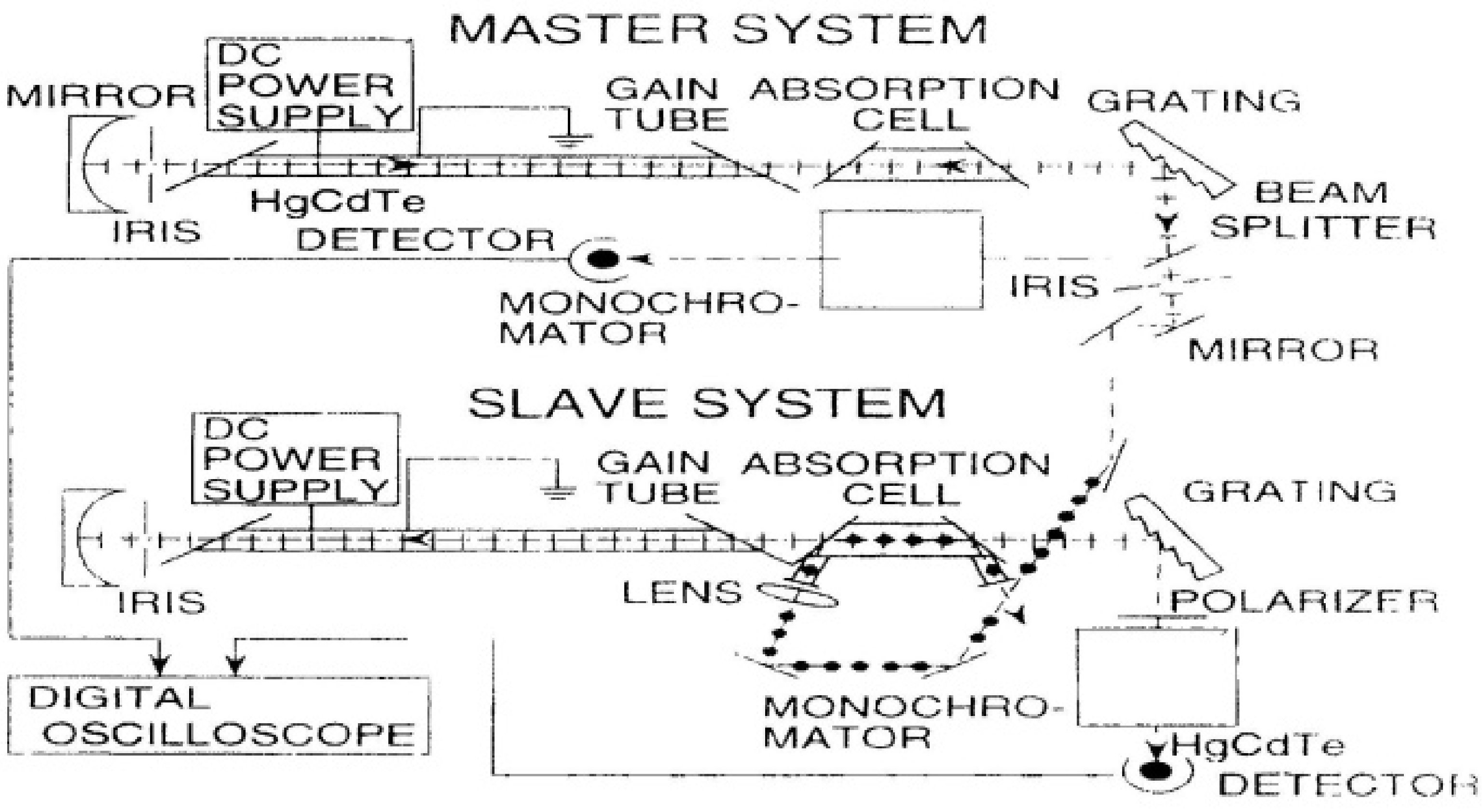
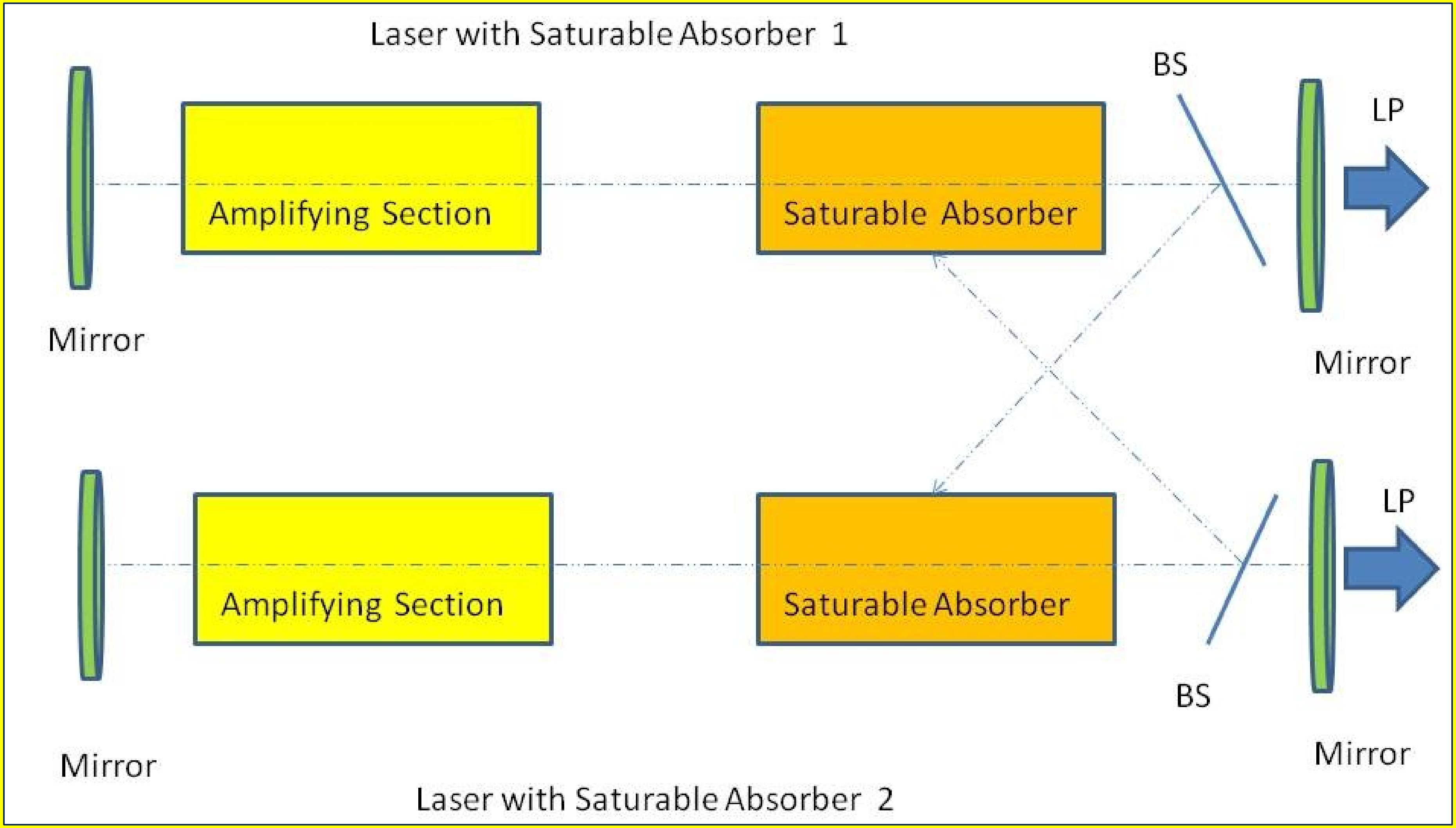


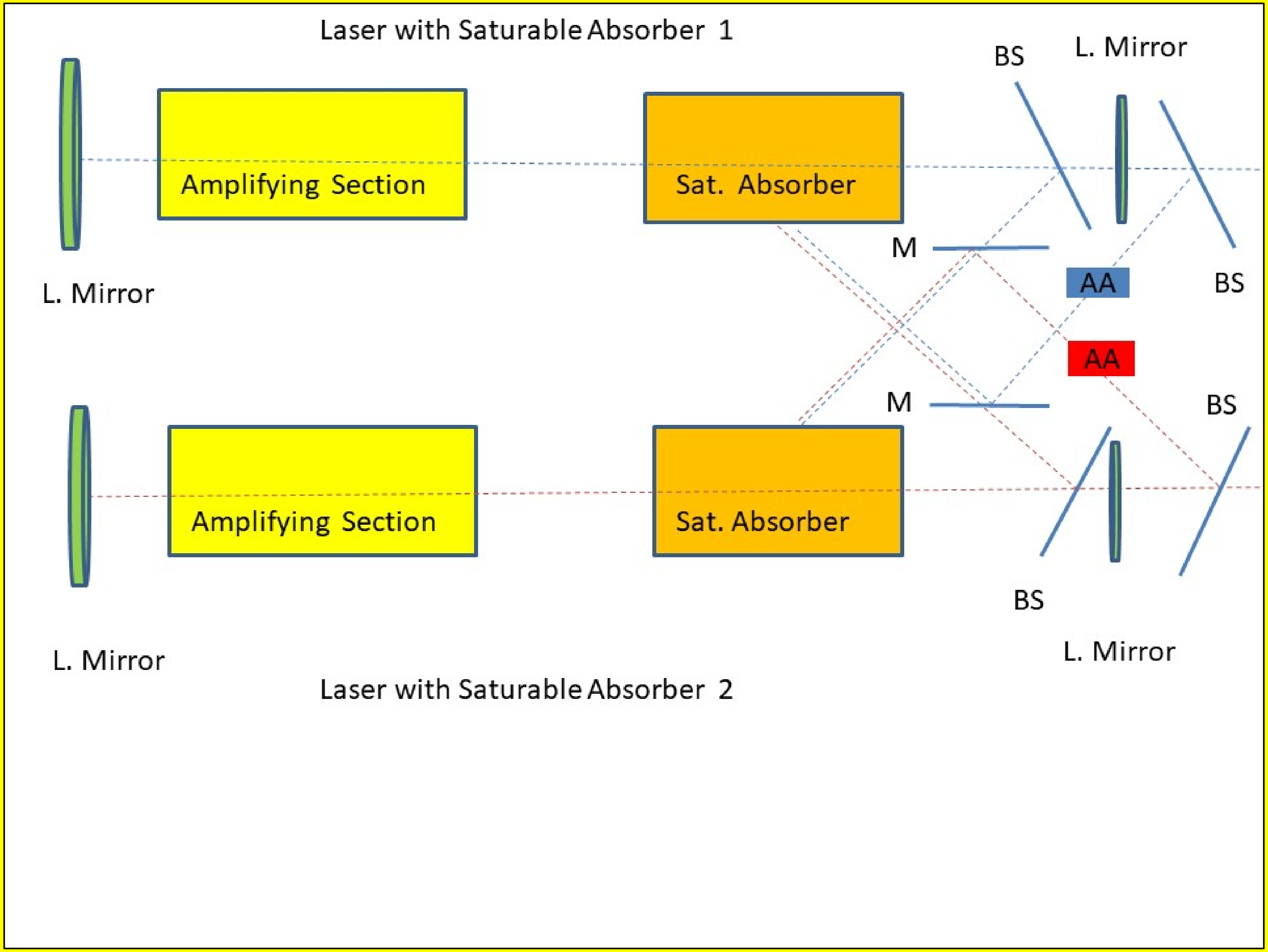
FIG. 1. Diagram of the experimental setup.

T. Sugawara, M. Tachikawa, et al. PRL v. 72, p. 3502, 1994,
Master-Slave Inhibitory Coupling of CO2 LSA.

Schematic of Two MUTUALLY Non-coherently Coupled CO_2 LSA. The Laser Fields E_1 & E_2 are not superimposed.



SchematicTwo COHERENTLY & MUTUALLY Coupled CO2 LSA.
The Laser Fields E_1 & E_2 are superimposed coherently (feedback)



In the COHERENTLY & MUTUALLY

coupled Lasers with a Saturable Absorber (LSA)

the lasers fields E_1 and E_2

at the Saturable Absorbers sites

can have 1) different phase shifts ϕ_1 & ϕ_2

and 2) frequency mismatches δ_1 & δ_2

For the sake of simplicity we asume that

in the coupled LSA that

the lasers fields E_1 and E_2

at the Saturable Absorbers sites

have 1) the same phase shifts $\phi_1 = \phi_2$,

& 2) frequency mismatches $\delta_1=\delta_2=0$

Thus, the lasers fields E_1 and E_2
at the Saturable Absorbers sites are

$$E_1 \approx [E_{02} + \mu \exp(i \phi) E_{01}] \exp(i \omega t)$$

$$E_2 \approx [E_{01} + \mu \exp(i \phi) E_{02}] \exp(i \omega t)$$

Intensities: $F=|E_1|^{(2)}$ & $G=|E_2|^{(2)}$

, where μ accounts for the Optical Feedback
(beam combining technique).

,while frequency mismatches $\delta_1=\delta_2=0$.

Single CO2 LSA Equations (Fast Absorber): Uncoupled Case.

$$\frac{dI}{dt} = -I + \frac{(z+1)\Omega}{z}(w-v) - \frac{\alpha I}{1+2\beta I} \; ,$$

$$\frac{dv}{dt} = -(\Omega + \gamma_1)v + \Omega w \; ,$$

$$\frac{dw}{dt} = \Omega v - (\Omega + \gamma_2)w + z\gamma_2 Q \; .$$

$$\Omega = \Omega_1 I \; , \qquad \Omega_1 = \frac{z+1}{(z+1)^2 + 2zI/\gamma_{R'}}$$

E.J. Doedel, B. Oldeman, C.L. Pando L., IJBC v. 21 p.305 (2011)

I is the laser intensity within the laser cavity, Fast Variable.

V, W are the effective atomic populations, Slow Variables.

$\Omega(I)$ is a rational function of I (laser intensity),

Model For Two Coherently & Mutually coupled CO2 LSA:

(Non-Symmetric Case Q1 ≠Q2 ; μ≠0)

$$\frac{dI_1}{dt} = I_1 \left(-1 + \frac{(z+1)\Omega_1}{z}(w_1 - v_1) - \frac{\alpha}{1 + 2\beta(I_1 + c\,F)} \right) ,$$

$$\frac{dv_1}{dt} = \Omega_1 I_1 (w_1 - v_1) - \gamma_1 v_1 ,$$

$$\frac{dw_1}{dt} = \Omega_1 I_1 (v_1 - w_1) - \gamma_2 w_1 + z\gamma_2 Q_1 .$$

$$\frac{dI_2}{dt} = I_2 \left(-1 + \frac{(z+1)\Omega_2}{z}(w_2 - v_2) - \frac{\alpha}{1 + 2\beta(c\,G + I_2)} \right) ,$$

$$\frac{dv_2}{dt} = \Omega_2 I_2 (w_2 - v_2) - \gamma_1 v_2 ,$$

$$\frac{dw_2}{dt} = \Omega_2 I_2 (v_2 - w_2) - \gamma_2 w_2 + z\gamma_2 Q_2 .$$

$$\Omega_i = \frac{z+1}{(z+1)^2 + 2zI_i/\gamma_R'} ,$$

Thus, the lasers fields E_1 and E_2
at the Saturable Absorbers sites are

$$E_1 \approx [E_{02} + \mu \exp(i \phi) E_{01}] \exp(i \omega t)$$

$$E_2 \approx [E_{01} + \mu \exp(i \phi) E_{02}] \exp(i \omega t)$$

Intensities: $F=|E_1|^{(2)}$ & $G=|E_2|^{(2)}$

, where μ accounts for the Optical Feedback
ω is the optical frequency.

,while frequency mismatches $\delta_1=\delta_2=0$.

Model For Two Mutually coupled CO2 LSA :

(Symmetric Case Q1=Q2=Q ; μ=0)

$$\frac{dI_1}{dt} = I_1\left(-1 + \frac{(z+1)\Omega_1}{z}(w_1 - v_1) - \frac{\alpha}{1+2\beta(I_1 + cI_2)}\right) \; ,$$

$$\frac{dv_1}{dt} = \Omega_1 I_1 (w_1 - v_1) - \gamma_1 v_1 \; ,$$

$$\frac{dw_1}{dt} = \Omega_1 I_1 (v_1 - w_1) - \gamma_2 w_1 + z\gamma_2 Q \; .$$

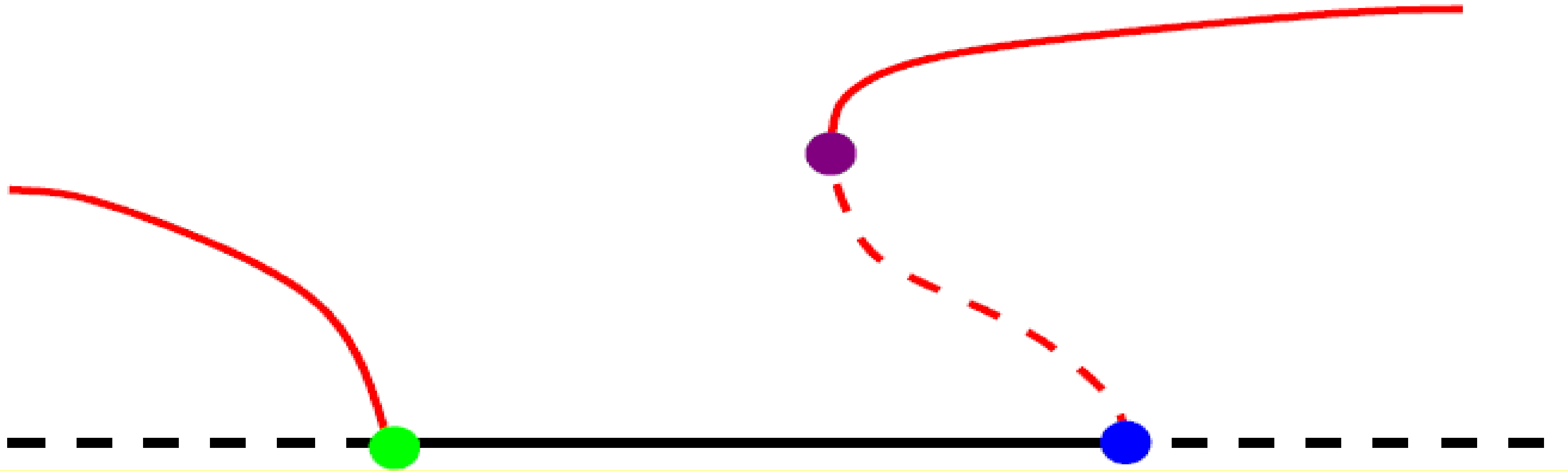
$$\frac{dI_2}{dt} = I_2\left(-1 + \frac{(z+1)\Omega_2}{z}(w_2 - v_2) - \frac{\alpha}{1+2\beta(cI_1 + I_2)}\right) \; ,$$

$$\frac{dv_2}{dt} = \Omega_2 I_2 (w_2 - v_2) - \gamma_1 v_2 \; ,$$

$$\frac{dw_2}{dt} = \Omega_2 I_2 (v_2 - w_2) - \gamma_2 w_2 + z\gamma_2 Q \; .$$

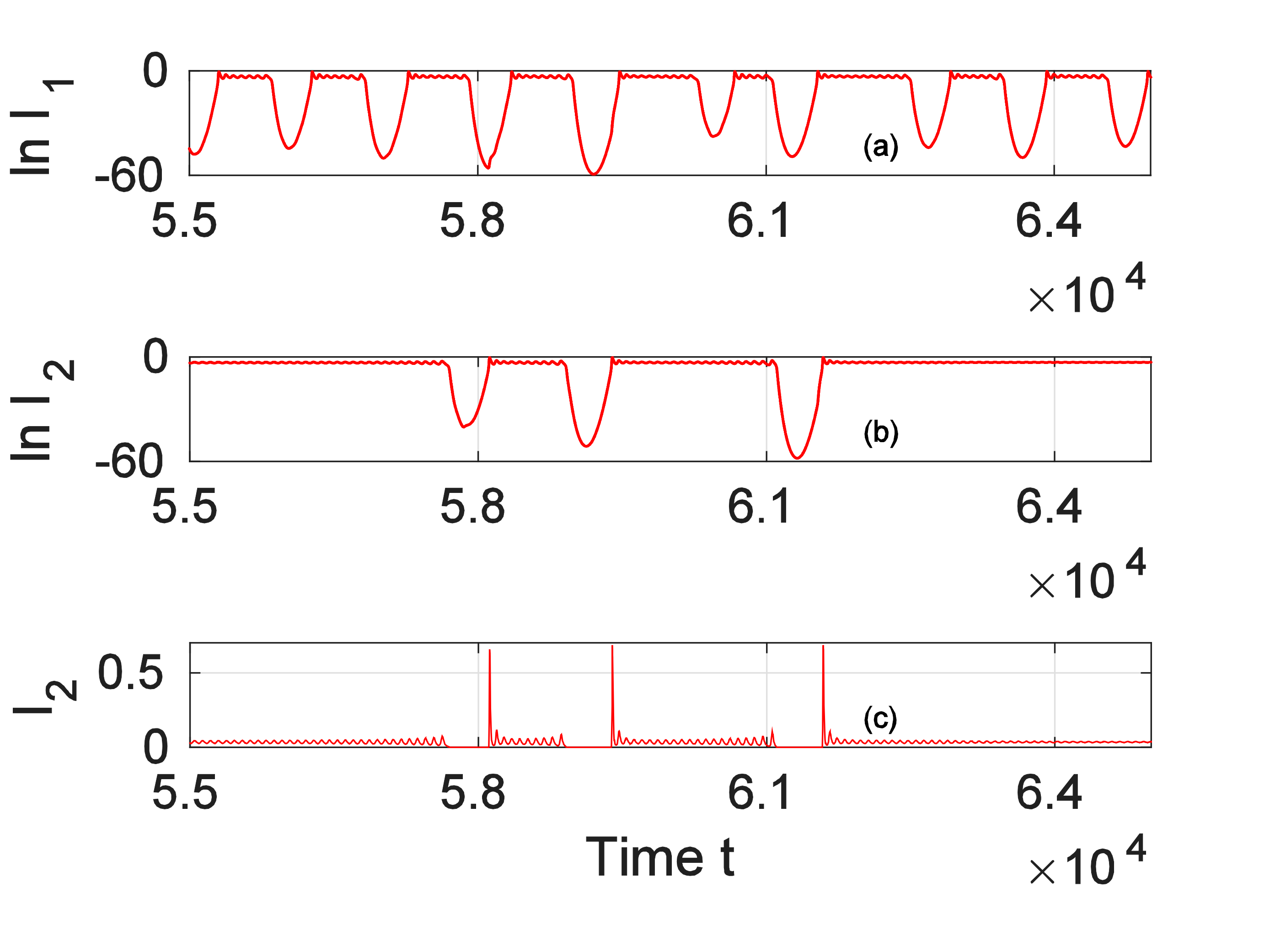
$$\Omega_i = \frac{z+1}{(z+1)^2 + 2zI_i/\gamma_R'} \; ,$$

3. WEAK MUTUAL COUPLING: Parameter c << 1

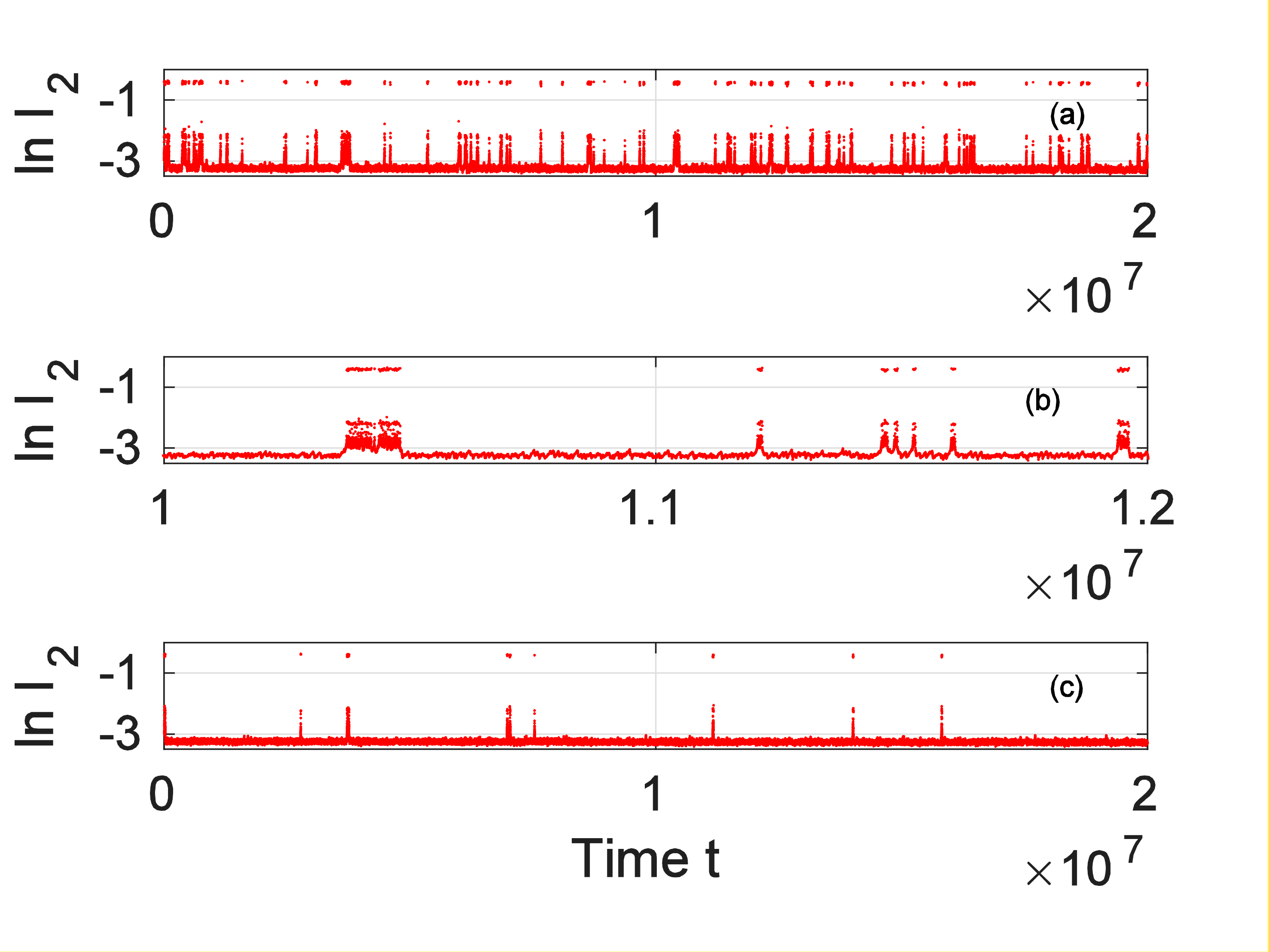


- Oscillator Death: X axis: Coupling & Y axis: Oscillator Norm.
- Black Line full (dashed) : Stable (unstable) Steady State.
- Red Line full (dashed): Stable (unstable) Limit Cycle.
- Dots: Bifurcations.

3 A. Asymmetric Case Q_1 ≠ Q_2: Rogue Waves



Q_1=2.27, Q_2=2.283, c=0.005, μ=0 ;C.L.Pando L, CS & F v.171 113462(2023)



Poincaré Section: Maxima of intensity I_2.

(a,b) Q_1=2.27, Q_2=2.283, c=0.005, μ=0 ; (c) Idem but with Q_2=2.234

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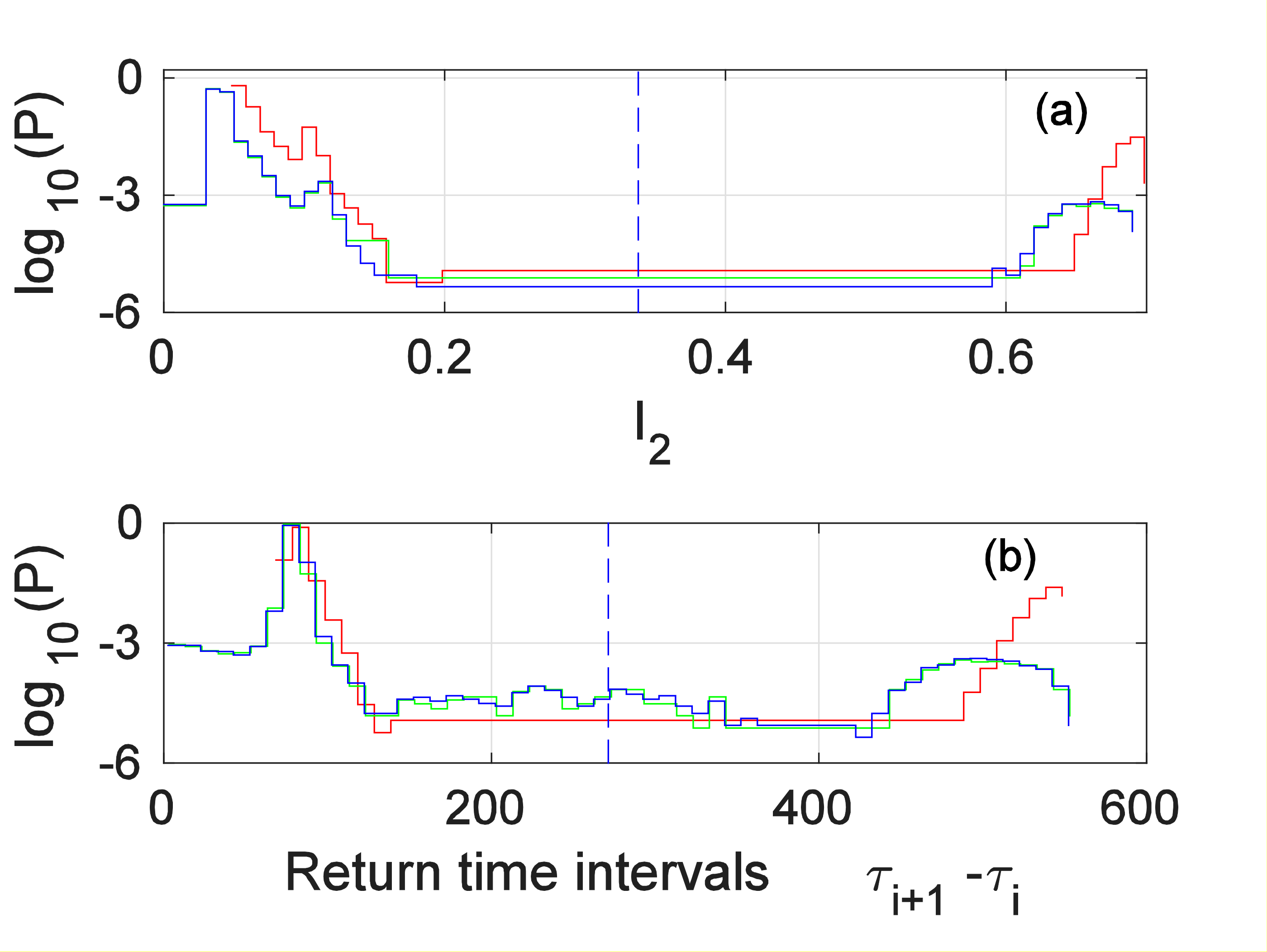

(a)Histograms for the Maxima of intensity I_2; (b)Histograms for Return times.
Q_1=2.27, Q_2=2.283, c=0 (red line) ; c=0.005, μ=0 (blue & green lines).
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CASE μ=0 VARIABLE	AVERAGE A	STANDARD DEVIATION S	LARGEST VALUE L	$R=\frac{L-A}{S}$
INTENSITY I_2	4.31x10^(-2)	3.68x10^(-2)	0.69	17.6
POINCARÉ RETURN TIMES	8.27x10^(1)	2.36x10^(-2)	552.2	19.9

8 – σ Criterion: If R > 8 then the times series shows Extreme Rare Events.
Q_1=2.27, Q_2=2.283, c=0.005, μ=0 ;
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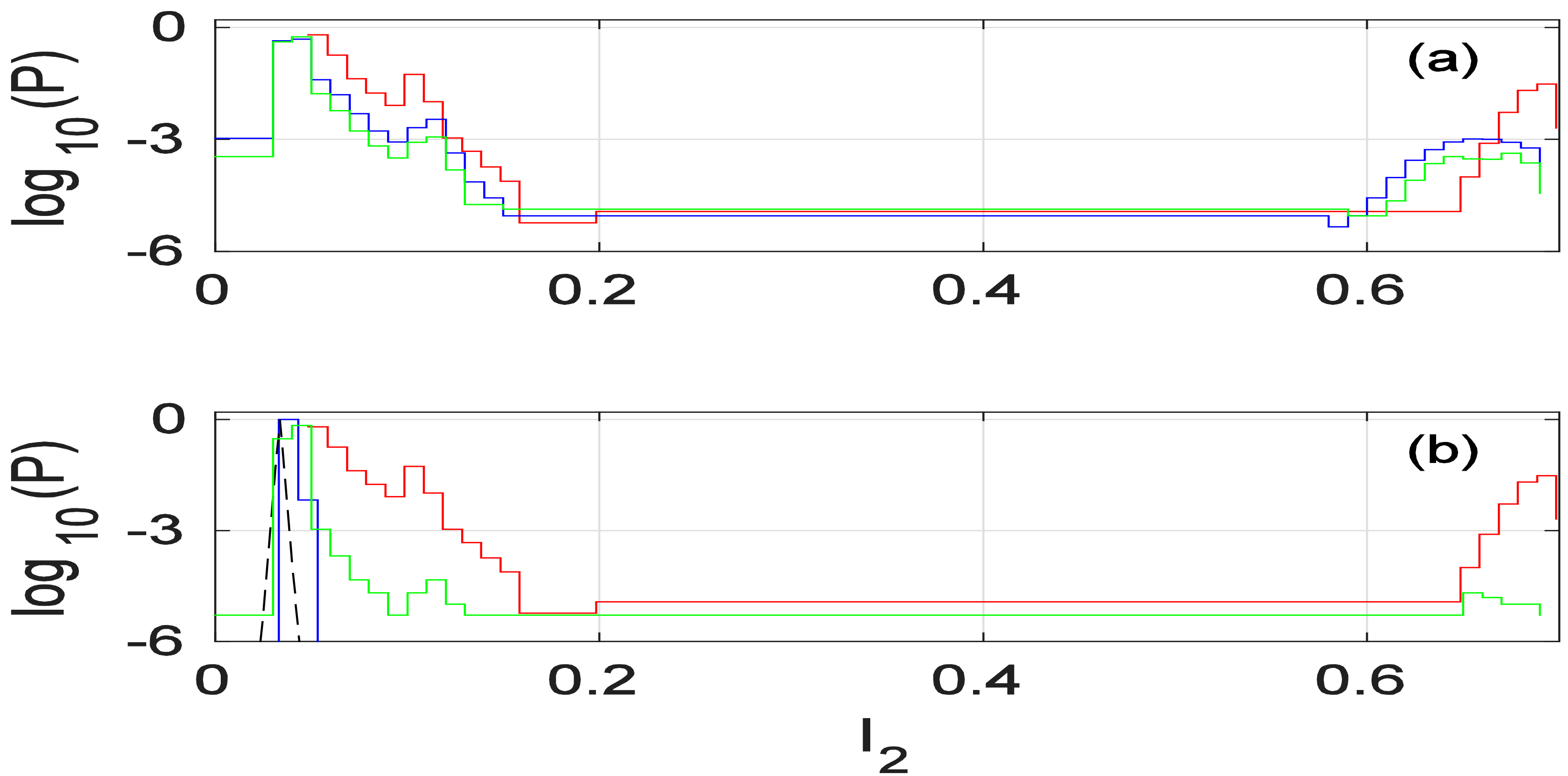
3 B. Rare Event Mitigation & Suppression via Optical Feedback : $\mu \neq 0$.



(a, b) Histograms for the Maxima of intensity I_2.
Q_1=2.27, Q_2=2.283, c=0 (red line). (a) μ=0.1 (μ=0.3) Blue (Green) Line.
(b) μ=0.9 (μ=0.75, μ=1) Blue (Green, Black dashed) Line.
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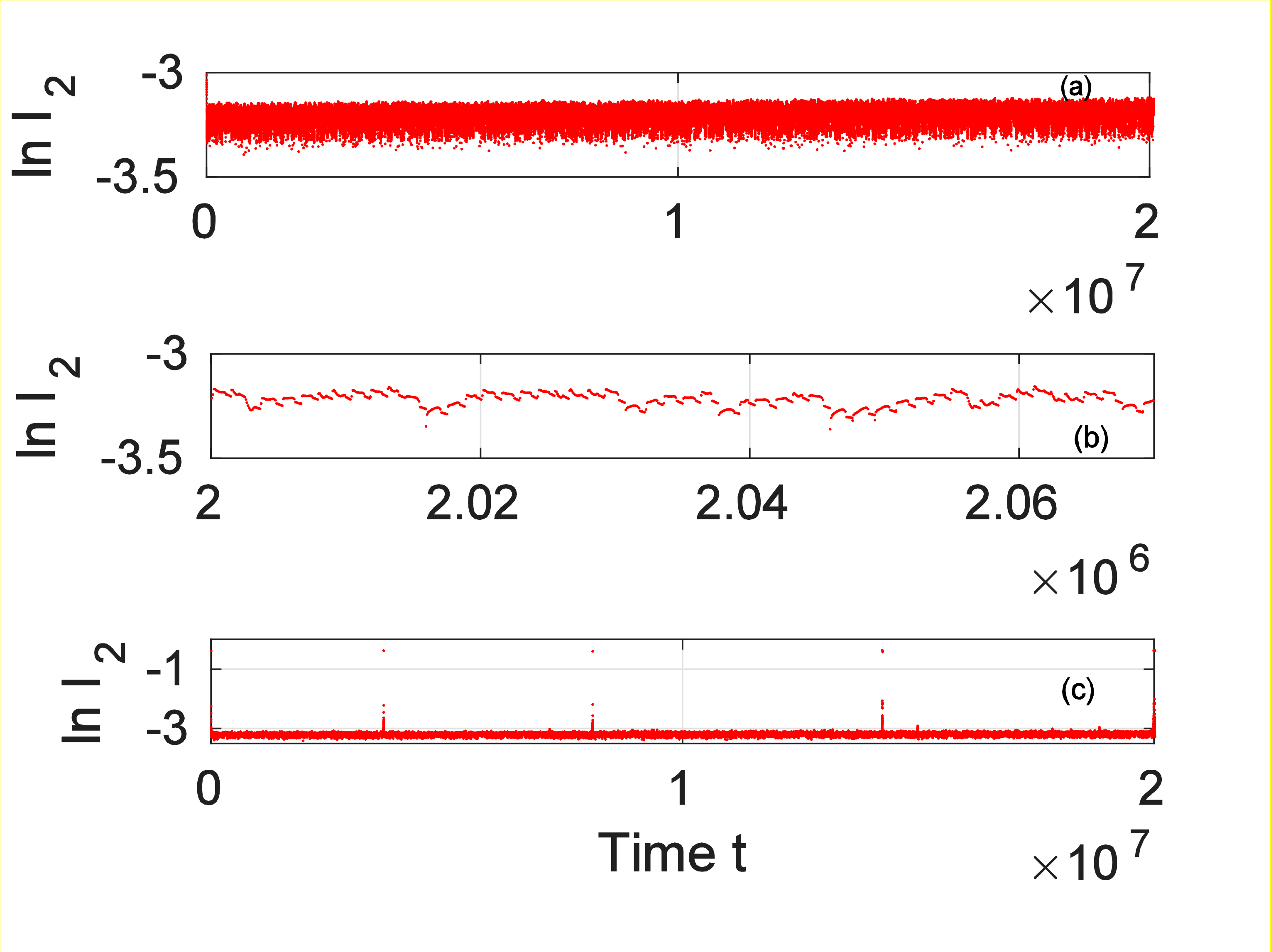

(a, b, c) Time series for the Maxima of intensity I_2.
Q_1=2.27, Q_2=2.283, c=0.005. (a,b) μ=1.0, (c) μ=0.75.
C.L. Pando L, Chaos, Solitons & Fractals v.171 , 113462 (2023)

Time series I_2 Parameter μ	AVERAGE A	STANDARD DEVIATION S	LARGEST VALUE L	$R=\frac{L-A}{S}$
0.3	4.53x10^(-2)	4.58x10^(-2)	0.69	14.08
0.75	4.09x10^(-2)	5.10x10^(-3)	0.69	127.3

8 – σ Criterion: If R > 8 then the times series shows Extreme Rare Events.
Q_1=2.27, Q_2=2.283, c=0.005, μ > 0 ;
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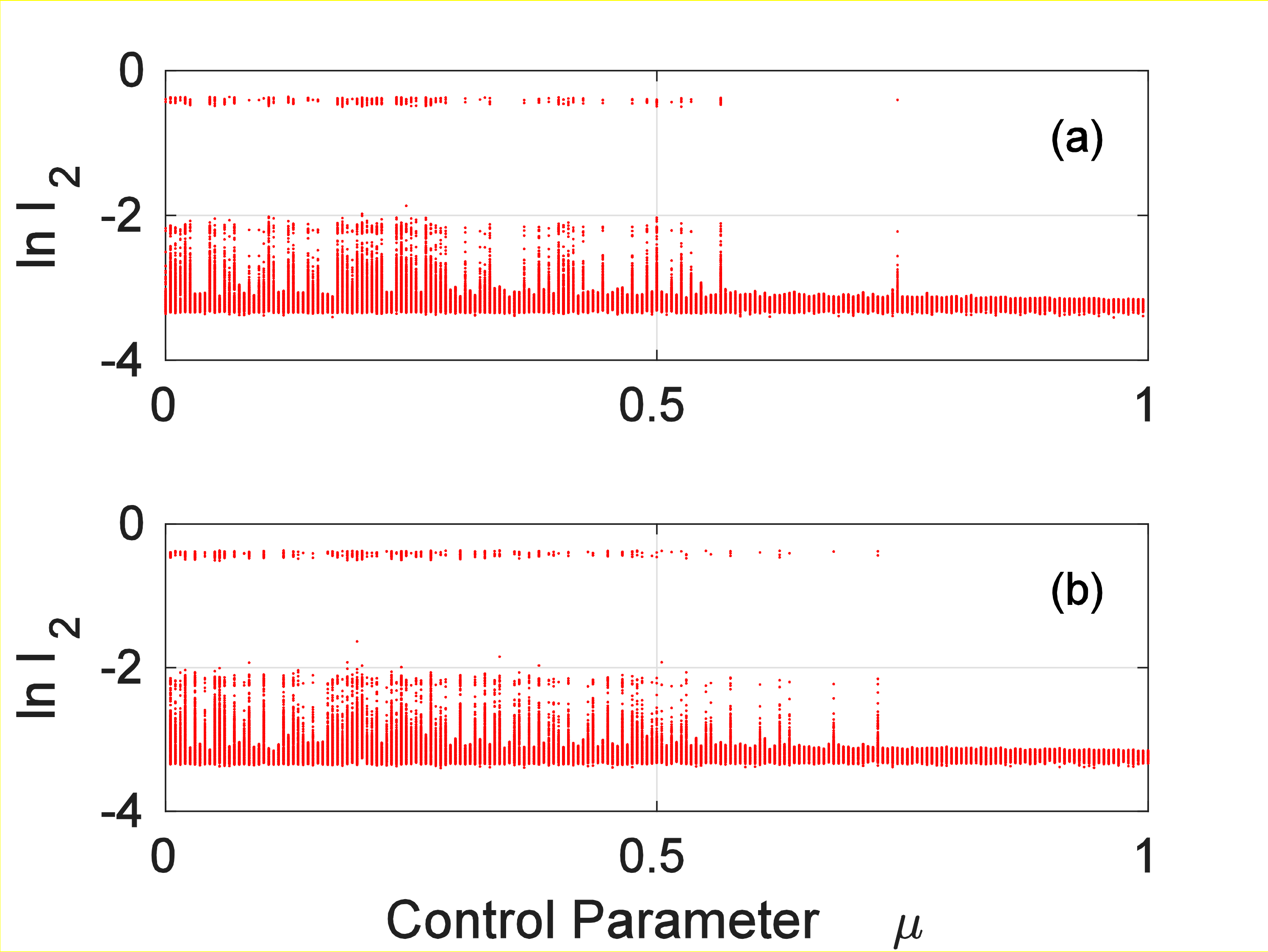

Maxima of intensity ln(I_2), Q_1=2.27, Q_2=2.283, c=0.005.
(a) Forward bifurcation diagram, (b) Backward bifurcation diagram.
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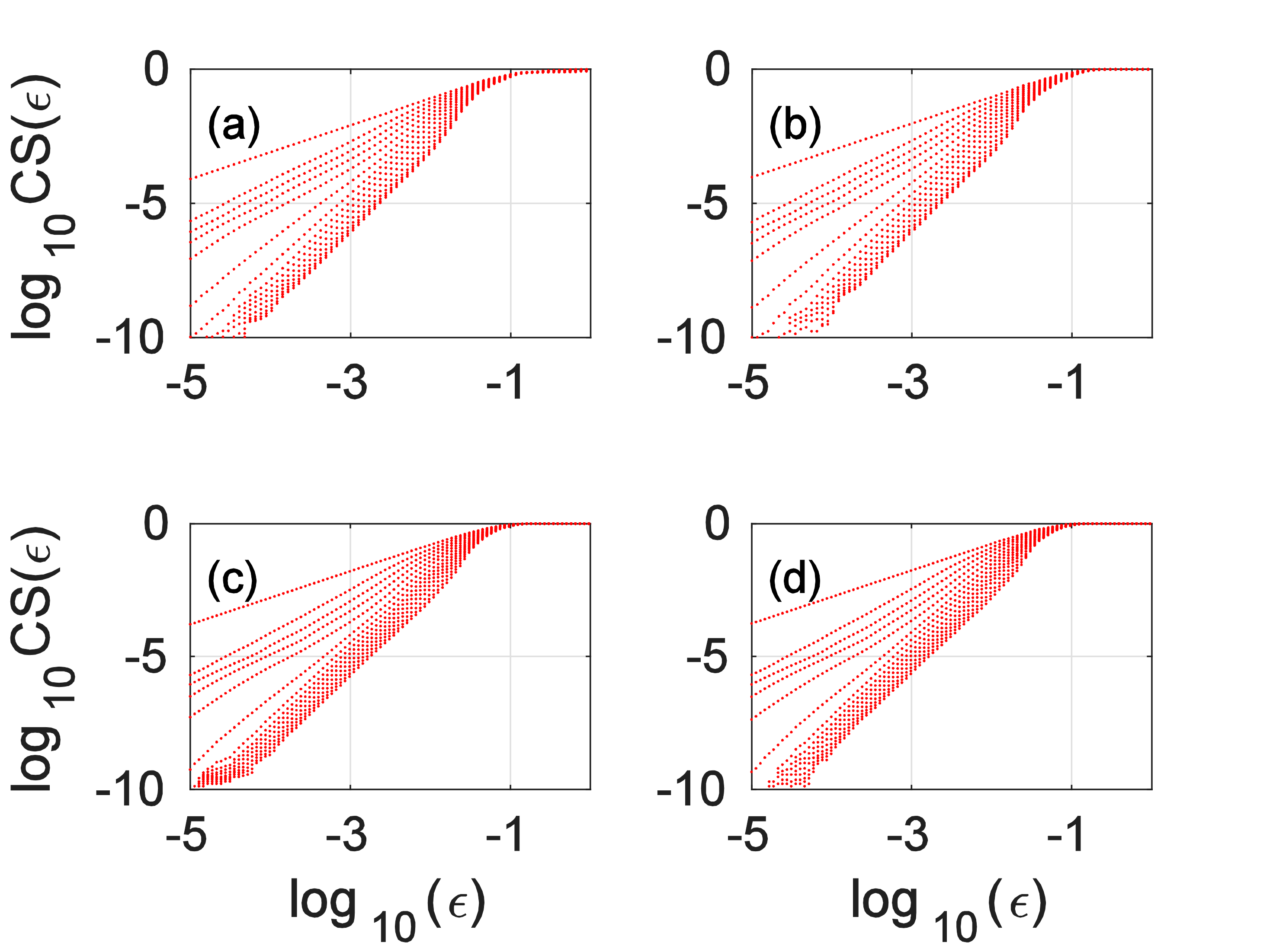

Correlation sum CS(ϵ) vs partition ϵ for embeddings (1-15) for I_2
when Q_1=2.27, Q_2=2.283, c=0.005,for (a)μ=0.1 ;(b)μ=0.3;(c)μ=0.9;(d)μ=1.0 .
C.L.Pando L, Chaos, Solitons & Fractals v.171 113462(2023) .

5. Conclusions

1) We extend a previous model of a pair of optically coupled lasers with saturable absorbers (SA) to account for Coherent Beam Combining of the two laser fields.

2) This pair of Coupled Lasers with SA show Extreme Rare Events, which can be Mitigated and Suppressed via a New Parameter, namely, a Relative Feedback Parameter, which can be tuned Optically & more Smoothly than the pump parameters.

3) Suitable Relative Feedback Parameters

can induce (Hyper) Extreme Rare Events with an "R index", where $R=(L-A)/\sigma$ (=127), which are much larger than that for the Rare Event threshold estimated via the 8-sigma criterion, where R=R_th=8.

Thank You very much for your kind attention!