IFUAP-BUAP Mini-curso en Nanoestructuras

Parte II.

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Supported by
US DOE & NSF NIRT

¿Preguntas?   ¿Más info?

ulloa@ohio.edu

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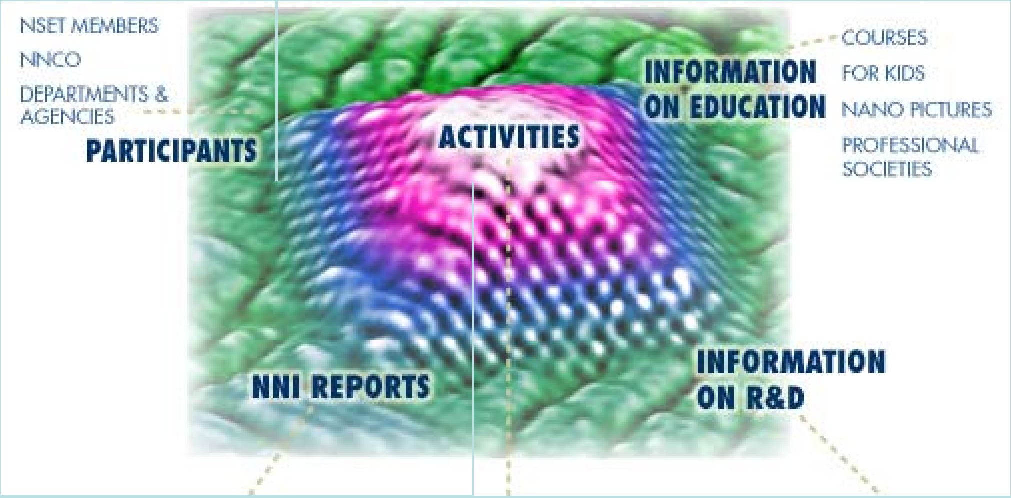
nano.gov



Resumen/Outline

- Quantum dots – confinement vs interactions
  - How to make / study them
- Coulomb blockade & assorted IV characteristics
- Optical effects – excitons: selection rules, field effects
- Transport in complex molecules: the case of DNA

Quantum Dots: L ~ λ

good things come in small packages

Confinement: KE~ k^2 ~ L^{-2}

Interactions: PE ~ q^2/L

For L~100nm, PE ~ 1meV
while **KE ~ 0.5meV** (in GaAs)

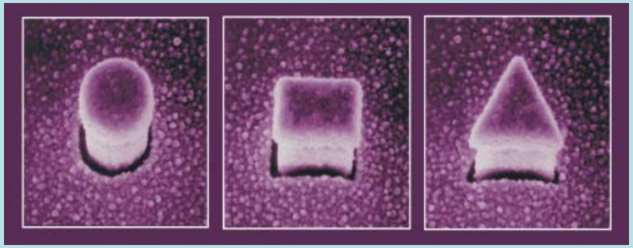
For L~5nm, PE ~ 20meV ; **KE ~ 200meV**

Total energy

E = KE + PE

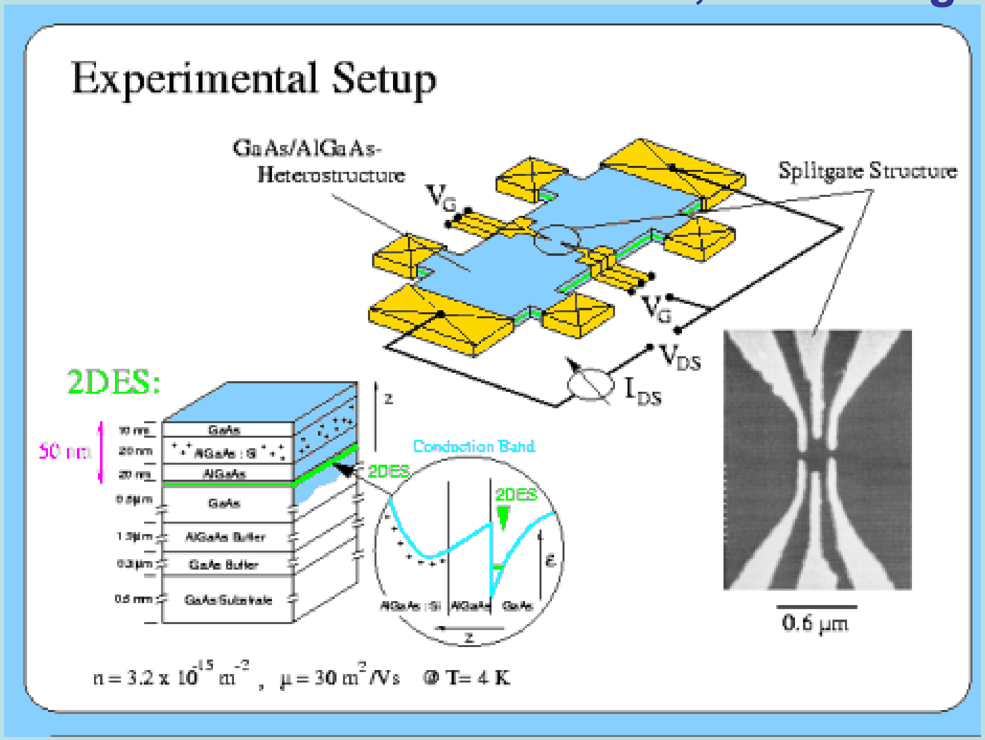
Quantum dot fabrication

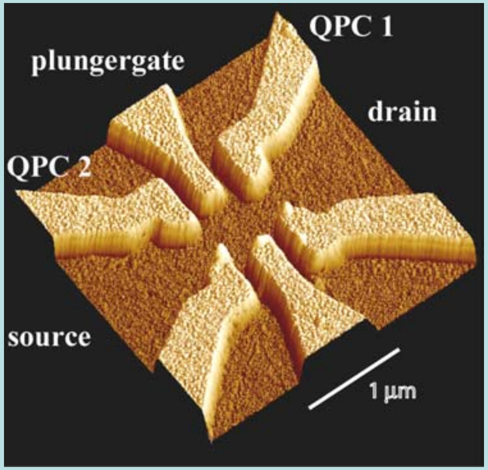
Lithographically



Kouwenhoven et al, U Delft

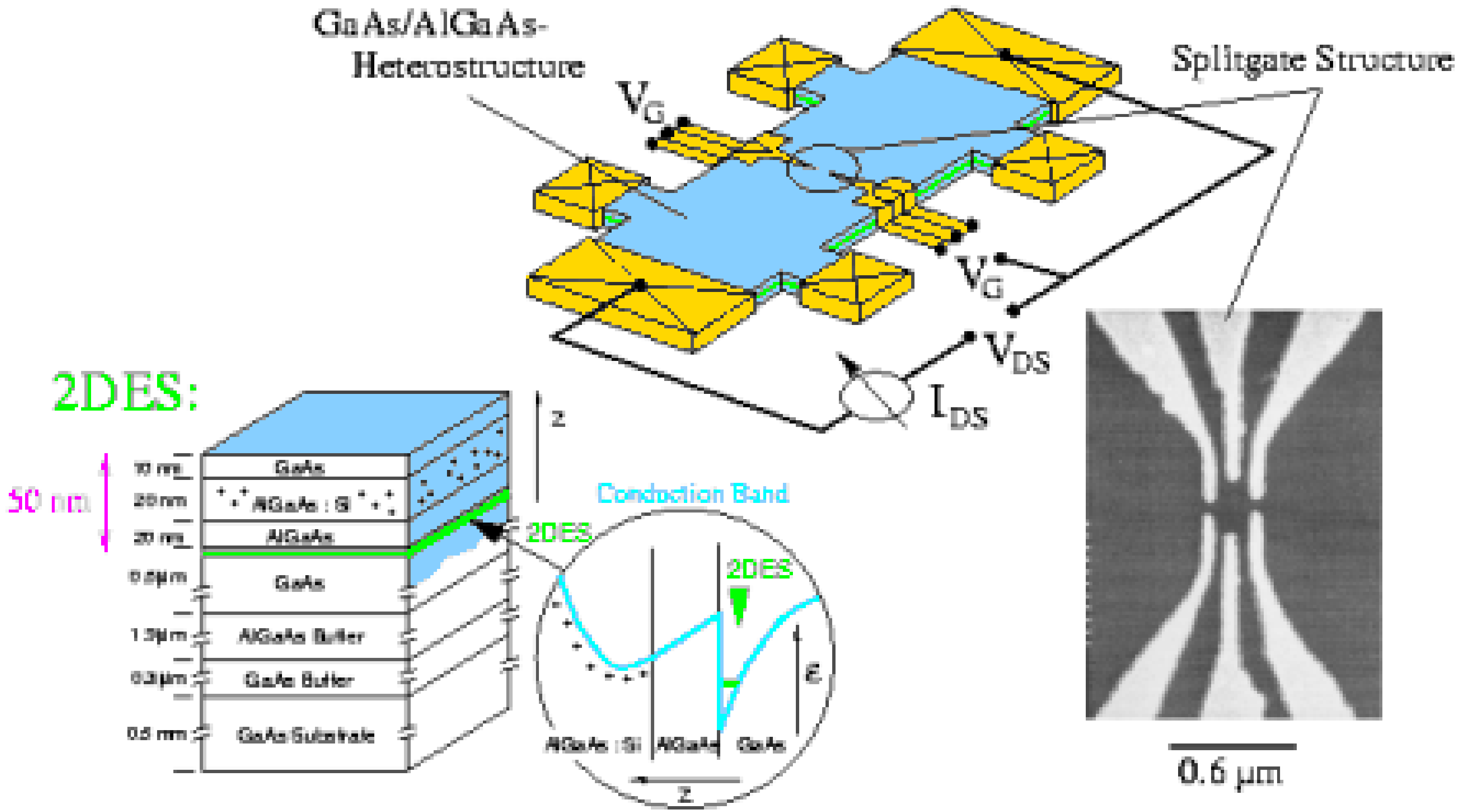
Weis et al, MPI Stuttgart





Ensslin et al, ETH Zurich

Experimental Setup

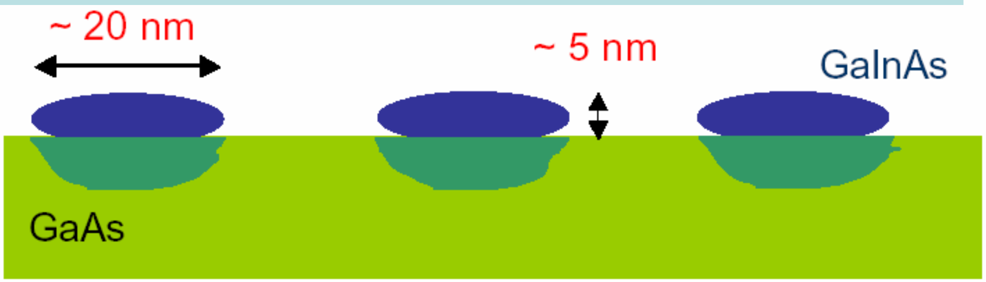


$n = 3.2 \times 10^{-15}\ m^{-2}$, $\mu = 30\ m^2/Vs$ @ T= 4 K

Quantum dot fabrication

Self-assembly

- Stranski – Krastanow islands
- MBE
- in-plane densities
  $\sim 10^{10}$ - 10^{11} cm^{-2}
- size variations < 10%
- sharp photoluminescence features, *frequency* $\propto$ size



Self-assembled quantum dots

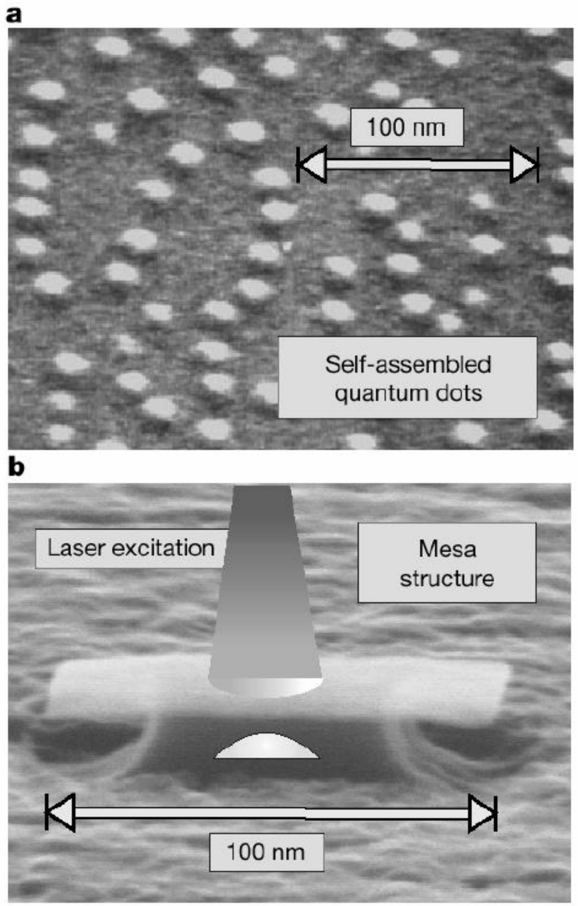


Figure 1 Scanning electron micrographs illustrating the experimental technique used for studying single self-assembled quantum dots. **a**, Scanning electron micrograph of a GaAs semiconductor layer on which $In_{0.60}Ga_{0.40}As$ self-assembled quantum dots with a density of about 10^{10} cm^{-2} have been grown by molecular beam epitaxy. To permit their microscopic observation these dots—unlike those used for spectroscopy—have not

Bayer et al, Nature, June 2000

AlAs and GaAs “*antidots*” in InAs

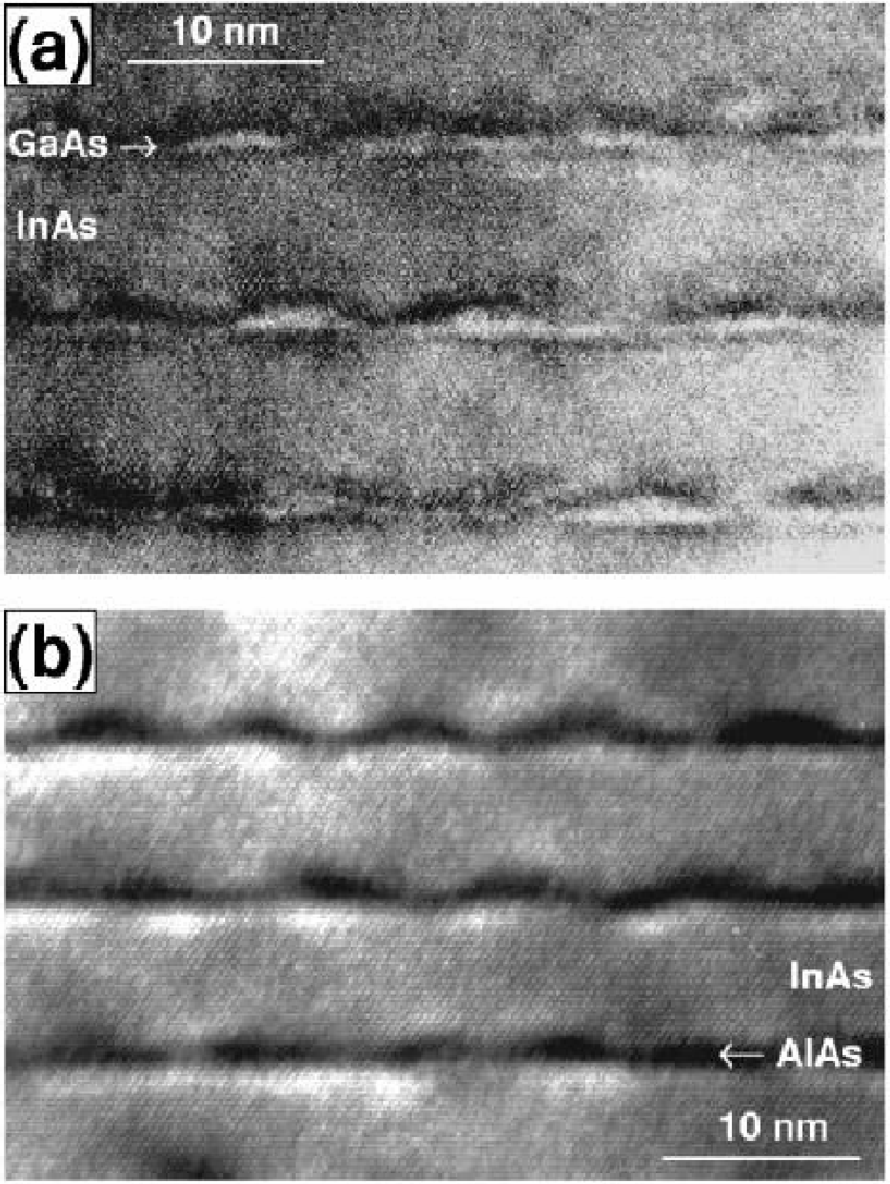
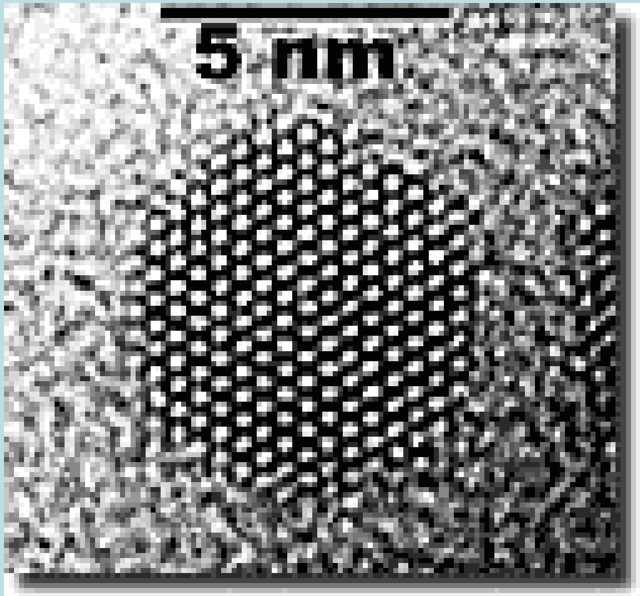


FIG. 1. High-resolution cross-sectional TEM images of samples *C* (a) and *D* (b).

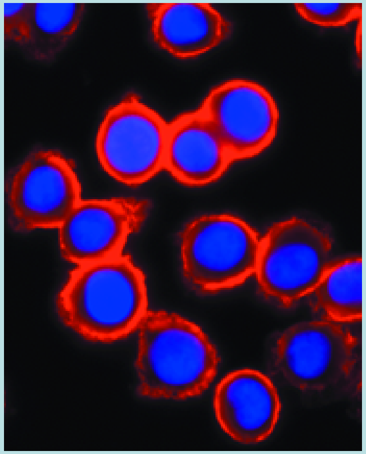
Tenne et al, PRB May 2000

Quantum dot fabrication

Colloidal dots



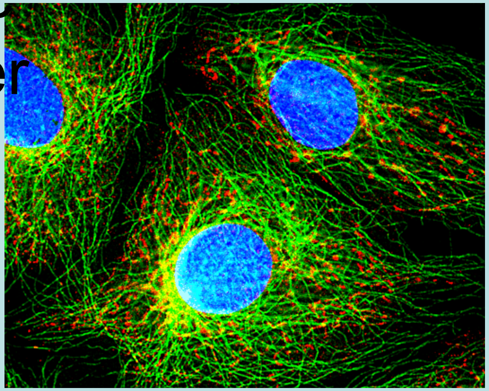
TEM by Andreas Kadavanich. Transmission electron micro-scopy shows the crystalline arrangement of atoms in a 5 nm CdSe Qdot particle.

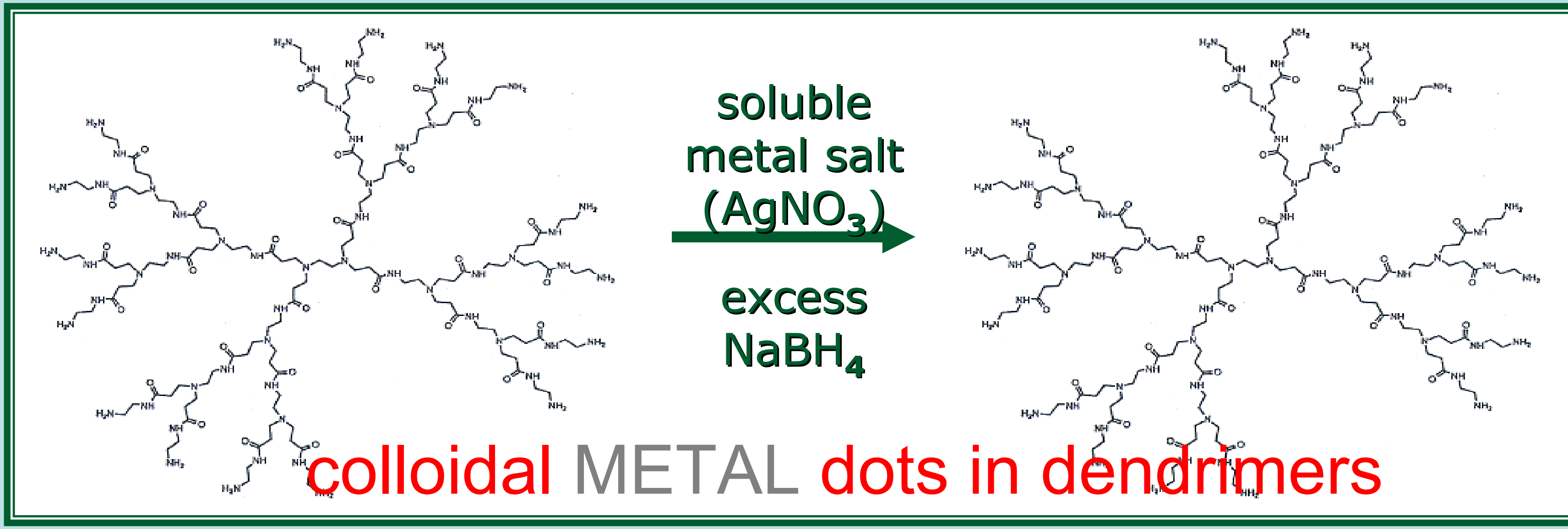


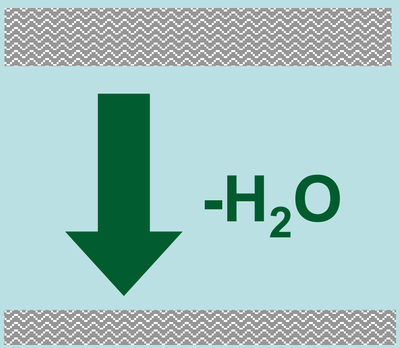
A family of Qdot particles can be made to emit a full spectrum of colors when excited with a single excitation source.

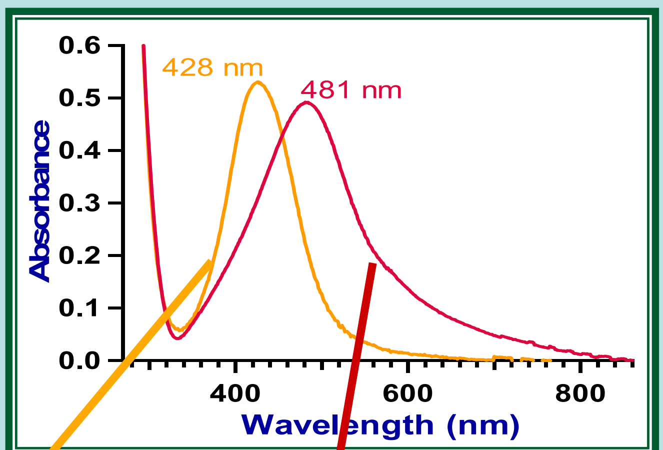
Quantum Dot Corp.

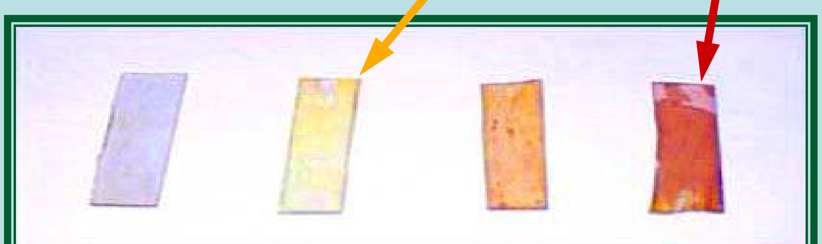
- Chemical synthesis
- CdSe, CdS, InP, etc
- Size ~ 5nm diameter
- Uses for biotags







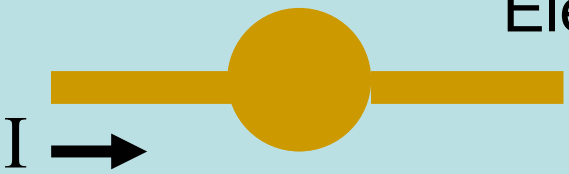




- **Au or Ag ~3-5nm dots**
- **pH or hydration changes interdot separation → aggregation changes interactions & color**

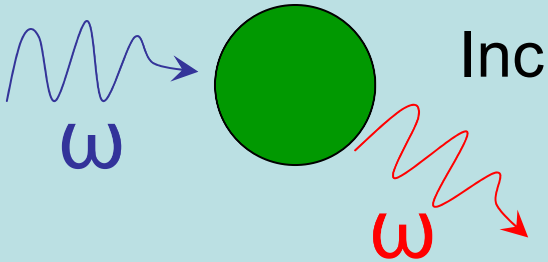
G. Van Patten, Ohio University

QD w/tunable size and e$^-$ number ~ artificial **atoms**



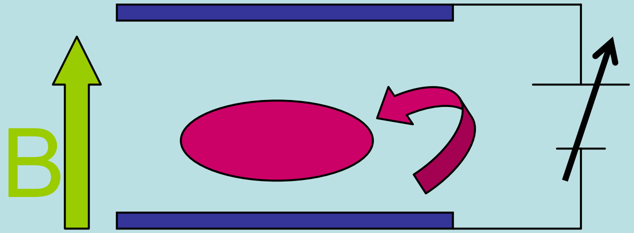
Electronic transport -

Low bias: ground state – Coulomb blockade
High bias: excited states – selection rules(!)
Rings, phases & resonances



Incident/outgoing photons -

Visible/optical: exciton
Far infrared: internal multi-electron excitations
Raman: excitons/confined phonons



Capacitance -

Ground state vs B-field
Combination w/optics & in-plane transport

Resumen/Outline

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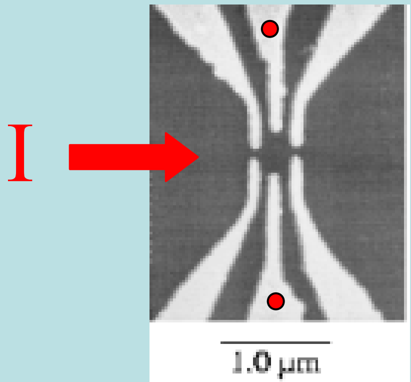
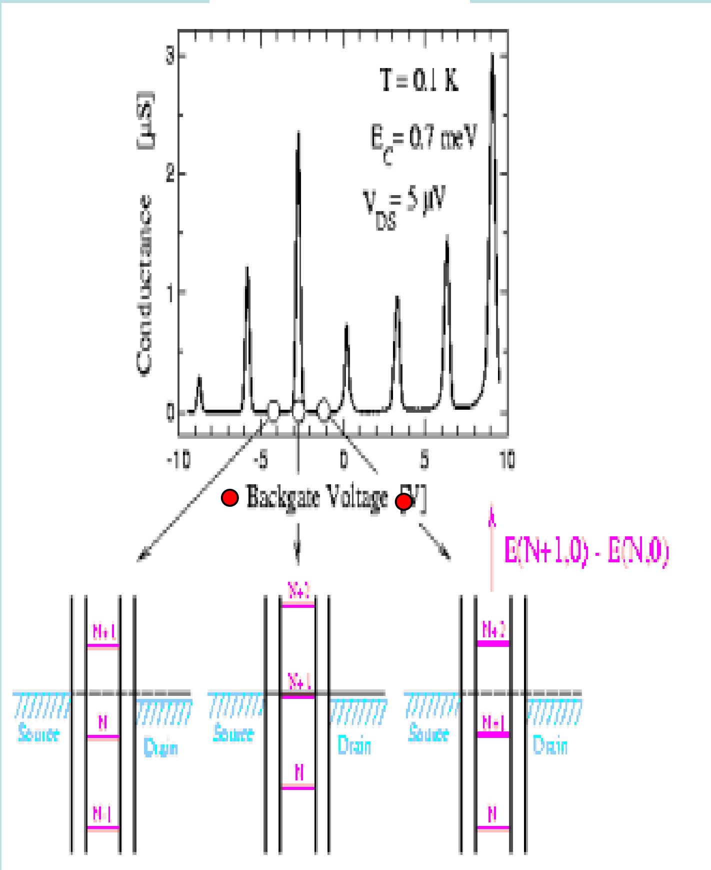
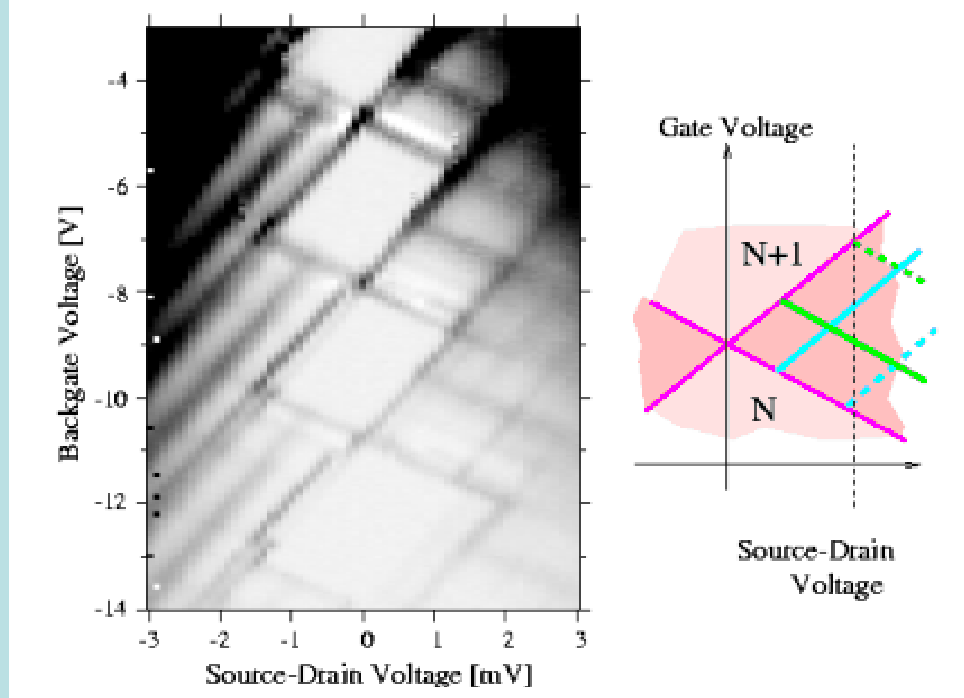
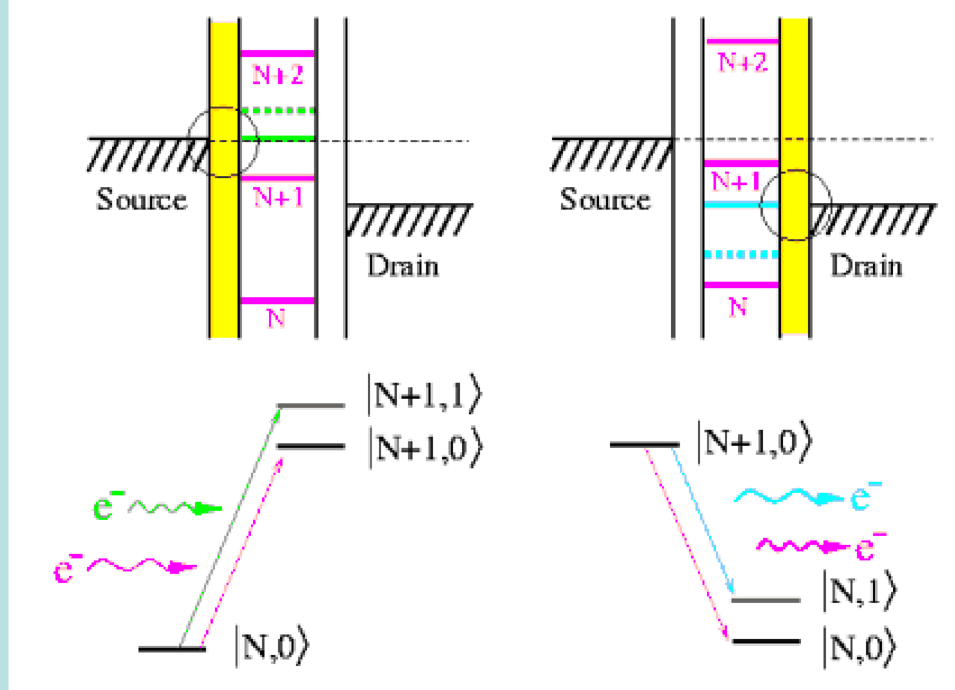
Coulomb blockade

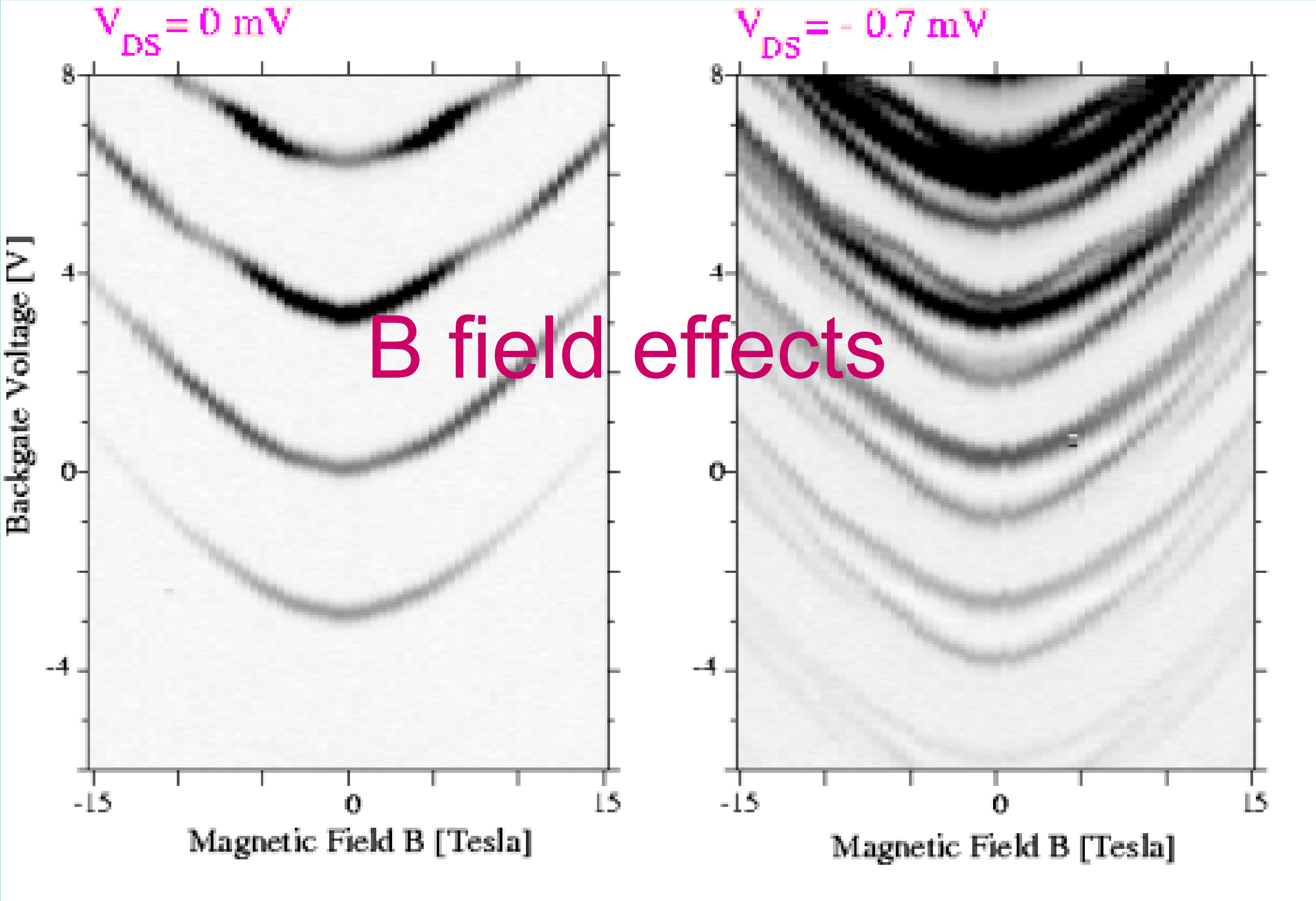








B field effects



Quantum Dot:
An Interacting N-Electron System

Hamiltonian of Quantum Dot with N Electrons:

L.D. Hallam, J. Weis, P.A. Maksym, PRB 53 (1996), 1452

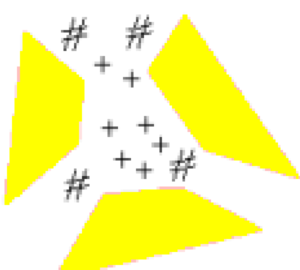
$$H = \sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m} - \sum_{i=1}^{N} e\cdot V_{ext}(\hat{r}_i) + \frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} e^2 \cdot G(\hat{r}_i, \hat{r}_j)$$

kinetic energy — confinement potential — electron-electron interaction in given environment

described by Green's function

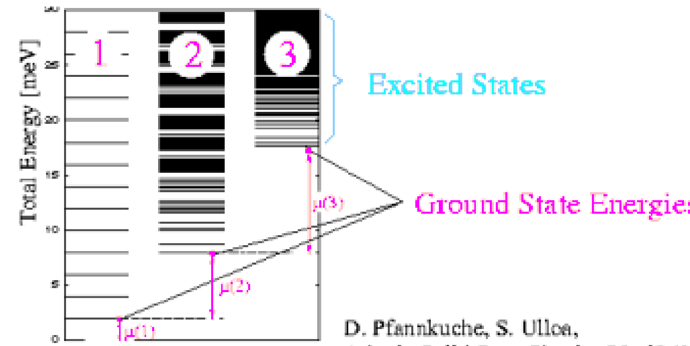
$$\left(\text{Metallic Island:} \quad -\Delta N e \sum_{g=1}^{G} \frac{C_g}{C_\Sigma} \cdot V_g + \frac{(\Delta N e)^2}{2\,C_\Sigma} \right)$$

Contributions to the External Confinement Potential:
('Binding Potential')



- electrostatic potential and arrangement of the electrodes
- ions and their image charges
- the image charges induced by the electron itself
- variation in the dielectrical properties
- difference in the conduction band minimum of of the different materials

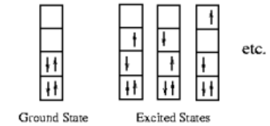
Total Energy Spectrum of One, Two and Three Electrons in a 2D Parabolic Confining Potential



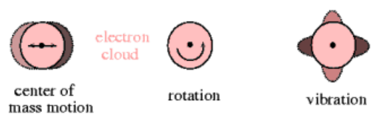
D. Pfannkuche, S. Ulloa, Adv. in Solid State Physics 35, 65 (1996)

What Kind of Excitations are Visible?

Single-Particle Excitations ?

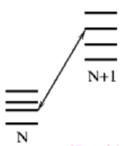


Collective Excitations ?



Selection Rules in Electrical Transport through a Quantum Dot

Spin Selection Rule:



$$S(N) \pm \frac{1}{2} = S(N+1)$$

$$S_z(N) \pm \frac{1}{2} = S_z(N+1)$$

D. Weinmann, W. Häusler, B. Kramer, Phys. Rev. Lett. 74, 984 (1995)

(Quasi-) Selection Rules by Correlations:

$$\sum_n \left| \langle N+1, j \,|\, a_n^+ \,|\, N, i \rangle \right|^2 < 1$$

D. Pfannkuche and S. Ulloa, Phys. Rev. Lett. 74, 1194 (1995)

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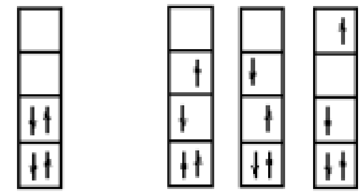
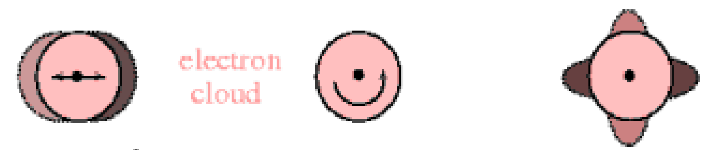


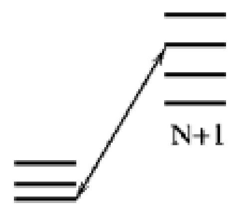
Ground State   Excited States   etc.

Collective Excitations ?



center of mass motion   rotation   vibration

Selection Rules in Electrical Transport through a Quantum Dot



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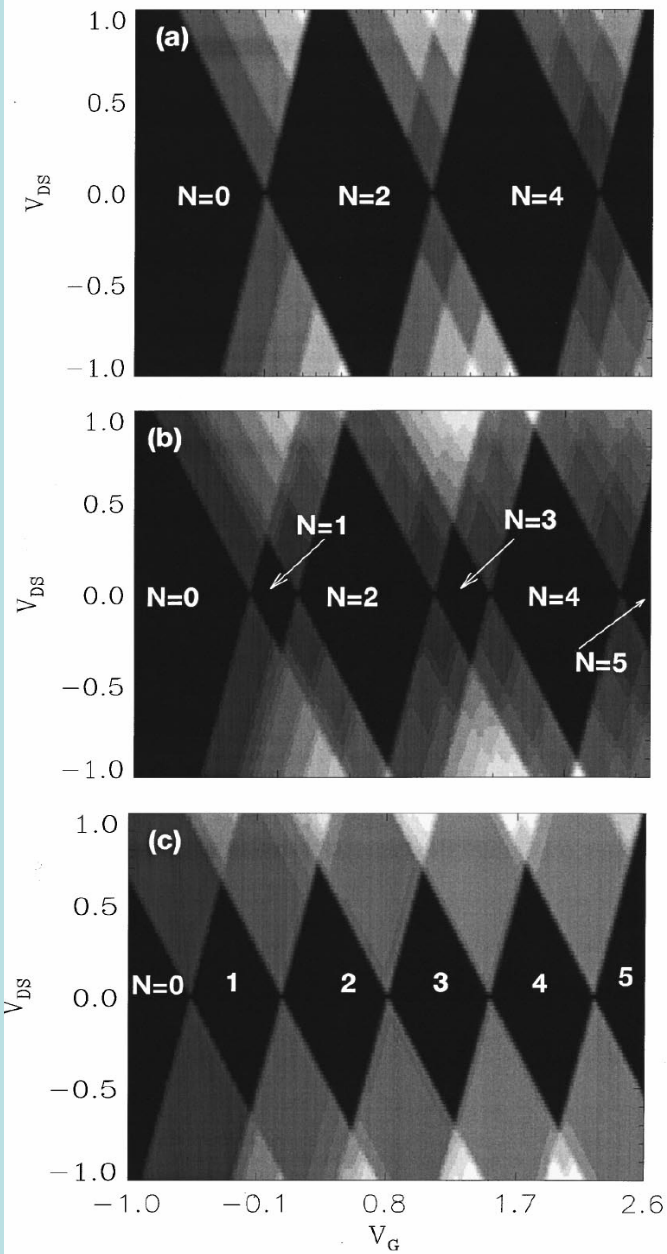
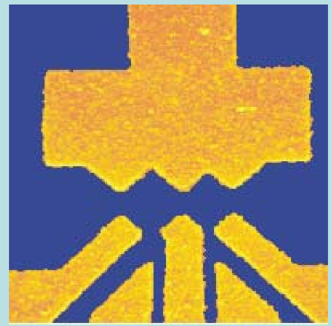
Phys. Rev. Lett. 74, 1194 (1995)

QD molecules

FIG. 5. Current as a function of gate voltage V_G and source-drain voltage V_{DS}, for different values of interdot tunneling: (a) $t=0.01$, (b) $t=0.1$, and (c) $t=0.2$. Symmetric DDS case.

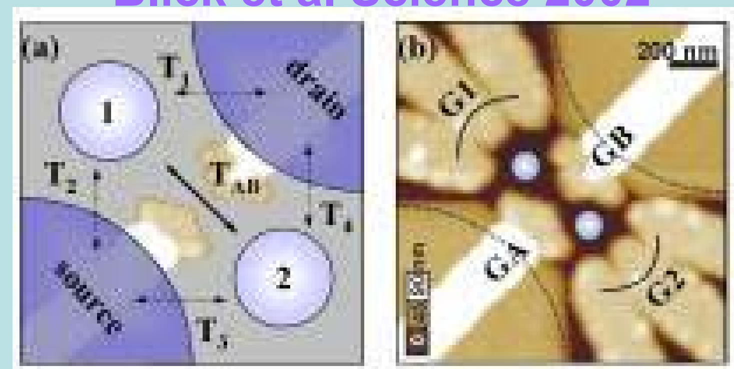


Ramirez et al, PRB 1999

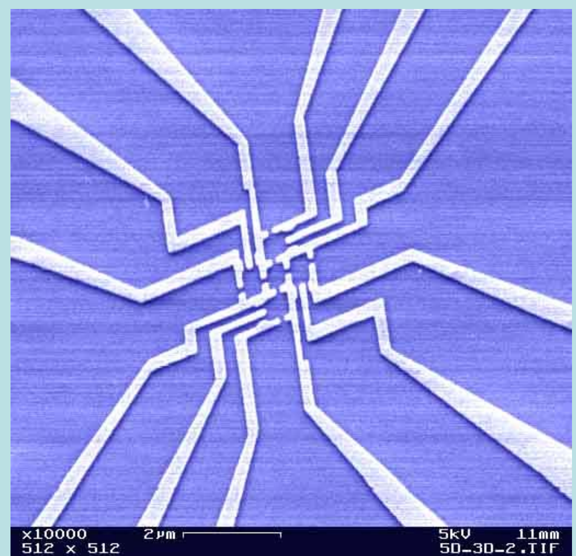


in series

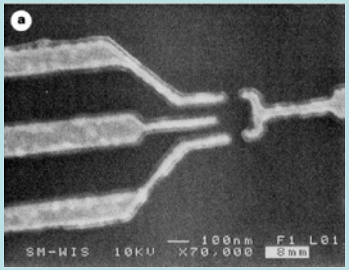
Blick et al Science 2002



in parallel

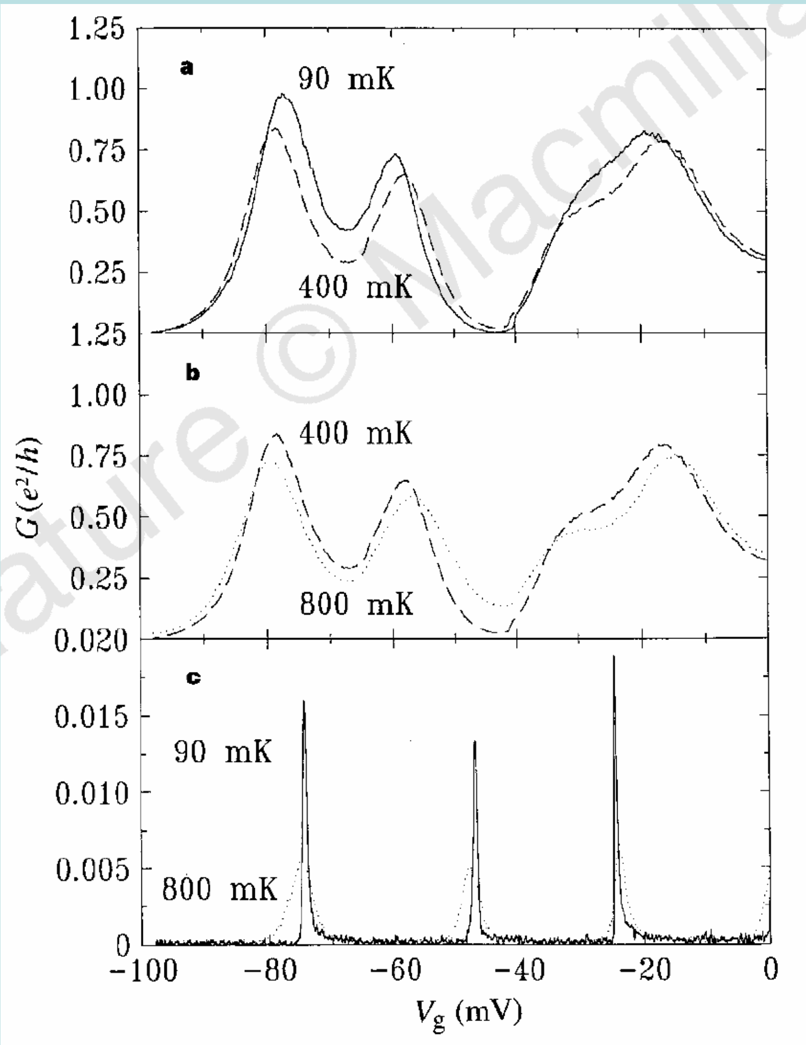


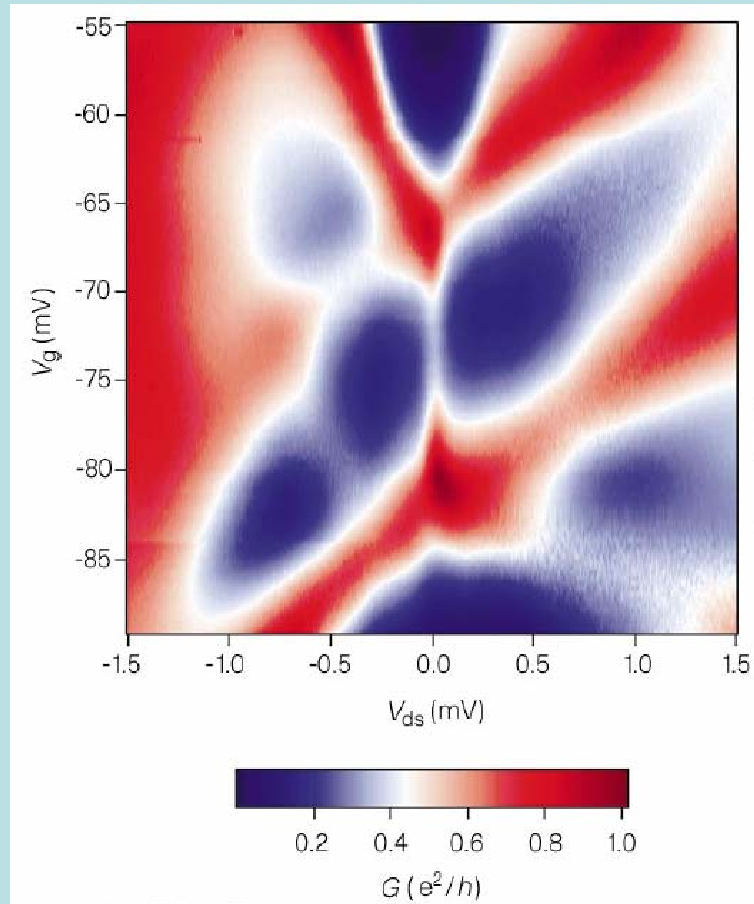
future?
multidots
for quantum computing



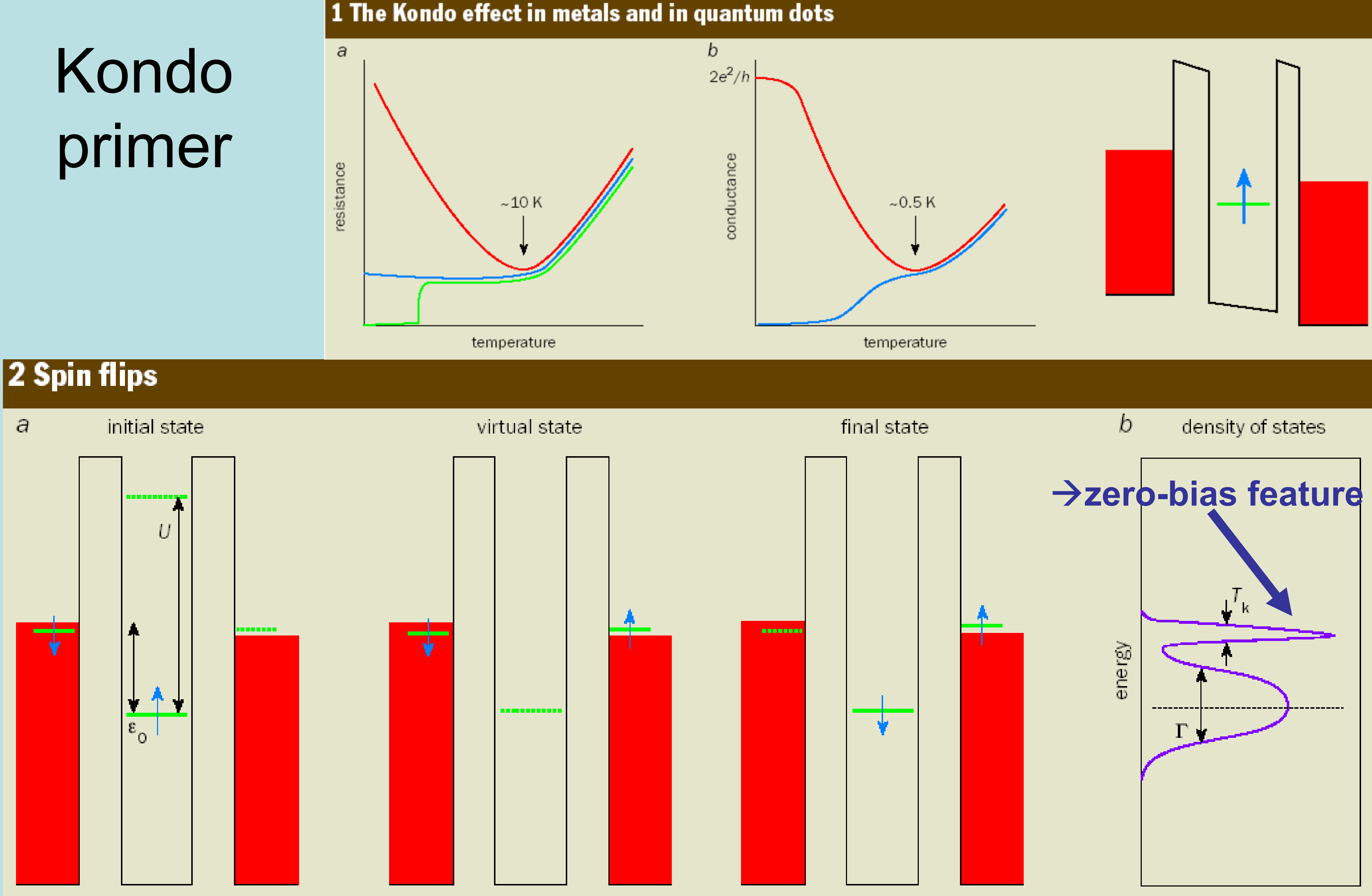
Kondo effect

Goldhaber-Gordon et al Nature 1998



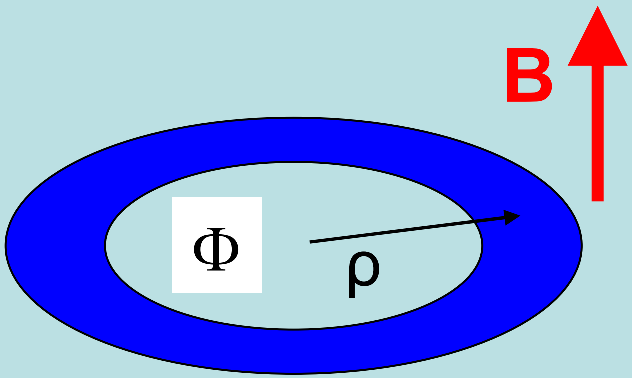


Kondo primer



Kouwenhoven & Glazman, Phys World 2001

Aharonov-Bohm effect



$$H = \frac{p^2}{2m} \to \frac{1}{2m}\left(p - \frac{e}{c}A\right)^2$$

$$B = \nabla \times A; \quad A = B\rho\hat{\varphi}$$

$$\Psi_N^{(e)}(\varphi_e) = \frac{1}{\sqrt{2\pi}} e^{iN\varphi_e}$$

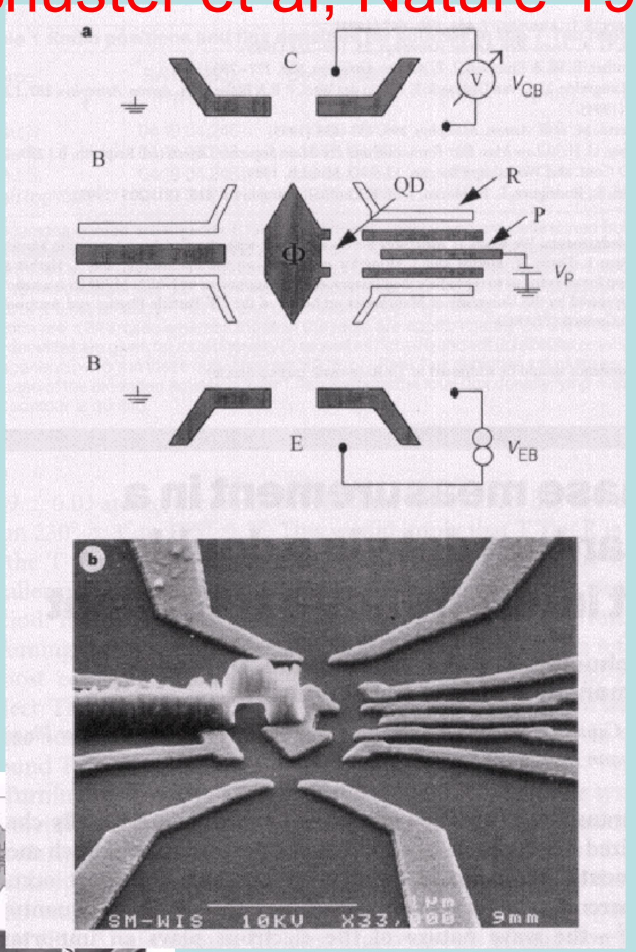
$$E_N^{(e)} = \frac{\hbar^2}{2m_e\rho^2}\left(N - \frac{\Phi}{\Phi_0}\right)^2$$

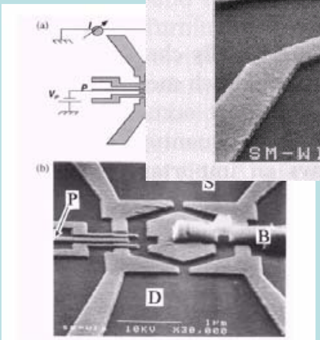
Phases, dots, “rings” and resonances

1. Phase in experiments → AB interferometer
2. Phase of a resonance (single particle)
3. CB in QD ~ SP resonance?
4. What’s the Fano effect / lineshape?
5. Fano as probe of coherence in QD?
6. QD in Kondo regime → phases in expts?
7. Fano+Kondo to probe coherence in QDK?

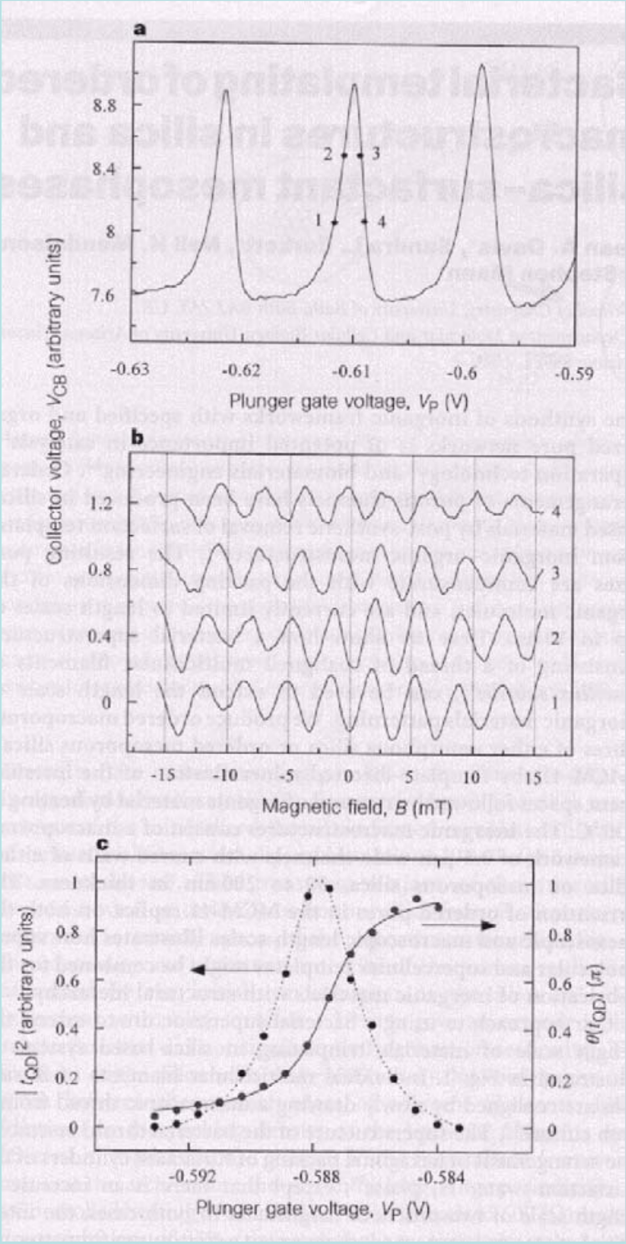
Phase in experiments → AB interferometer

Schuster et al, Nature 1997





Yacoby/Heiblum PRL 1995



~ CB peaks

AB oscill's

$|t_{QD}|\, e^{i\theta}$

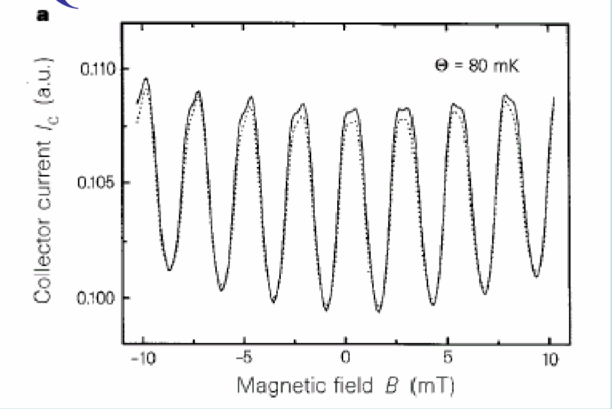
weaker AB signal when QPC is on

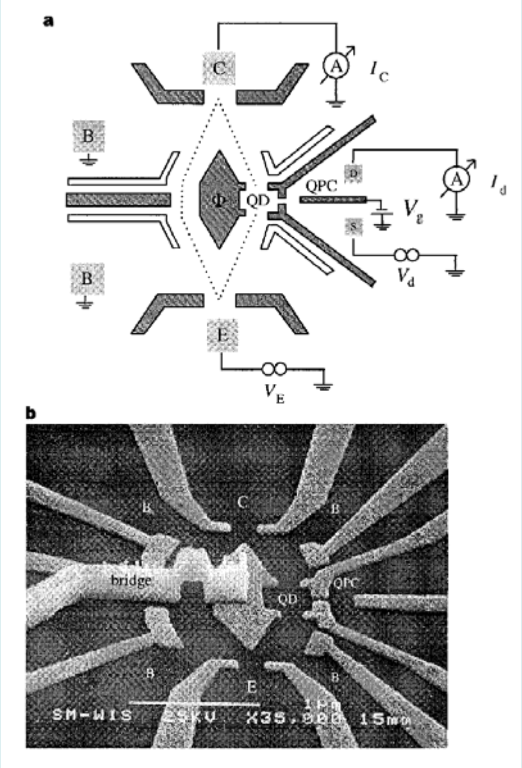
Dephasing in electron interference by a 'which-path' detector

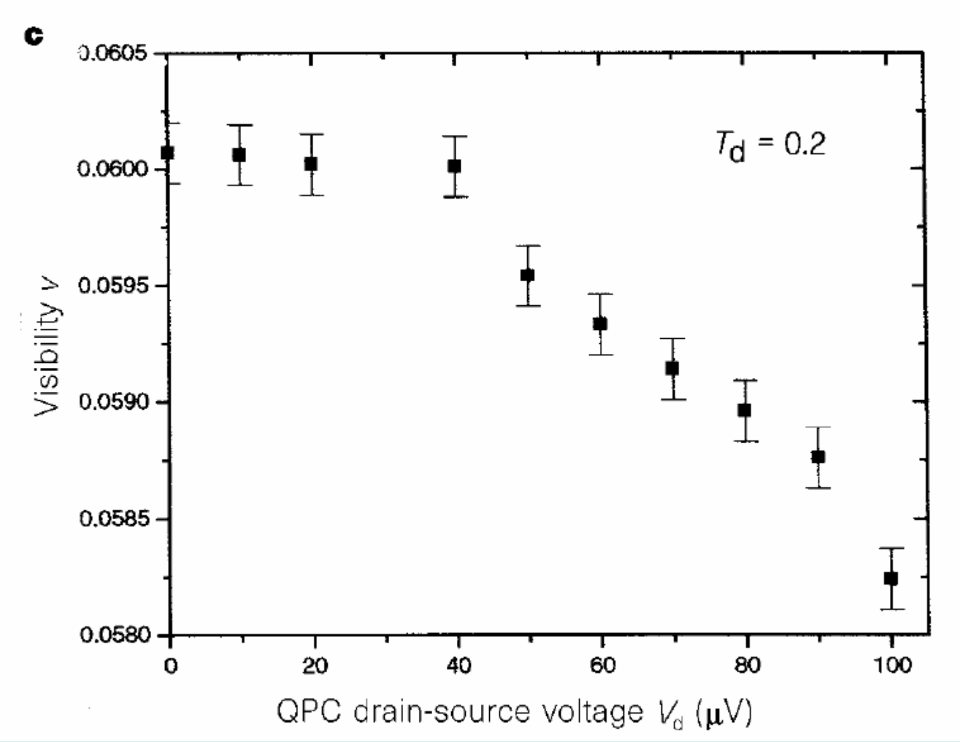
E. Buks, R. Schuster, M. Heiblum, D. Mahalu & V. Umansky

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

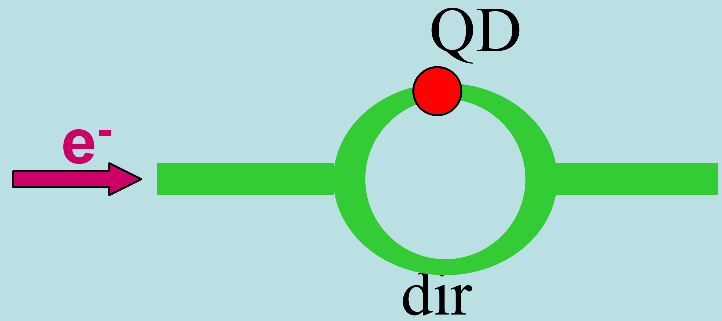
Nature 1998







AB interferometer: “phase measurer”



$$G \sim |t_{CE}|^2 = |t_{QD} + e^{i\Delta\varphi} t_{dir}|^2$$

$$\Delta\varphi = 2\pi\Phi/\Phi_0$$

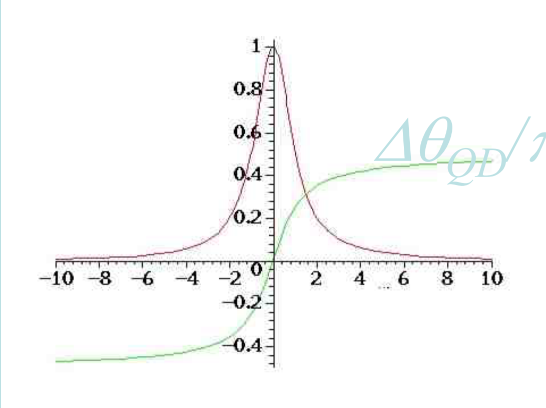
$$G \sim |t_{QD}|^2 + |t_{dir}|^2 + 2|t_{QD}||t_{dir}|\cos(\theta_{QD} - \theta_{dir} - \Delta\varphi)$$

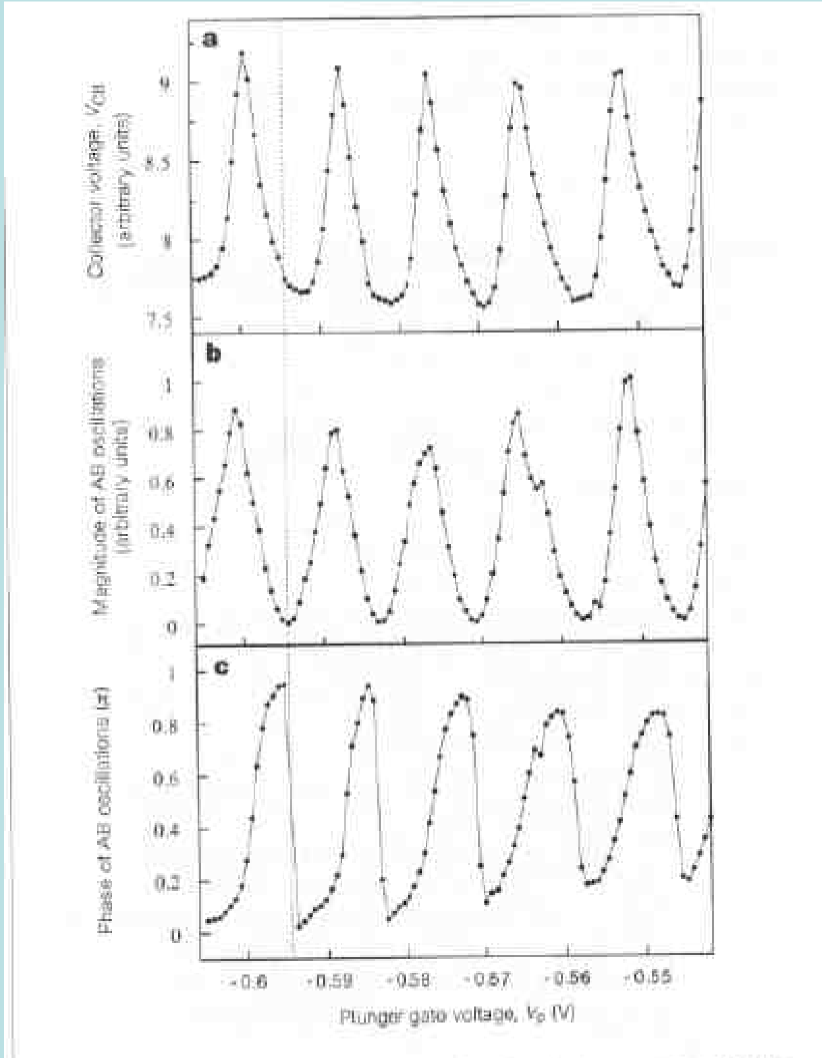
If θ_{dir} is indep of V_{gate} → θ_{QD} can be measured

A “pure” resonance has the Breit-Wigner form:

$$t_{QD} = C\frac{i\Gamma/2}{E - E_n + i\Gamma/2} = |t_{QD}|e^{i\theta_{QD}}$$

$$\theta_{QD} = \theta_C + \tan^{-1}\frac{2(E - E_n)}{\Gamma}$$





BW "SP" resonances OK….
→ coherent propagation through QD
→ CB charging ~ not important

but….

sequence of BW resonances would be expected to accumulate while phase "resets" to 0 as $t_{QD} \sim 0$!!?

overlapping resonances … BW??

e-e interactions?

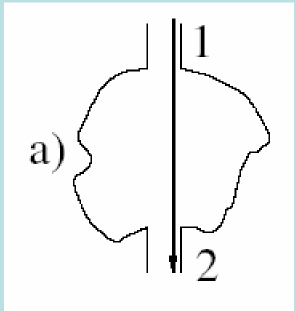
G(B)=G(-B) not valid here
(open device → $\Delta\theta \neq 0,\pi$)

Why? … N theory papers …
nice disc:
Aharony et al cond-mat/0205268

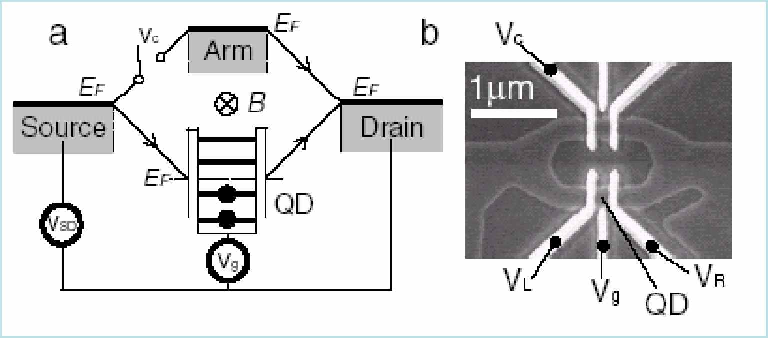
Fano effect / lineshape

Ugo Fano, PR 1961

interference of discrete “autoionized” state with a continuum → asymmetric peaks in atomic excitation spectra



In QD;
expt: Göres PRB 2000
th: Clerk PRL 2001 …



In AB interf + QD;
expt: Kobayashi PRL 2002

Fano …

$$\underset{\varphi}{\text{—}} \quad \& \quad \{\psi_E\} \quad \xrightarrow[V_E]{\text{mix}} \quad \Psi_E = a\varphi + \sum_{E'} b_{E'}\psi_{E'}$$

$$a = \frac{1}{\pi V_E}\sin\Delta(E); \qquad b_{E'} = \frac{V_{E'}}{\pi V_E}\frac{\sin\Delta(E)}{E-E'} - \cos\Delta(E)\,\delta(E-E')$$

$$\Delta(E) = -\tan^{-1}\frac{\pi|V_E|^2}{E-E_\varphi-F(E)}; \qquad F(E) = \sum_{E'}\frac{|V_{E'}|^2}{E-E'} = \text{shift of resonance due to continuum}$$

"excitation" of Ψ_E via an operator T from an initial state I yields:

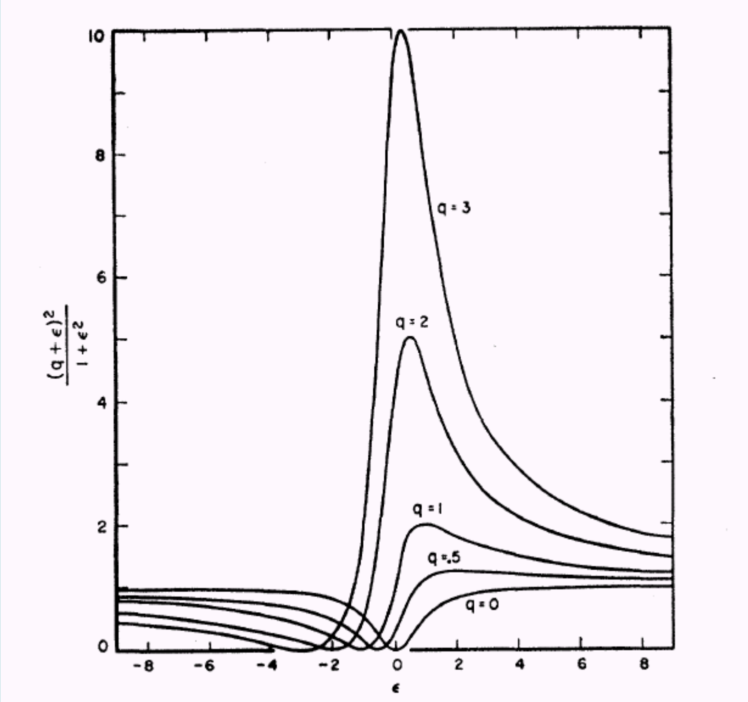
$$\langle\Psi_E|T|I\rangle = \frac{\sin\Delta}{\pi V_E^*}\langle\Phi|T|I\rangle - \langle\psi_E|T|I\rangle\cos\Delta$$

$$\Phi = \varphi + \sum_{E'}\frac{V_{E'}\psi_{E'}}{E-E'} \rightarrow \text{modified state due to mix}$$

Fano lineshape

$$q = \frac{\langle \Phi | T | I \rangle}{\pi V_E^* \langle \psi_E | T | I \rangle}; \qquad \varepsilon = -\cot\Delta = \frac{E - E_\varphi - F}{\Gamma/2}; \qquad \Gamma = 2\pi |V_E|^2$$

$$\frac{|\langle \Psi_E | T | I \rangle|^2}{|\langle \psi_E | T | I \rangle|^2} = \frac{(q+\varepsilon)^2}{1+\varepsilon^2} = \text{ transition prob via "autoionized" state}$$



Notice prob can vanish
due to interference

Is $G \sim |t_{QD} + t_{dir}|^2$ a Fano line?

$$t_{dir} = e^{i\beta}\sqrt{G_d}; \quad t_{QD} = \frac{z}{\varepsilon + i}$$

$$G = G_d \frac{|\varepsilon + q|^2}{\varepsilon^2 + 1}$$

with

$$q = i + \frac{z}{\sqrt{G_d}} e^{-i\beta}$$

Clerk PRL 2001

Fano as probe (proof?) of coherence in QD (CB)

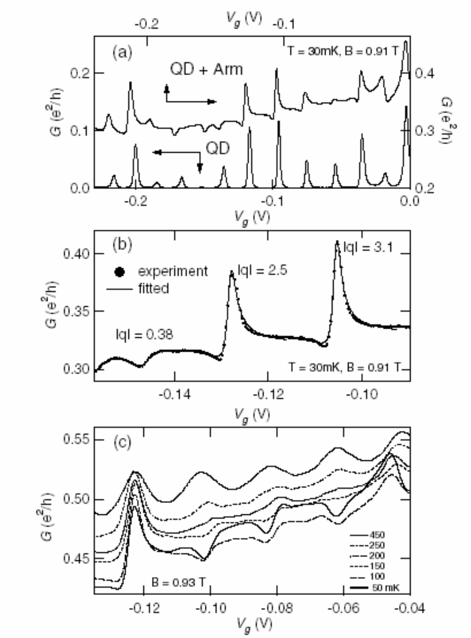


FIG. 2. (a) Typical Coulomb oscillation at $V_C = -0.12$ V with the arm pinched off, and asymmetric Coulomb oscillation

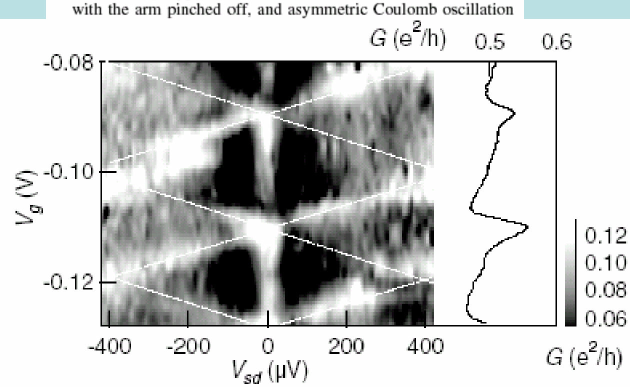
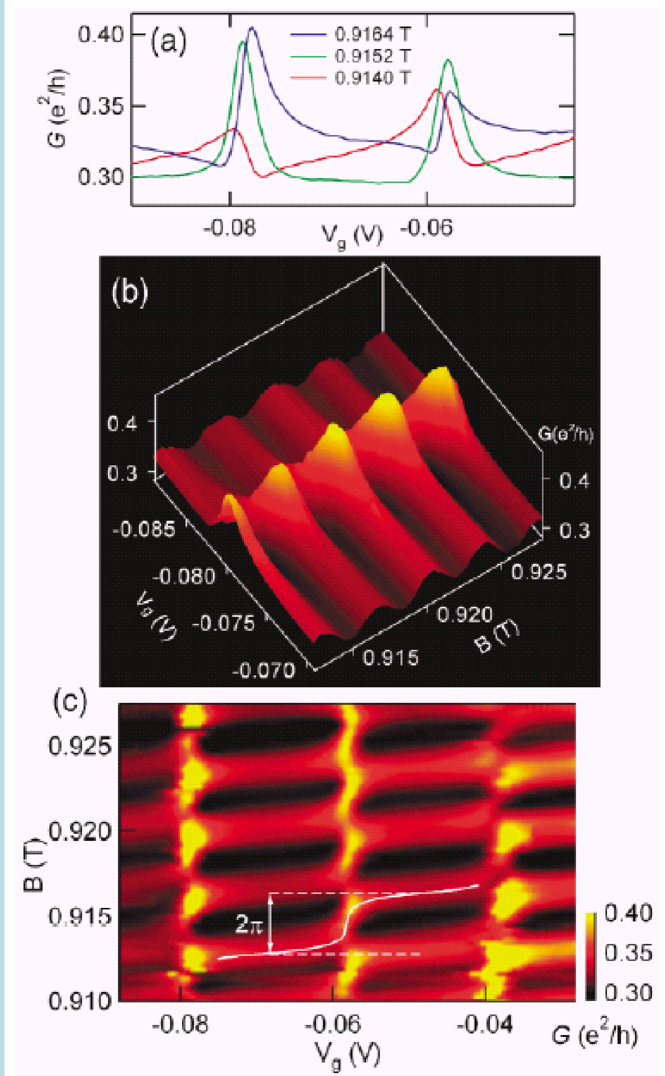


FIG. 3. Differential conductance obtained as a function of V_g at $T = 30$ mK and $B = 0.92$ T. The corresponding Fano line shape is also shown in the right panel. The zero-bias conductance peak exists in the CB region with a Coulomb diamond superimposed. The edge of the CB region is emphasized with white dashed lines. Incoherent contribution from the differential conductance of the upper arm, which shows slight non-Ohmic behavior at finite V_{sd}, has been subtracted from the data.



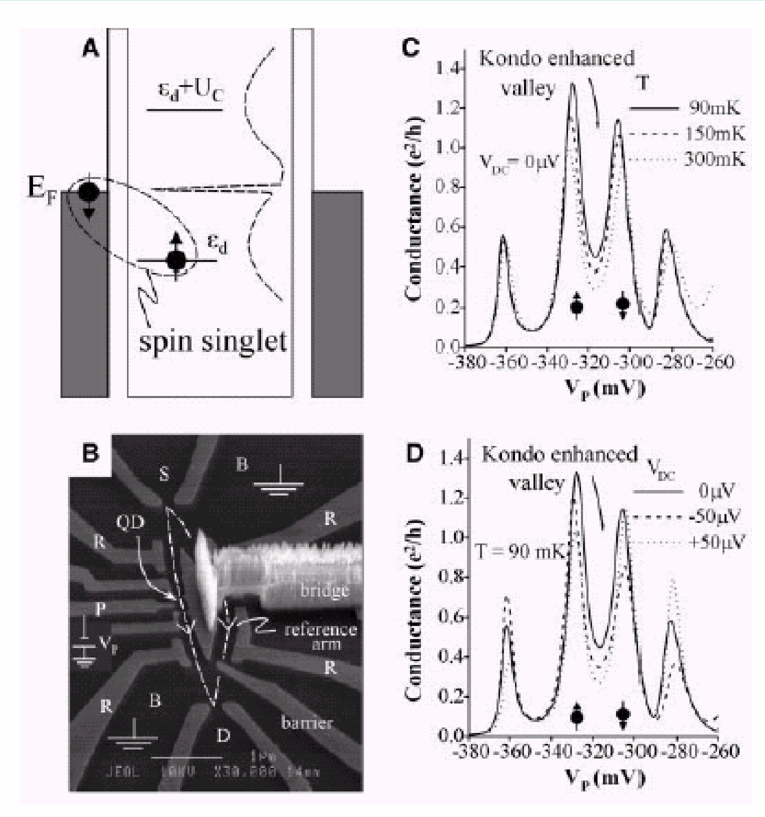
Kobayashi PRL 2002

direct paths “through” QD?

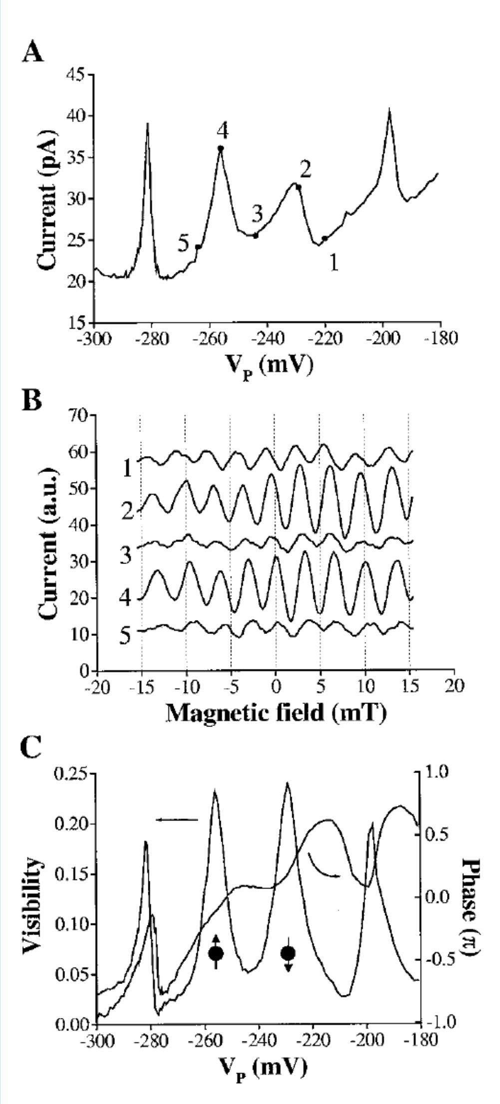
how is QD “intrinsic” width affected by ring?

decoherence?
1 vs N passes?

QD in Kondo regime → phases in expts?



Ji, Heiblum, Science 2000

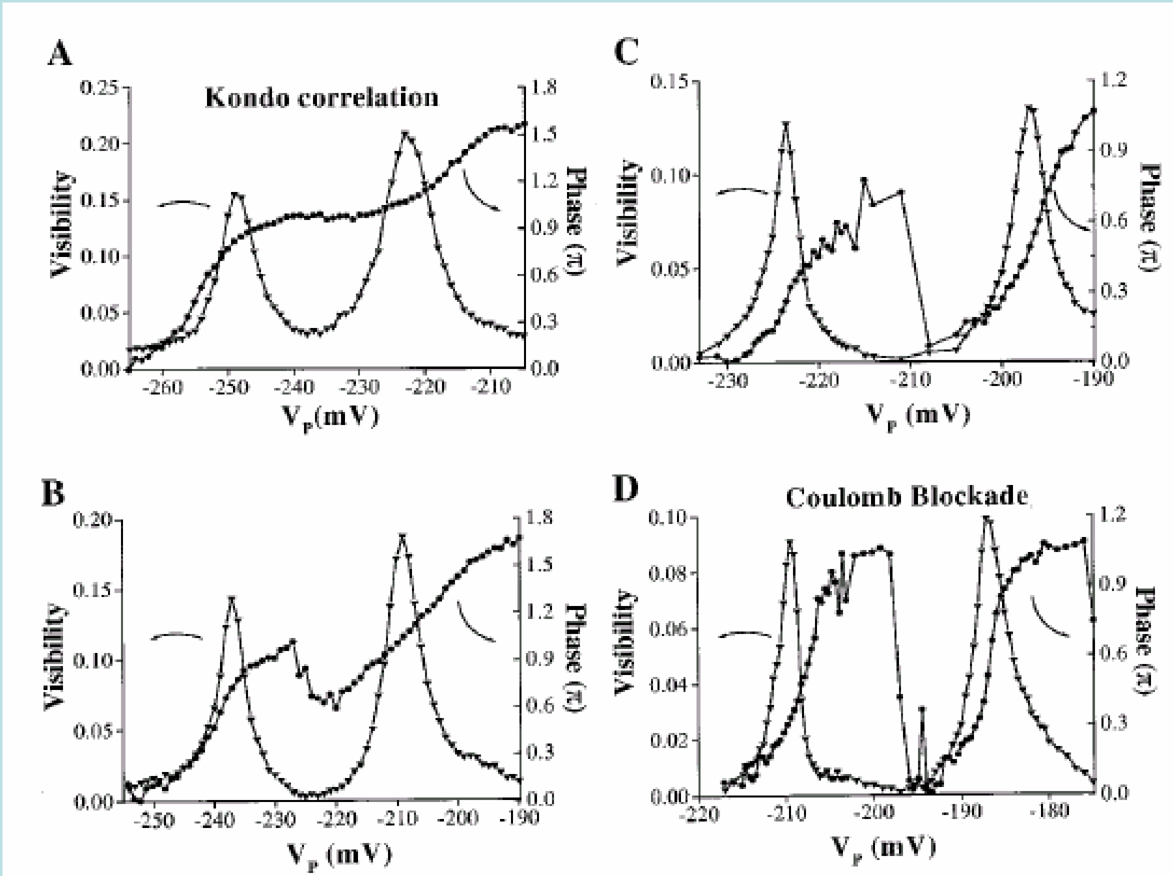


peaks deform

AB osc's clear

phase lapses in CB
but
plateaus in K valley
~ π

phases in Kondo regime



phase evolves from plateaus ~ π

to

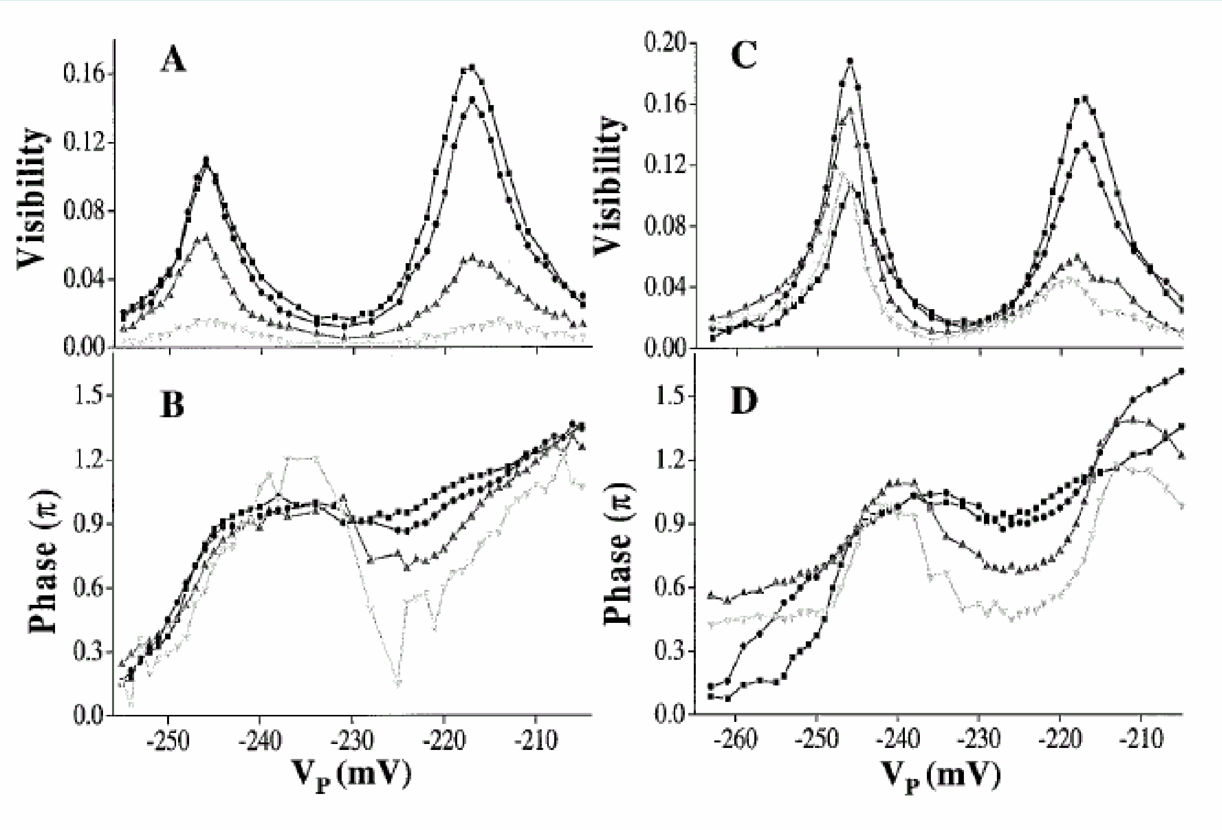
lapses to 0 as Γ decreases

th: Gerland PRL 2000

K res phase shift ~ π/2

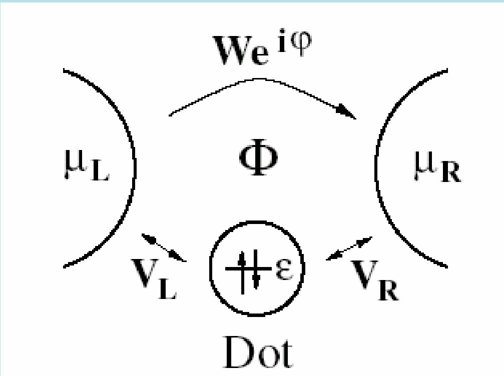
(NRG)

phases in Kondo regime



w/temperature w/DC bias

Fano+Kondo to probe coherence in QDK?



Hofstetter PRL 2001
(~Bulka PRL 2001)

direct ~ W

$$T(\omega) = T_\mathrm{b} + \sqrt{\alpha T_\mathrm{b} R_\mathrm{b}}\cos\varphi\,\bar{\Gamma}\,\mathrm{Re}G^\mathrm{r}(\omega) - \tfrac{1}{2}[\alpha(1 - T_\mathrm{b}\cos^2\varphi) - T_\mathrm{b}]\bar{\Gamma}\,\mathrm{Im}G^\mathrm{r}(\omega)$$

AB phase/flux

and for $\mathrm{G}^\mathrm{r}(\omega) = [\omega - \varepsilon_\mathrm{dot} - \Sigma(\omega)]^{-1}$

$$\mathrm{e} = 2[\varepsilon_\mathrm{dot} + \Re\Sigma(0)]/\bar{\Gamma}$$

$$\mathrm{g} = \mathrm{T_b}\frac{(\mathrm{e}+\mathrm{q})^2}{\mathrm{e}^2+1} + \alpha\frac{\sin^2\varphi}{\mathrm{e}^2+1}$$ "generalized Fano form"

$$\alpha = 4\Gamma_\mathrm{R}\Gamma_\mathrm{L}/\bar{\Gamma}^2; \qquad \mathrm{q} = -\sqrt{\alpha(1-\mathrm{T_b})/\mathrm{T_b}}\cos\varphi$$

references on phases/Kondo/resonances

- Yacoby, PRL **74**, 4047 (1995)
- Schuster, Nature **385**, 417 (1997)
- Aharony, cond-mat/0205268
- Fano, PR **124**, 1866 (1961)
- Clerk, PRL **86**, 4636 (2001)
- Kobayashi, PRL **88**, 256806 (2002)
- Ji, Science **290**, 779 (2000)
- Gerland, PRL **84**, 3710 (2000)
- Bulka, PRL **86**, 5128 (2001)
- Hofstetter, PRL **87**, 156803 (2001)

IFUAP–BUAP – Minicurso en Nanoestructuras
Tarea # 1 — Entrega: 22 Julio 2003

1. **[40pts]** (a) Calcule el espectro para una partícula de masa m confinada en una caja "rectangular" en 3D y 2D de radio R. Suponga que la caja tiene paredes infinitamente duras.
(b) Describa como varía la energía de confinamiento con respecto a R.
(c) Escriba la forma completa de los eigenstados en 3D/2D.
(d) ¿Cuál es el valor de la energía (en eV) para los dos primeros estados en 2/3D si la masa de la partícula es $0.067m_0$ (con m_0 la masa del electrón libre), y el radio es $R = 5, 50, 100$nm?
Hint: funciones de Bessel.

2. **[60pts]** (a) Estime el efecto de la interacción de Coulomb entre electrones en un punto cuántico de radio R, utilizando $U = e^2/R\epsilon$, donde ϵ es la constante dieléctrica del material (≈ 12 para materials típicos), si el radio del punto es 5 o 100nm. Compare esta estimación con la diferencia entre los dos primeros niveles en un punto como se modeló en 1 arriba, $\Delta = E_2 - E_1$. ¿Para qué radio R son estas dos cantidades iguales ($U = \Delta$)? Haga la estimación tanto en 2D como en 3D.
(b) Use teoría de perturbaciones para hacer la estimación más confiable:
$$\langle \Psi_n(1)\Psi_m(2)|V(1-2)|\Psi_k(1)\Psi_l(2)\rangle,$$
en donde $V(1-2) = e^2/\epsilon|r_1 - r_2|$ es la interacción de Coulomb entre dos partículas, y las varias Ψ_j son las funciones de onda (modeladas/escritas en 1(c) arriba). Use cualquier método de integración, numérica incluso, para evaluar (o al menos *estimar*) la integral en el caso que los dos electrónes están en el estado base (y diferente spin, por supuesto) en un punto en 3D. Compare con (a) y comente.
Hint (aunque no esencial):
$$\frac{1}{|r|} = \int \frac{4\pi}{q^2} e^{-iq\cdot r} d^3q$$

IFUAP-BUAP Mini-curso en Nanoestructuras

Parte II. – Toma 2

Sergio E. Ulloa – Ohio University

Department of Physics and Astronomy, **CMSS**,

and **Nanoscale and Quantum Phenomena Institute**

Ohio University, Athens, OH

Supported by
US DOE & NSF NIRT

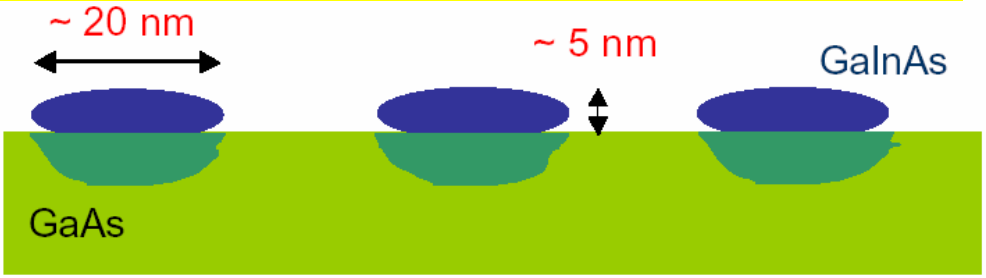
Resumen/Outline

- Quantum dots – confinement vs interactions
  - How to make / study them
- Coulomb blockade & assorted IV characteristics
- Optical effects – excitons: selection rules, field effects
- Transport in complex molecules: the case of DNA

Quantum dot fabrication

Self-assembly

- Stranski – Krastanow islands
- MBE
- in-plane densities
  $\sim 10^{10} - 10^{11}$ cm^{-2}
- size variations < 10%
- sharp photoluminescence features, *frequency* $\propto$ size



Self-assembled quantum dots

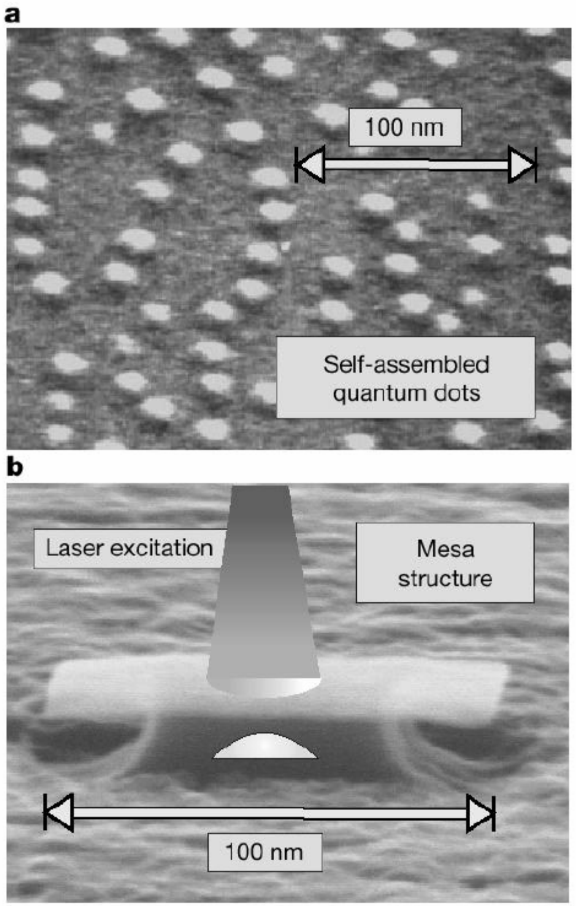


Figure 1 Scanning electron micrographs illustrating the experimental technique used for studying single self-assembled quantum dots. **a**, Scanning electron micrograph of a GaAs semiconductor layer on which $In_{0.60}Ga_{0.40}As$ self-assembled quantum dots with a density of about 10^{10} cm^{-2} have been grown by molecular beam epitaxy. To permit their microscopic observation these dots—unlike those used for spectroscopy—have not

Bayer et al, Nature, June 2000

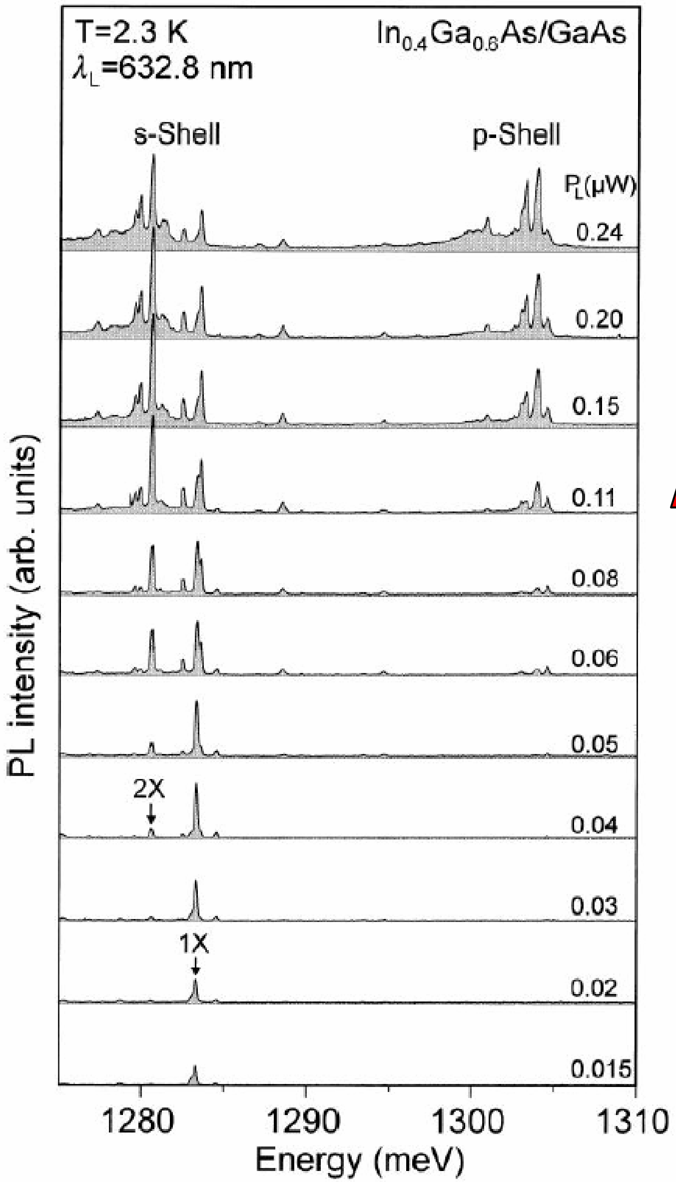


Fig. 1. Power dependent PL spectra from a single isolated quantum dot at zero magnetic field. Contributions from the s-shell and p-shell can be clearly distinguished. In the spectral region of the s-shell, the single exciton (1X) and biexciton lines (2X) are labelled.

Photoluminescence in SINGLE QD

- Excitons
- *Excited* excitons (s- and p-shell)
- Biexcitons, X^-, X^{--}, etc.

laser power

Findeis et al, Sol. State Comm, 2000

Excitons and dot shapes

- **Dot asymmetries reflected in exciton properties:**
  - **Binding energy vs dot size**
  - **Oscillator strength / optical response**
  - **Influence of magnetic field**

- **Raman differential cross section and intensity ---experiments and phonon mode confinement:**
  - **Selection rules**
  - **Carrier masses**
  - **Scanning experiments**

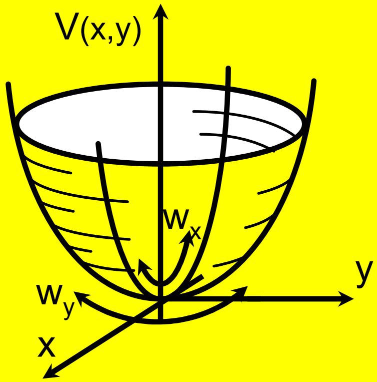
see part I

Trallero & Ulloa PRB 1999

- **Quantum rings: excitonic Aharonov-Bohm effect for neutral/polarizable entity**

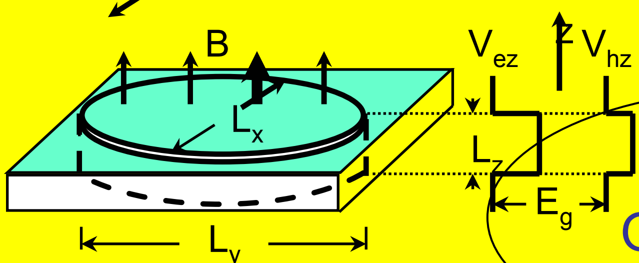
Simple geometry: Parabolic confinement

Song& SU; Pereyra & SU PRB 2000



$$H = H_e + H_h + H_{eh}$$

$$H_j = \frac{1}{2m_j}\left(\mathrm{p}_j \pm \frac{e}{c}\mathrm{A}_j\right)^2 + \frac{1}{2}m_j\left(\omega_x^2 x_j^2 + \omega_y^2 y_j^2\right) + V(z_j)$$



$$H_{eh} = -\frac{e^2}{\varepsilon\,|\mathrm{r}_e - \mathrm{r}_h|}$$

Harmonic Oscillator in 2D

$$\mathbf{A}_e = \frac{1}{2}\mathbf{B}\times(\mathbf{r}_e - \mathbf{r}_h) \text{ and } \mathbf{A}_h = \frac{1}{2}\mathbf{B}\times(\mathbf{r}_h - \mathbf{r}_e).$$

$$H = H_{COM} + H_{rel} + H_c$$

$$H_{COM} = \frac{P^2}{2M} + \frac{M}{2}(w_x^2 X^2 + w_y^2 Y^2),$$

$$H_c = -\frac{ie\hbar}{Mc}B\left(x\frac{\partial}{\partial Y} - y\frac{\partial}{\partial X}\right).$$

$$H_{rel} = \frac{p^2}{2\mu} + \frac{1}{2}\mu(\tilde{w}_x^2 x^2 + \tilde{w}_y^2 y^2) + \frac{1}{2}\gamma w_{c\mu} l_z - \frac{e^2}{\varepsilon r},$$

$$\boxed{\tilde{w}_{x(y)}^2 = w_{x(y)}^2 + w_{c\mu}^2/4}$$

effective confinement increases with B field

$$\langle n_x' n_y' | H_{rel} | n_x n_y \rangle$$

$$= \hbar \tilde{w}_x \left(n_x + \frac{1}{2} \right) + \hbar \tilde{w}_y \left(n_y + \frac{1}{2} \right) - \langle n_x' n_y' | \frac{e^2}{\varepsilon r} | n_x n_y \rangle$$

$$+ i \frac{1}{4} \gamma \hbar w_{c\mu} \langle n_x' n_y' | (a_x^\dagger a_y^\dagger - a_x a_y) \left(\tilde{\eta} - \frac{1}{\tilde{\eta}} \right) | n_x n_y \rangle$$

$$+ i \frac{1}{4} \gamma \hbar w_{c\mu} \langle n_x' n_y' | (a_x a_y^\dagger - a_x^\dagger a_y) \left(\tilde{\eta} + \frac{1}{\tilde{\eta}} \right) | n_x n_y \rangle ,$$

off-diagonal Coulomb interaction

exact expression: Song & SU PRB 1995

off-diagonal one-particle elements

$$\tilde{\eta} = \sqrt{\tilde{\omega}_x / \tilde{\omega}_y} \xrightarrow{B=0} \eta = \sqrt{\omega_x / \omega_y}$$

$$\xrightarrow{B>>1} \eta \simeq 1 \text{ (circular limit)}$$

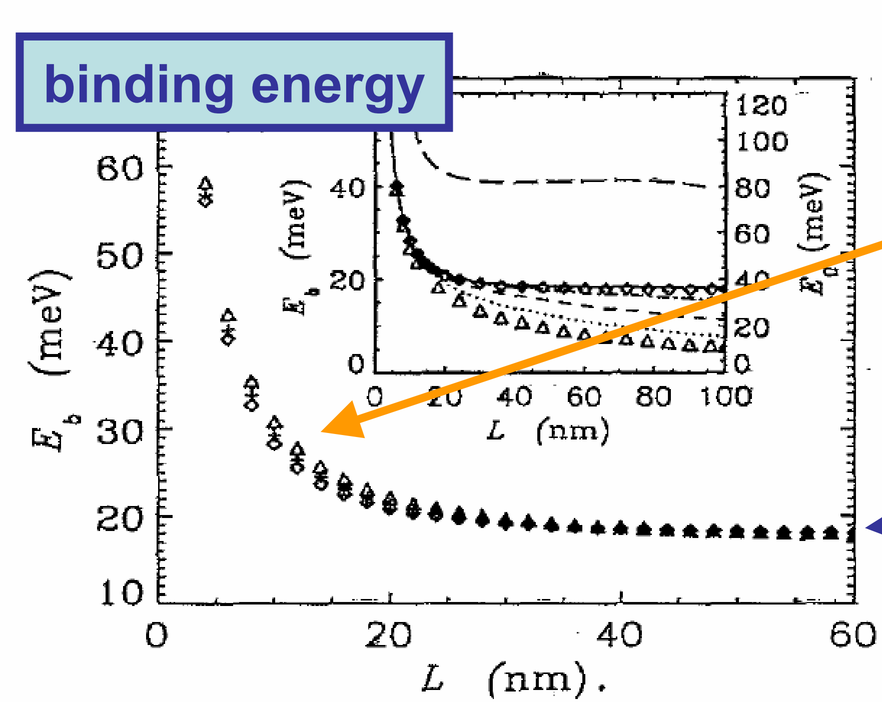
$$r_s^2 = \langle \psi | r^2 | \psi \rangle = \sum_{n_x n_y} \left[\frac{\hbar}{\mu \Omega_y} (n_y + \tfrac{1}{2}) + \frac{\hbar}{\mu \Omega_x} (n_x + \tfrac{1}{2}) \right] |a_{n_x, n_y}|^2$$

$$+ \frac{1}{2} \frac{\hbar}{\mu \Omega_y} \left[\sqrt{(n_y + 2)(n_y + 1)}\, a^*_{n_x, n_y + 2} + \sqrt{n_y (n_y - 1)}\, a^*_{n_x, n_y - 2} \right] a_{n_x, n_y}$$

$$+ \frac{1}{2} \frac{\hbar}{\mu \Omega_x} \left[\sqrt{(n_x + 2)(n_x + 1)}\, a^*_{n_x + 2, n_y} + \sqrt{n_x (n_x - 1)}\, a^*_{n_x - 2, n_y} \right] a_{n_x, n_y} ,$$

Coulomb interactions in harmonic oscillator basis

$$
\begin{aligned}
\left\langle n_x', n_y' \left| \frac{e^2}{\epsilon\sqrt{x^2+y^2}} \right| n_x, n_y \right\rangle = {} & \frac{e^2}{\epsilon\pi}\sqrt{\frac{\mu\omega_y}{\hbar}}\left(2^{s_x+s_y}\, n_x'!\, n_x!\, n_y'!\, n_y!\right)^{-1/2} \sum_{\alpha=0}^{[n_x'/2]} \sum_{\beta=0}^{[n_x/2]} \sum_{\gamma=0}^{[n_y'/2]} \sum_{\delta=0}^{[n_y/2]} \\
& \times \frac{(-1)^{\eta}\, n_x'!\, n_x!\, n_y'!\, n_y!}{\alpha!(n_x'-2\alpha)!\beta!(n_x-2\beta)!\gamma!(n_y'-2\gamma)!\delta!(n_y-2\delta)!}\left(\frac{\omega_y}{\omega_x}\right)^{\frac{s_y}{2}-\gamma-\delta} (-1)^{s_y}\, 2^{s_x+s_y-2\eta} \\
& \times F\left[\frac{1}{2}(s_y+1)-\gamma-\delta, \frac{1}{2}(s_x+s_y+1)-\eta; \frac{1}{2}(s_x+s_y)-\eta+1; 1-\frac{\omega_y}{\omega_x}\right] \\
& \times \Gamma\left[\tfrac{1}{2}(s_x+s_y+1)-\eta\right]\Gamma\left[\tfrac{1}{2}(s_x+1)-\alpha-\beta\right] \\
& \times \Gamma\left[\tfrac{1}{2}(s_y+1)-\gamma-\delta\right]/\Gamma\left[\tfrac{1}{2}(s_x+s_y)-\eta+1\right], \qquad (A
\end{aligned}
$$

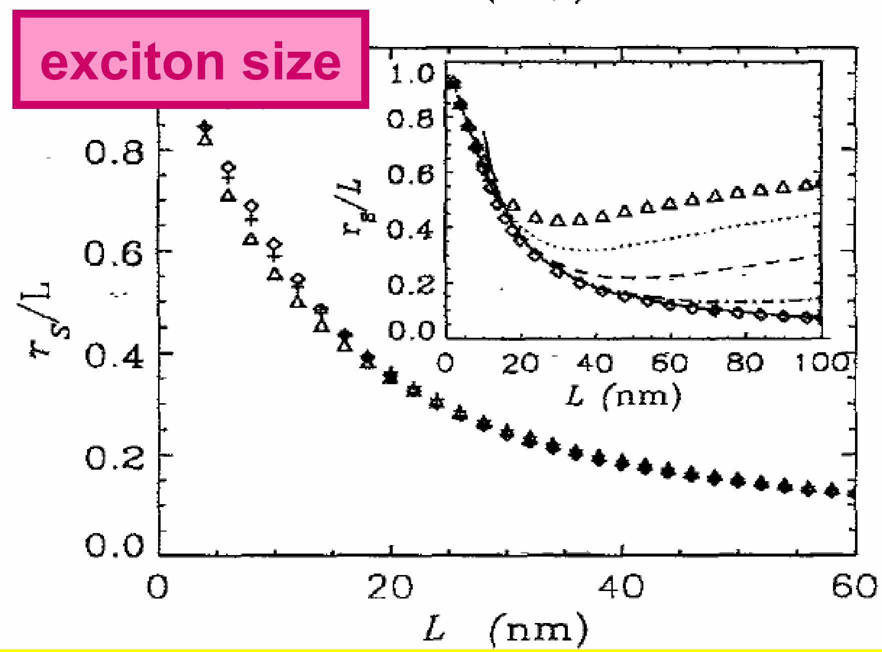


binding energy = $E_{exc} - E_0 = E_b$

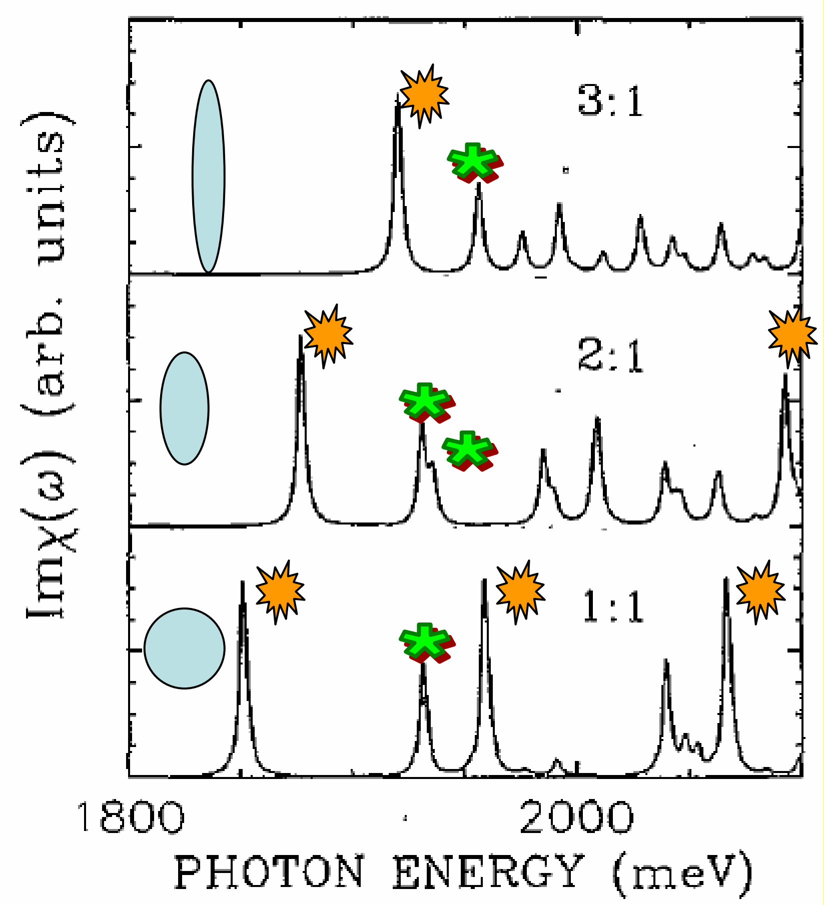
$E_b \sim 1/L$

L_x/L_y = 1:1 ◊ 2:1 + 3:1 △

saturates to 2D value
for $L \gg a_B$ & all shapes



exciton "shrinks" as $L \rightarrow 0$, $r_S \rightarrow L$
but
$r_s \rightarrow a_B$ as $L \gg a_B$



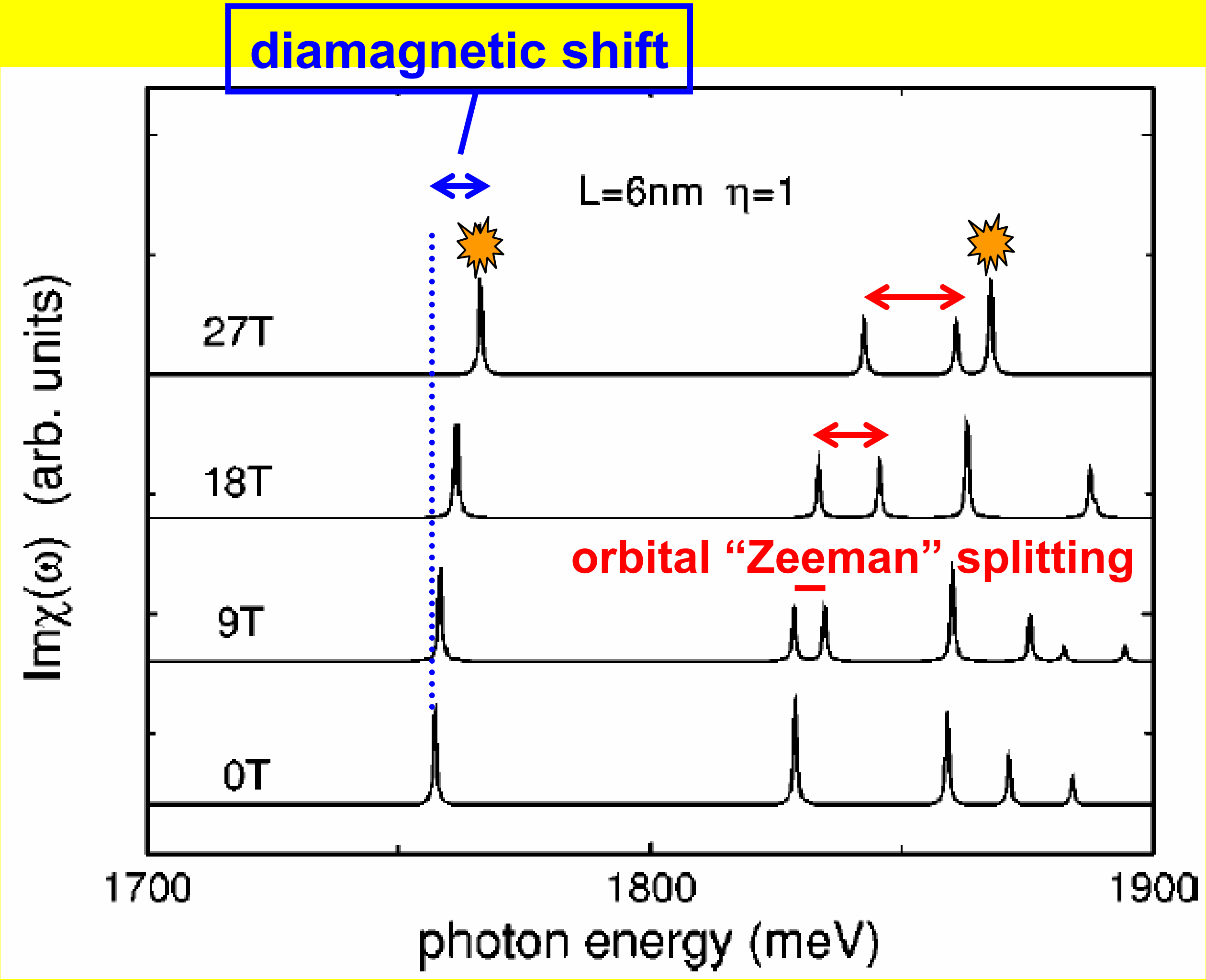
χ = linear optical susceptibility
~ PLE signal ~ <u>optical response</u>

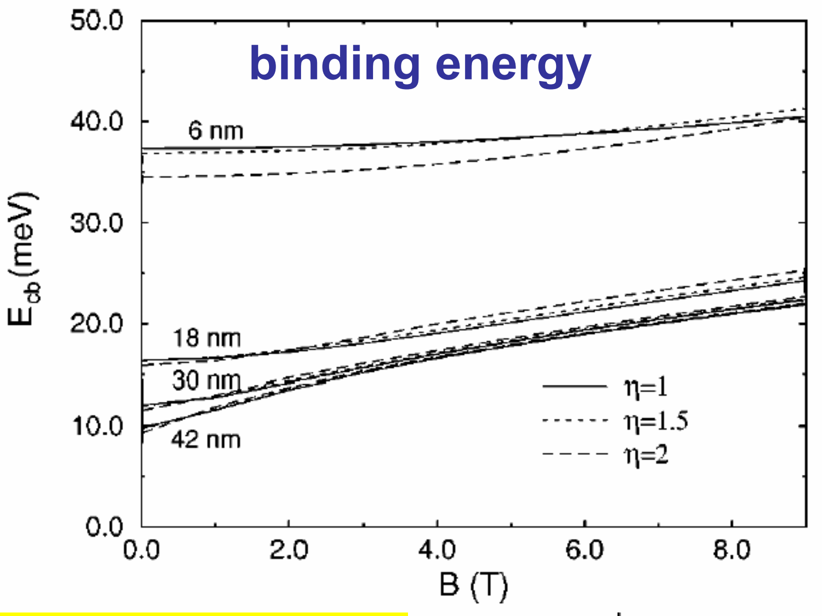
COM gnd state & replicas

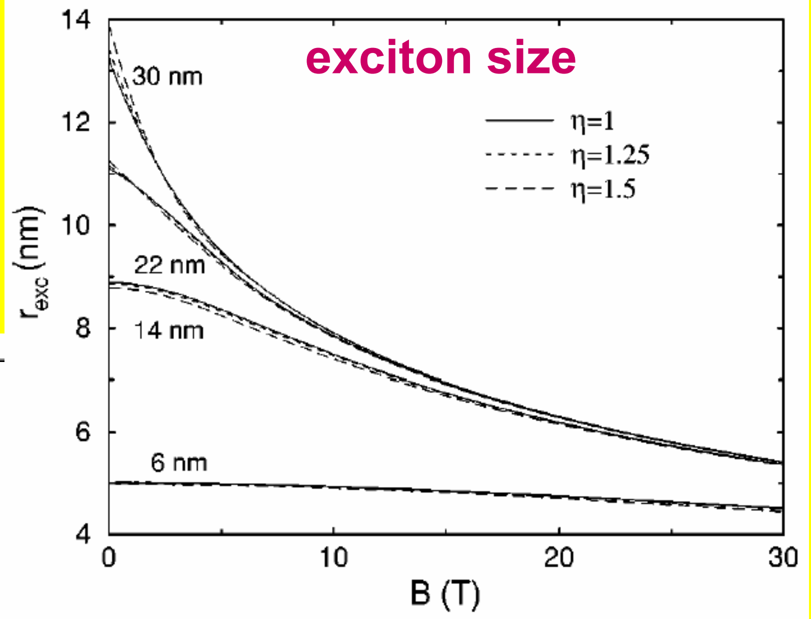
excited states of internal degrees of freedom

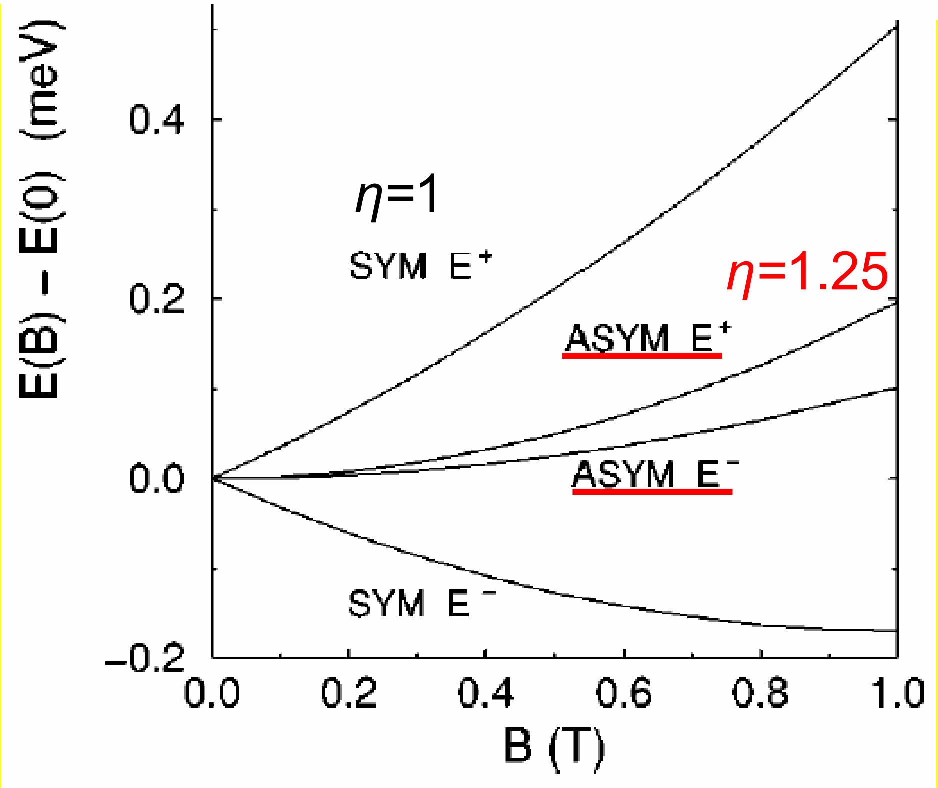
area of dots = 5x5 nm^2

Magnetic field effects





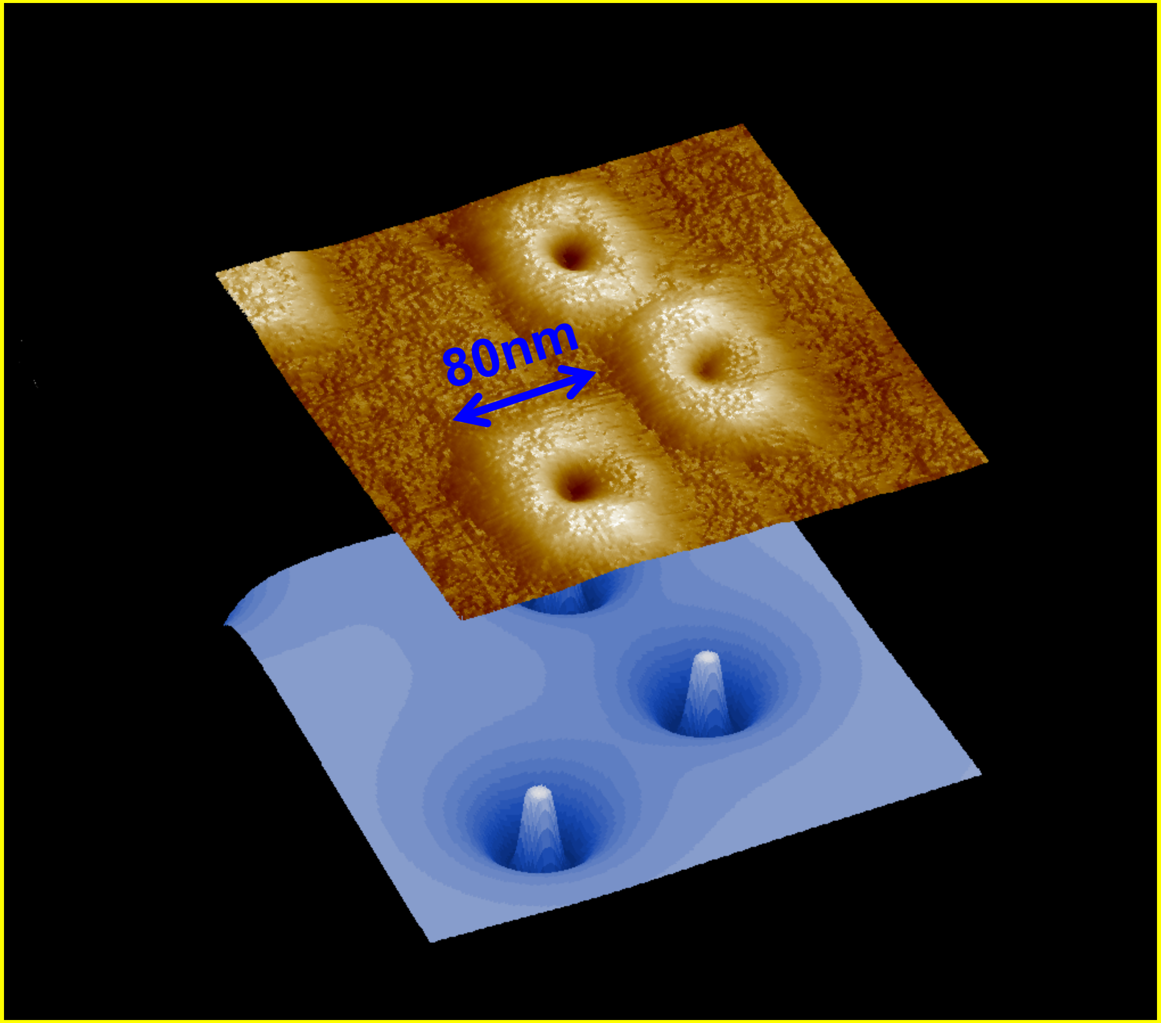




suppression of Zeeman orbital splitting →

asymmetry of structure

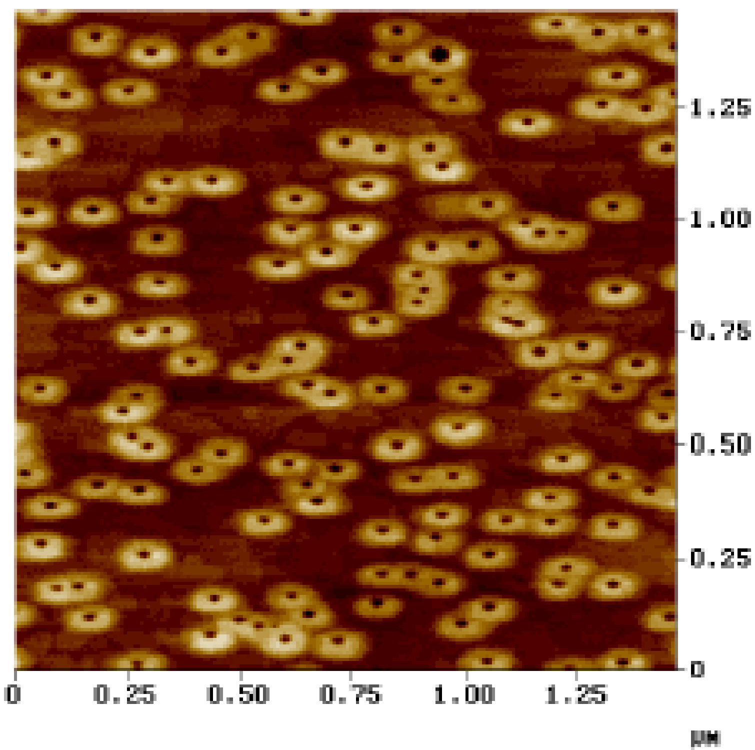
SAQD RINGS!!



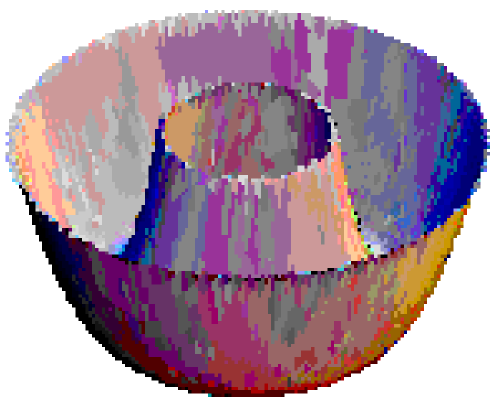
smallest COHERENT ring potential for electrons and/or holes

Lorke et al PRL 1999

AFM-picture of InAs-quantum rings

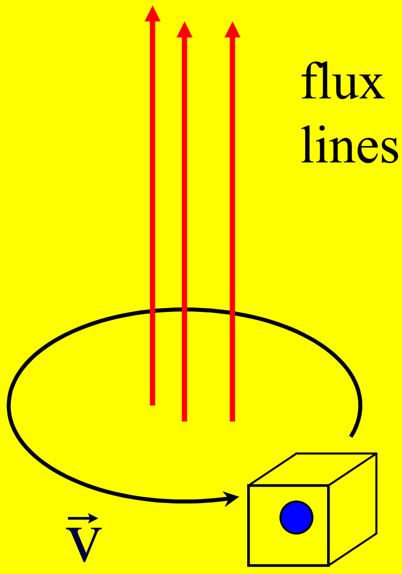


parabolic ring-potential



J Garcia 1999

Is there AB effect for excitons?



$$\Delta\phi = \frac{e}{c\hbar}\oint \vec{A}\, d\vec{l} = \frac{e}{c\hbar}\Phi$$

$\vec{v} \rightarrow 0$

AB is the nonadiabatic version of Berry phase in a flux

exciton charge = ZERO $\longrightarrow$ no ABE?

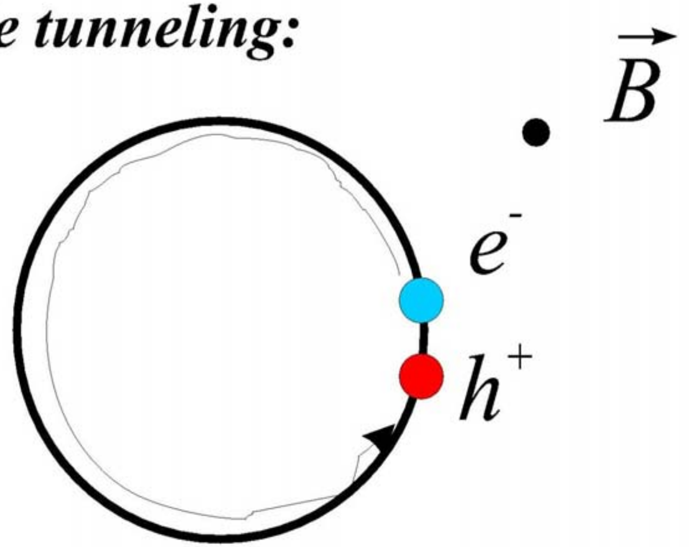
M. Berry, Proc. R. Soc. Lond. A **392**, 45 (1984)

BUT...

Ground state of 1D excitons (Romer & Raikh 2000):

$$\Delta_0^0 = -\frac{\pi^2 V_0^2}{\varepsilon_0}\left[1 + 4\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\exp\left(-\frac{2\pi^2|V_0|}{\varepsilon_0}\right)\right]$$

Electron-to-hole tunneling:

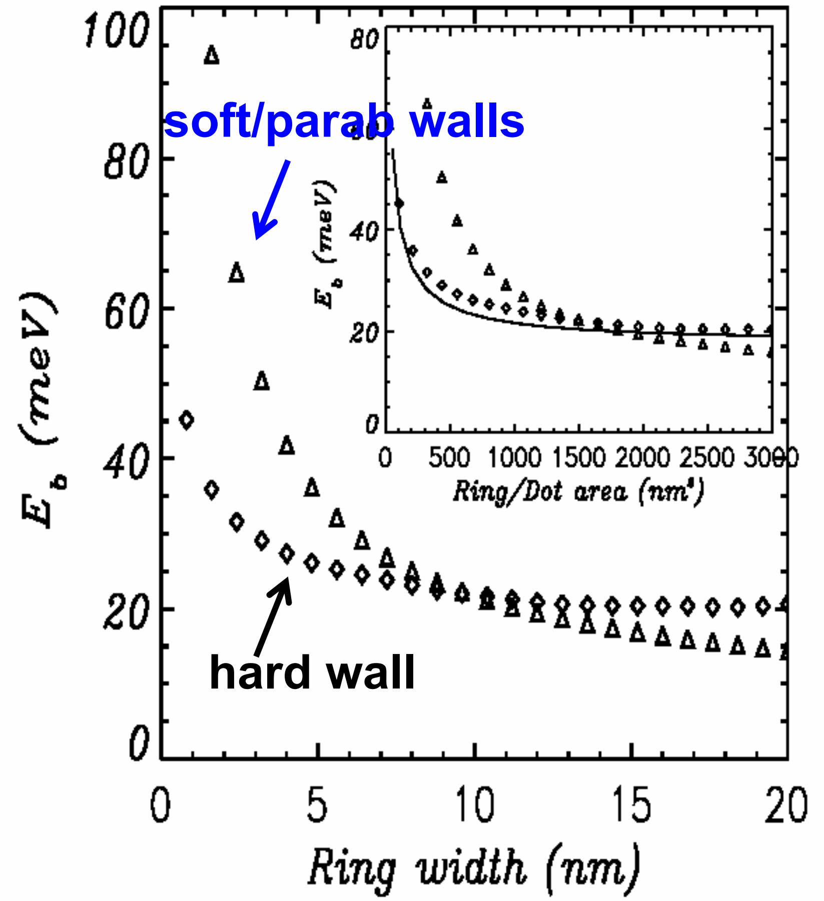


$$\Phi = \pi R^2 B > \Phi_0/2$$

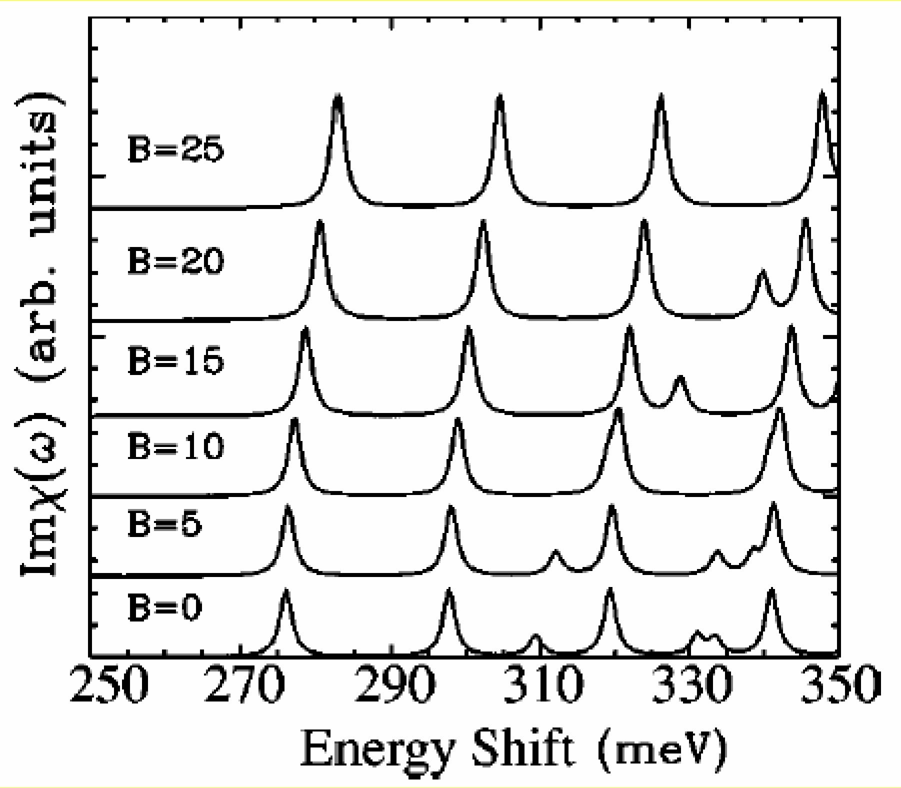
$$\Delta E_{exc}(B) = \delta E_{exc}(0) exp[-2\pi^2 V_0/\varepsilon_0] cos[2\pi\Phi/\Phi_0]$$

Results for microscopic calculation

Song & SU PRB 2001

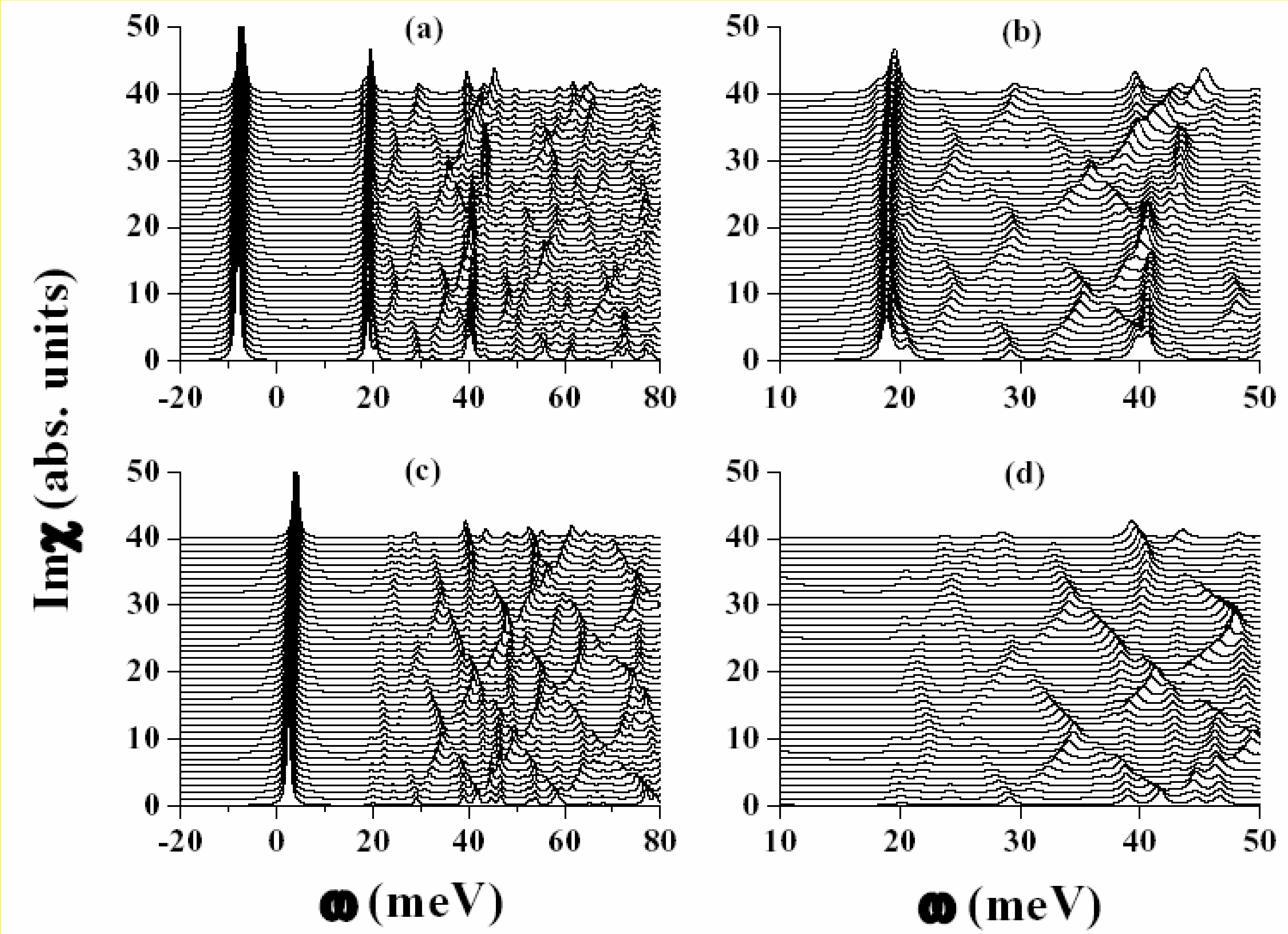


binding energy



NO AB oscillations!!

AB oscillations seen if ring is narrower ~ 1D like width = 10nm

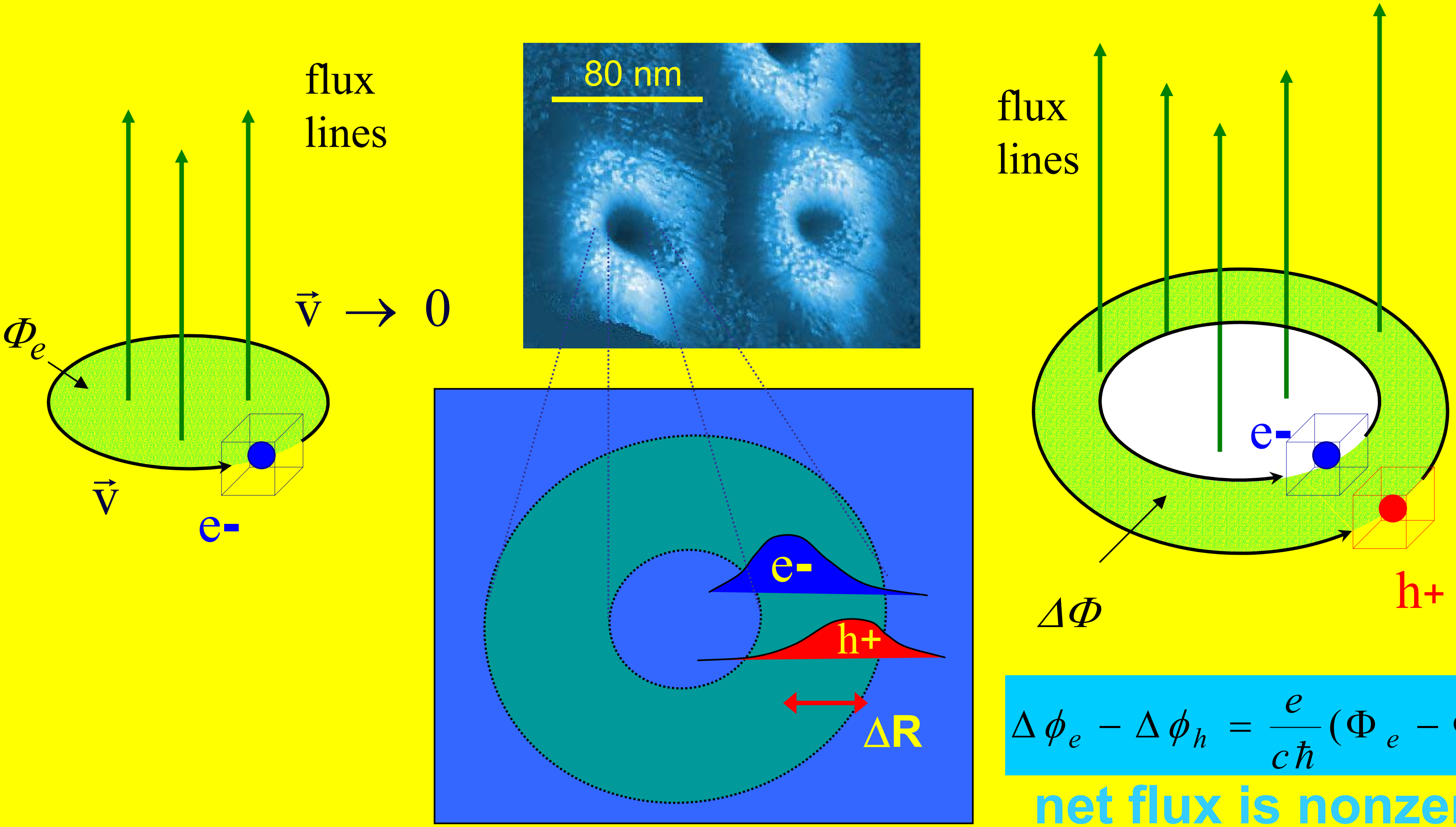


heavy hole

light hole

Hu et al PRB 2001

BUT excitons are polarizable!

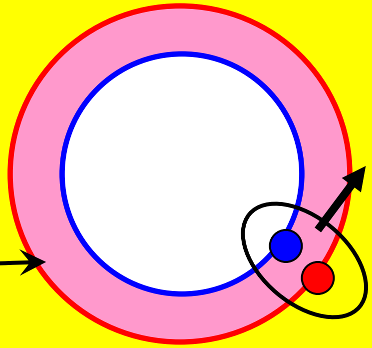


$$\Delta\phi_e - \Delta\phi_h = \frac{e}{c\hbar}(\Phi_e - \Phi_h)$$

net flux is nonzero!

strong Coulomb interaction limit: $R_0 = (R_e + R_h)/2 \gg a_0^*$

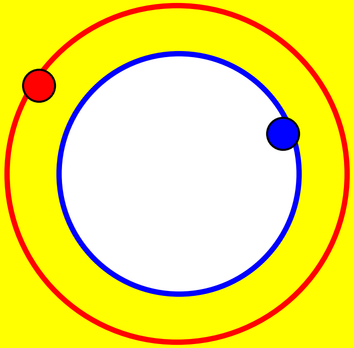
$$E_{exc} = \frac{\hbar^2}{2\bar{M}R_0^2}\left(L + \frac{\Delta\Phi}{\Phi_0}\right)^2 + E_1, \quad \Delta\Phi = \pi(R_e^2 - R_h^2)B.$$



correlated motion

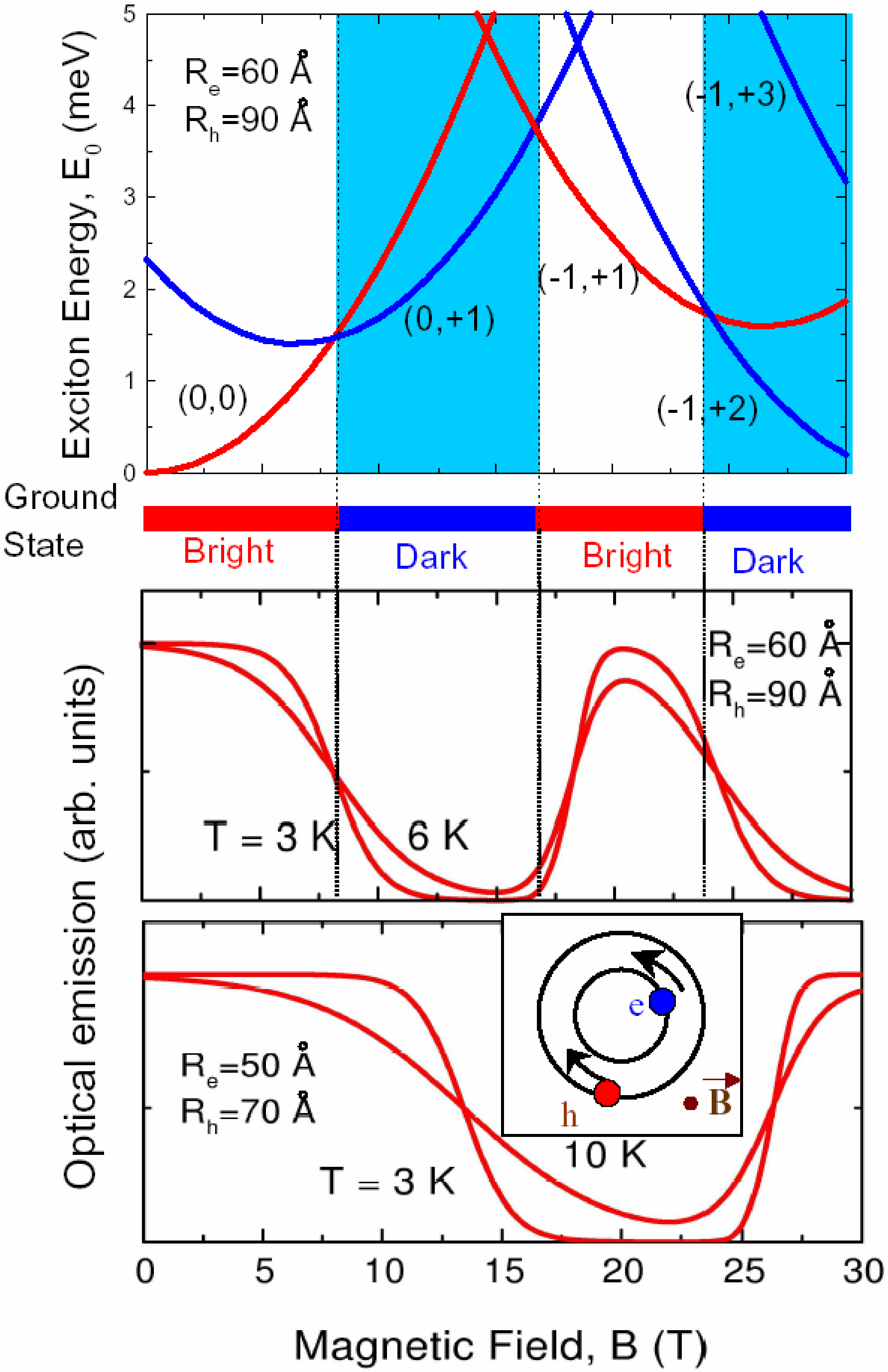
weak interaction limit: $R_0 \ll a_0^*$

$$E_{exc} = \frac{\hbar^2}{2m_eR_e^2}\left(L_e + \frac{\Phi_e}{\Phi_0}\right)^2 + \frac{\hbar^2}{2m_hR_h^2}\left(L_h - \frac{\Phi_h}{\Phi_0}\right)^2,$$



$$\Phi_{e(h)} = \pi R_{e(h)}^2 B, \qquad L = L_e + L_h$$

independent motion



weak interaction limit

optical emission strongly suppressed in B field windows

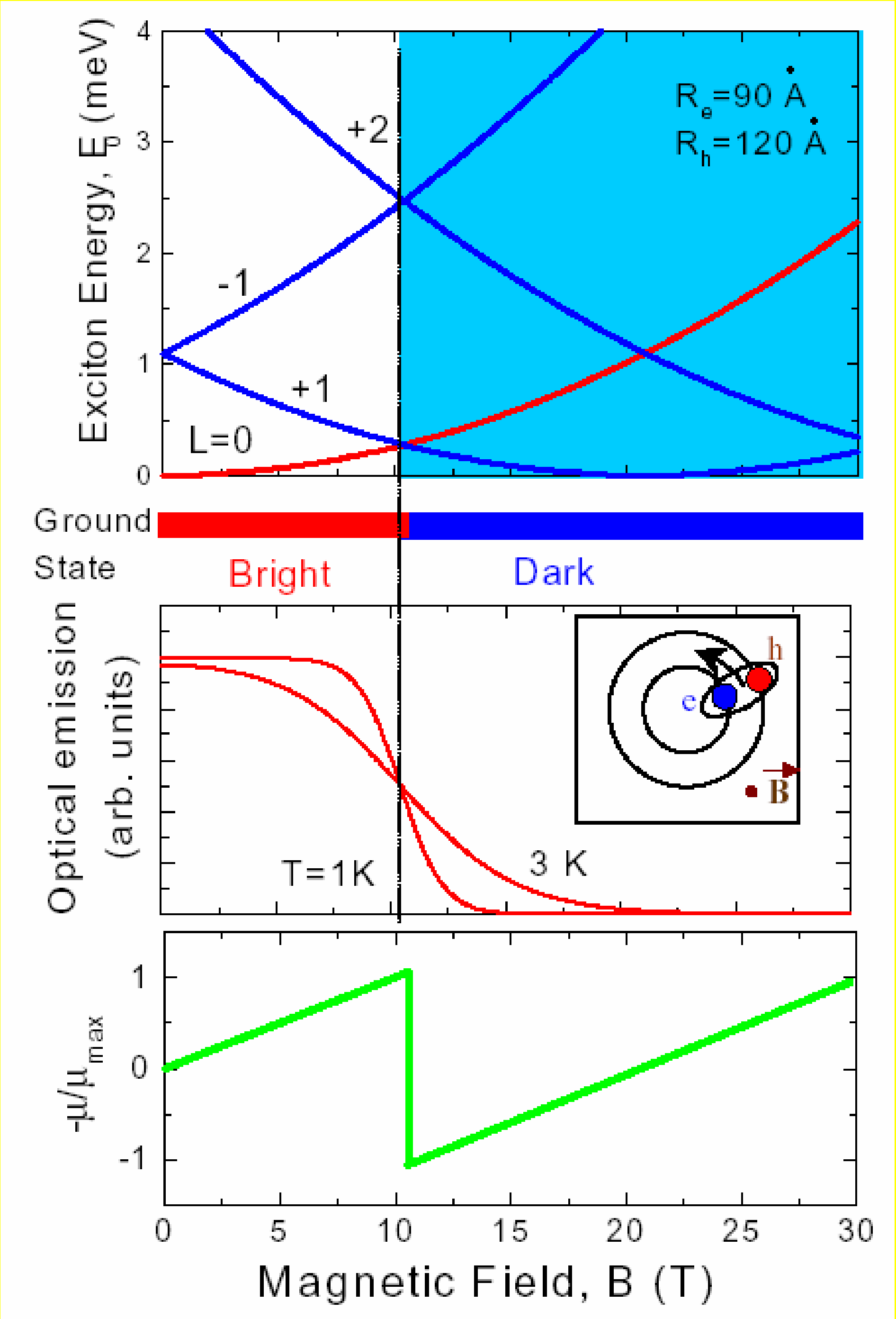
dark window L_{tot} non zero

bright window L_{tot} = 0

Govorov et al PRB 2002

strong interaction limit

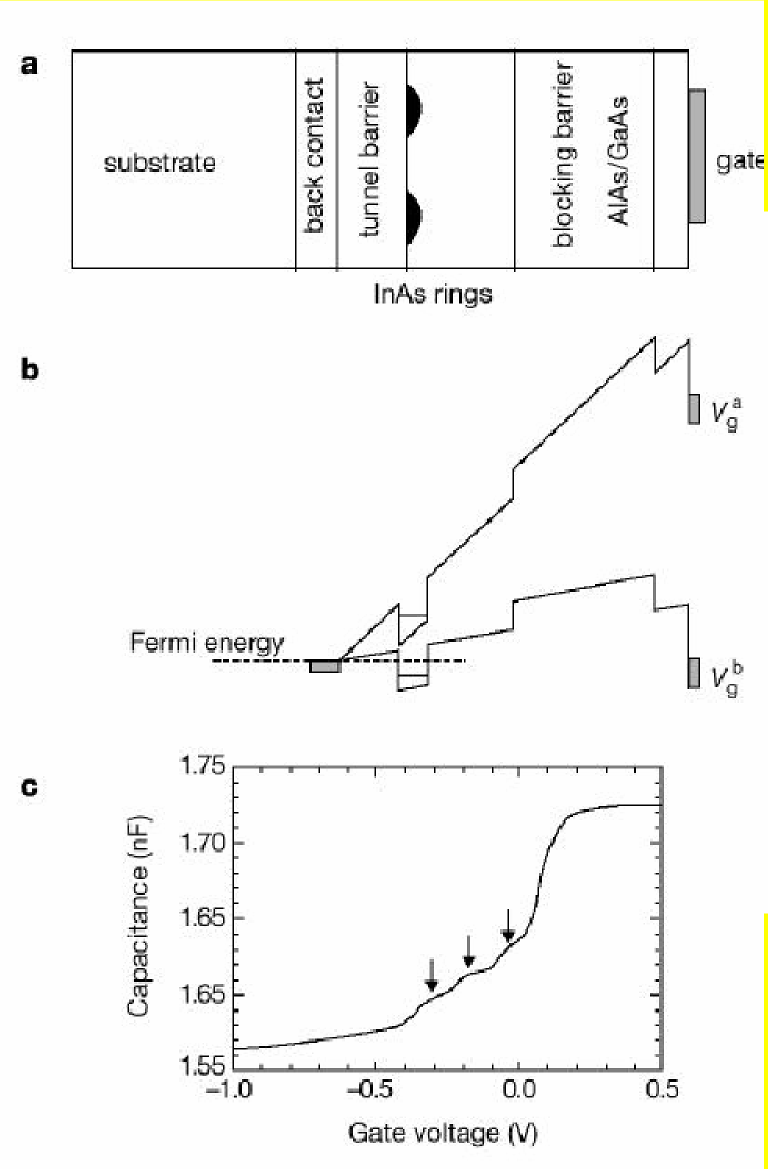
optical emission strongly suppressed after critical B field

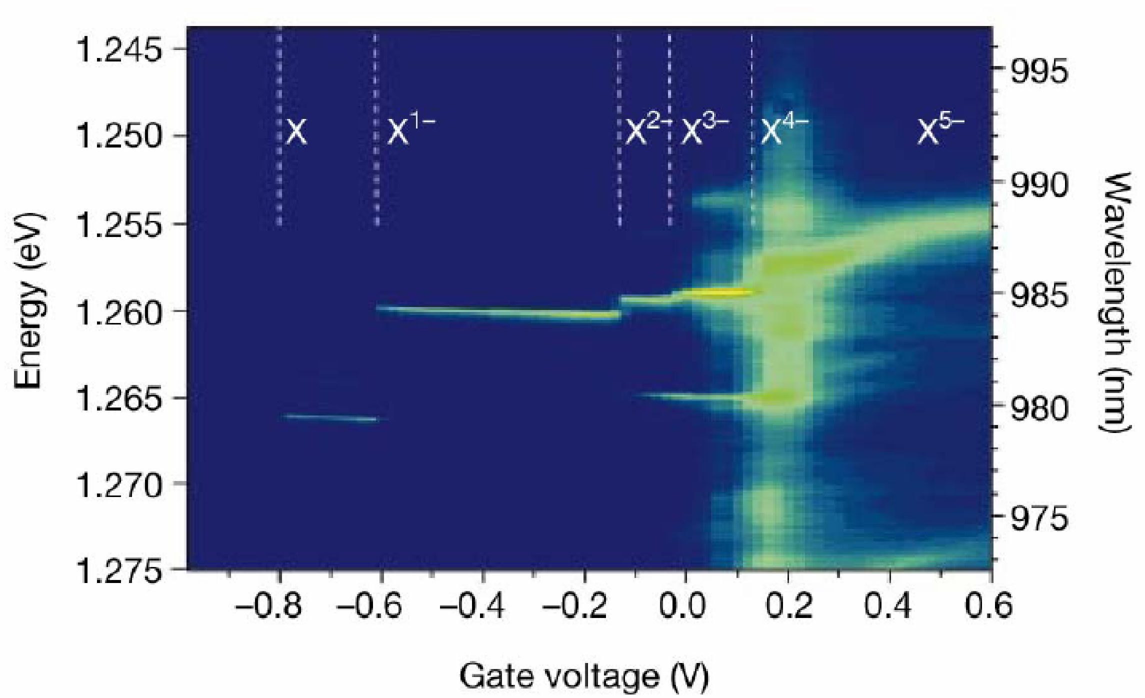


effective persistent current associated with Berry´s phase and angular momentum in ground state

Optical emission from a charge tunable quantum ring

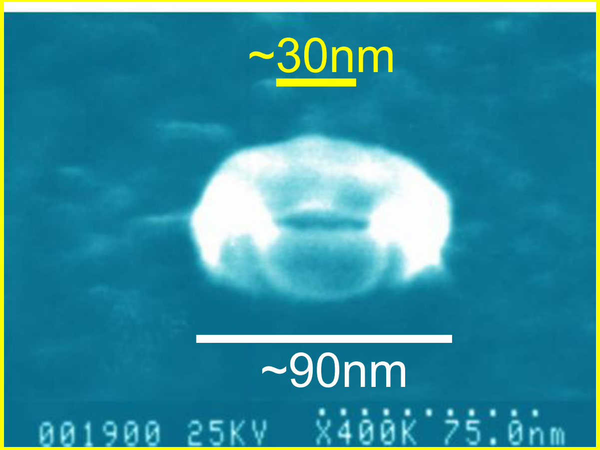
Warburton et al Nature 2000



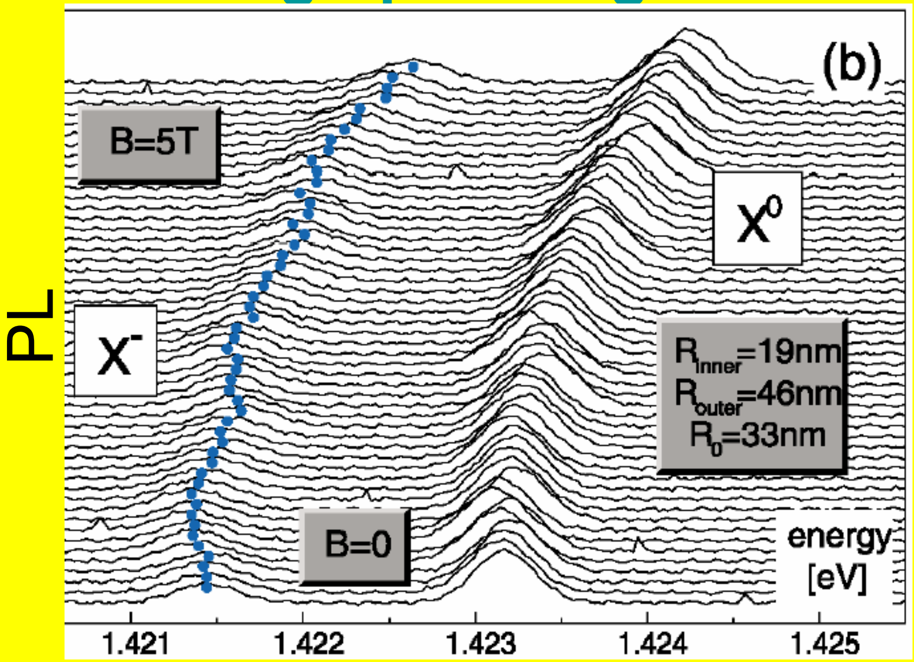


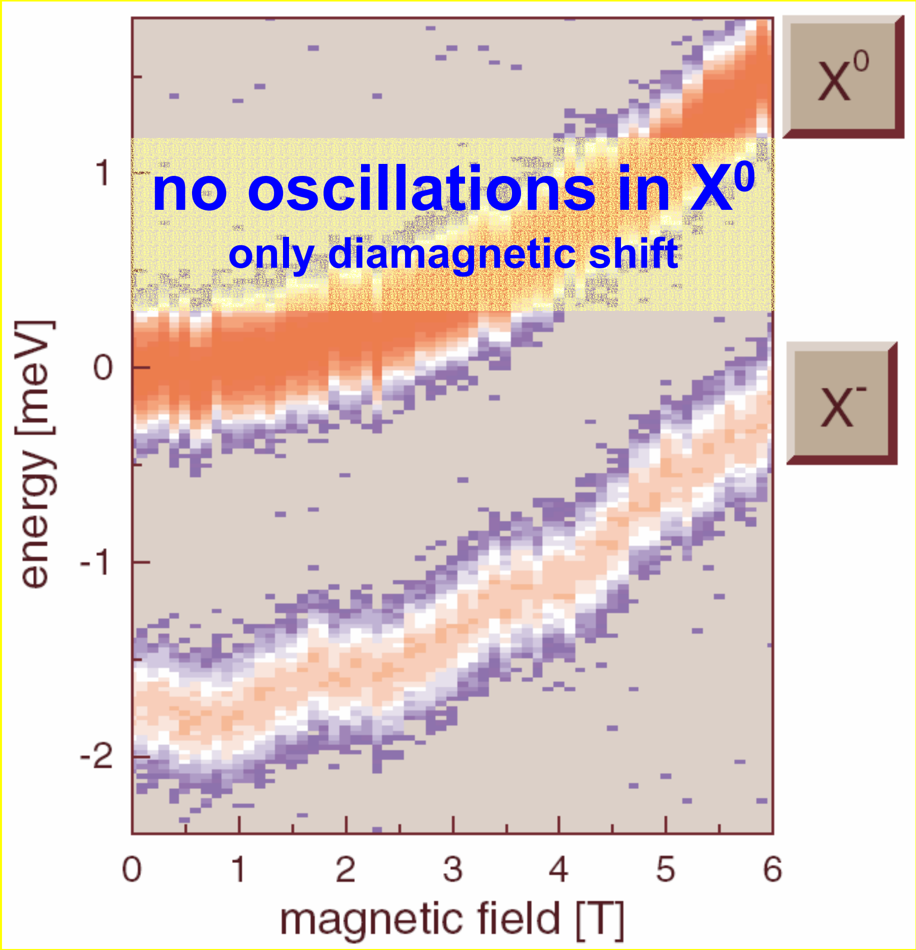
Optical detection of the AB effect in a charged particle

Bayer et al PRL 2003

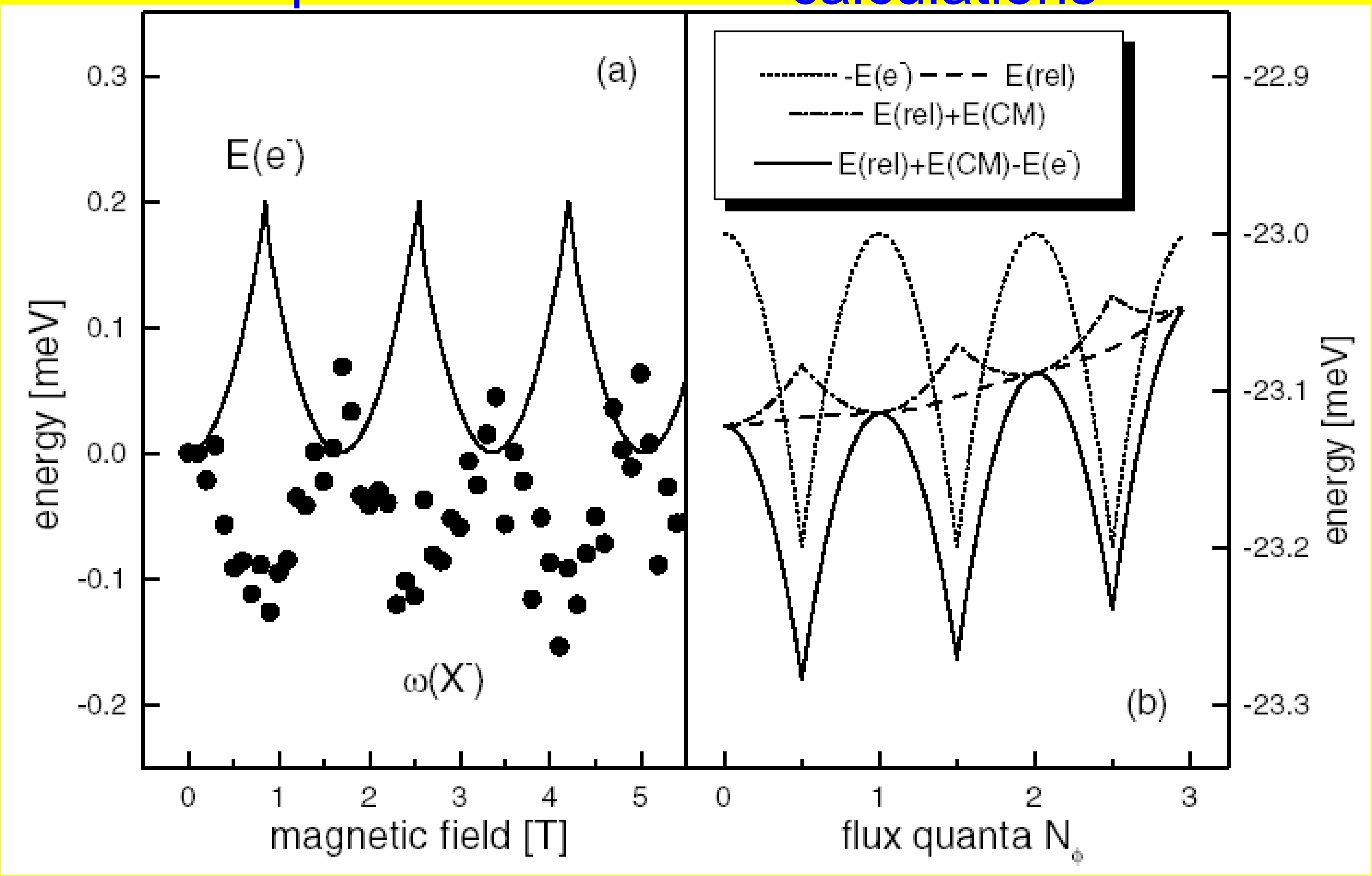


lithographic ring





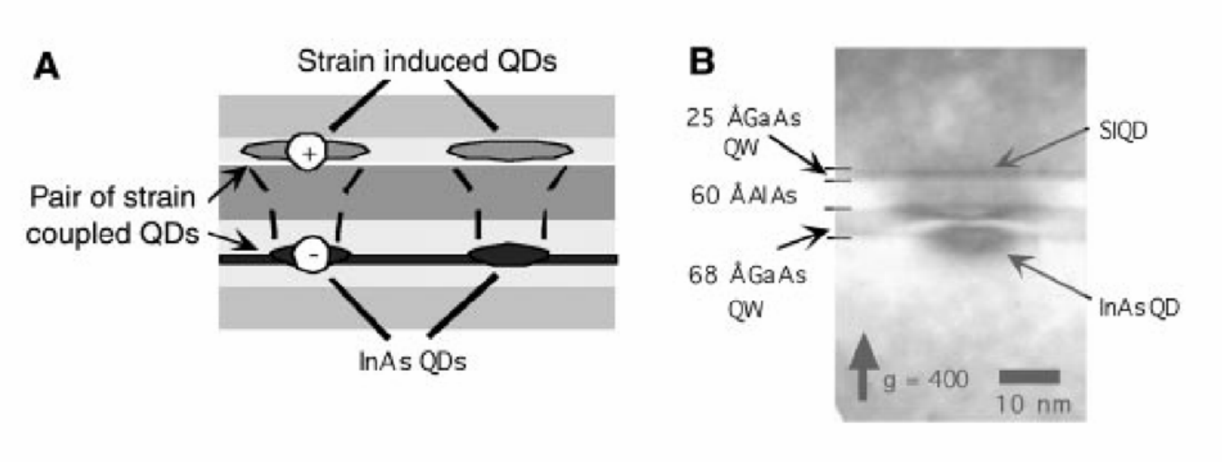
experiments calculations

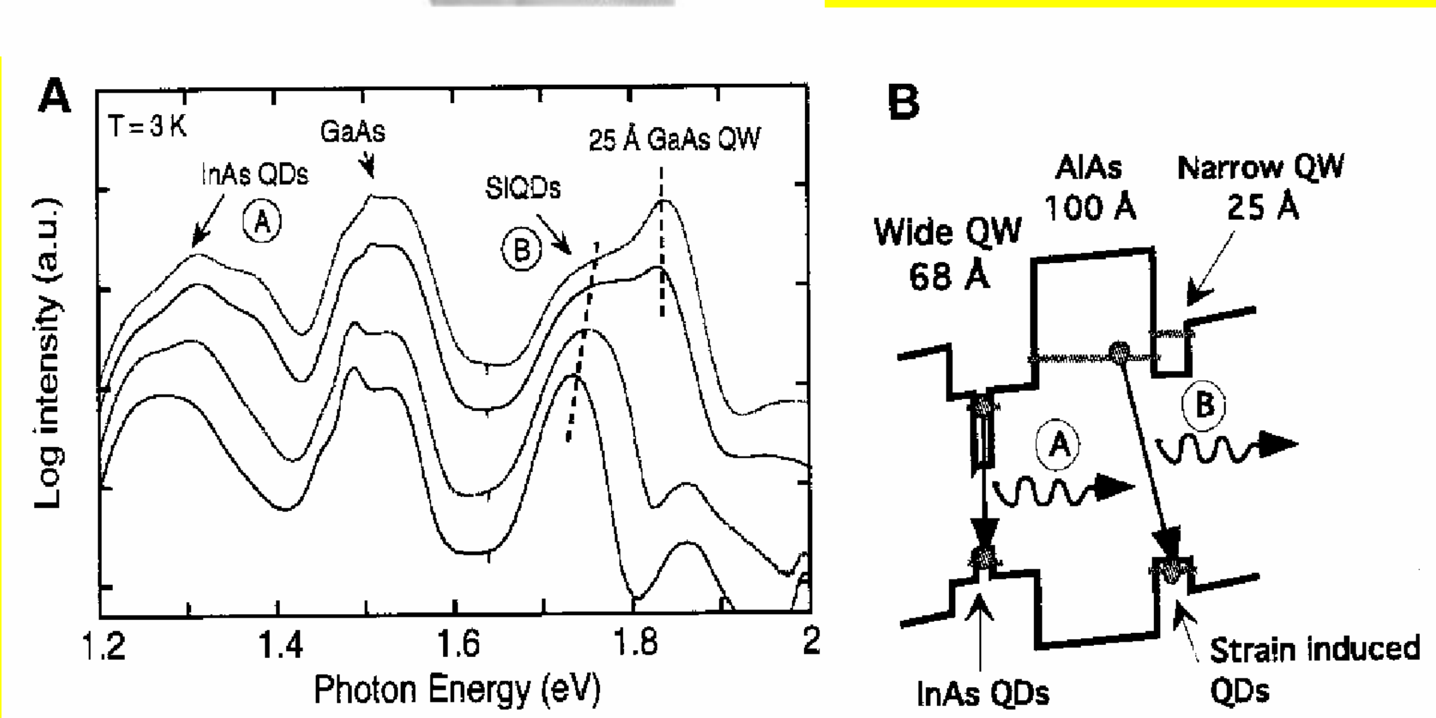


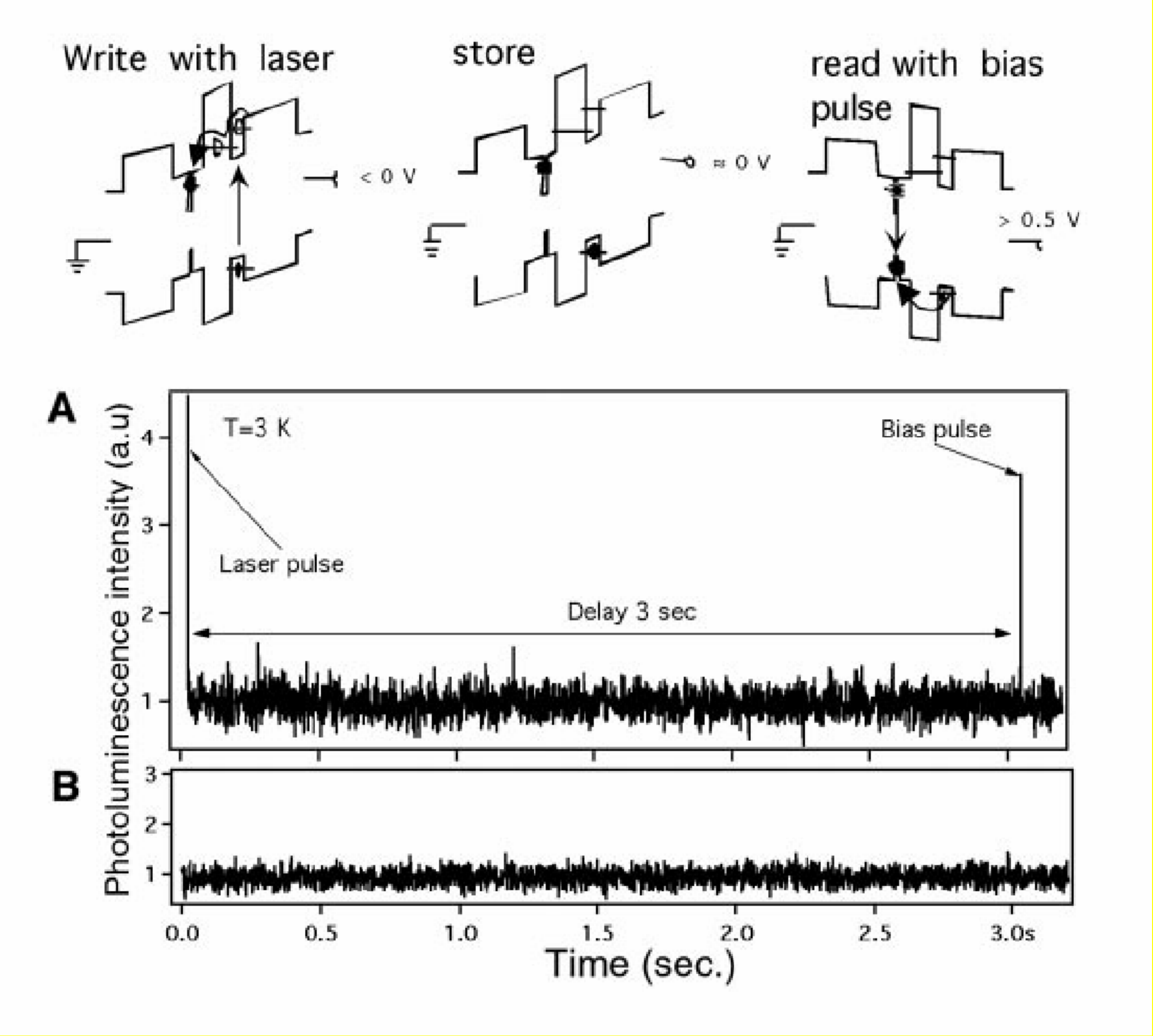
reasonable agreement w/calculations → interactions do not change period and only slightly the amplitude

Exciton Storage in Semiconductor Self-Assembled Quantum Dots

T. Lundstrom, W. Schoenfeld, H. Lee, P. M. Petroff* Science 2000







Write with laser
< 0 V
store
≈ 0 V
read with bias pulse
> 0.5 V
A
T=3 K
Bias pulse
Laser pulse
Delay 3 sec
B
Photoluminescence intensity (a.u)
0.0
0.5
1.0
1.5
2.0
2.5
3.0s
Time (sec.)

IFUAP–BUAP – Minicurso en Nanoestructuras
Tarea # 2

Efecto Aharonov-Bohm en un anillo conductor.

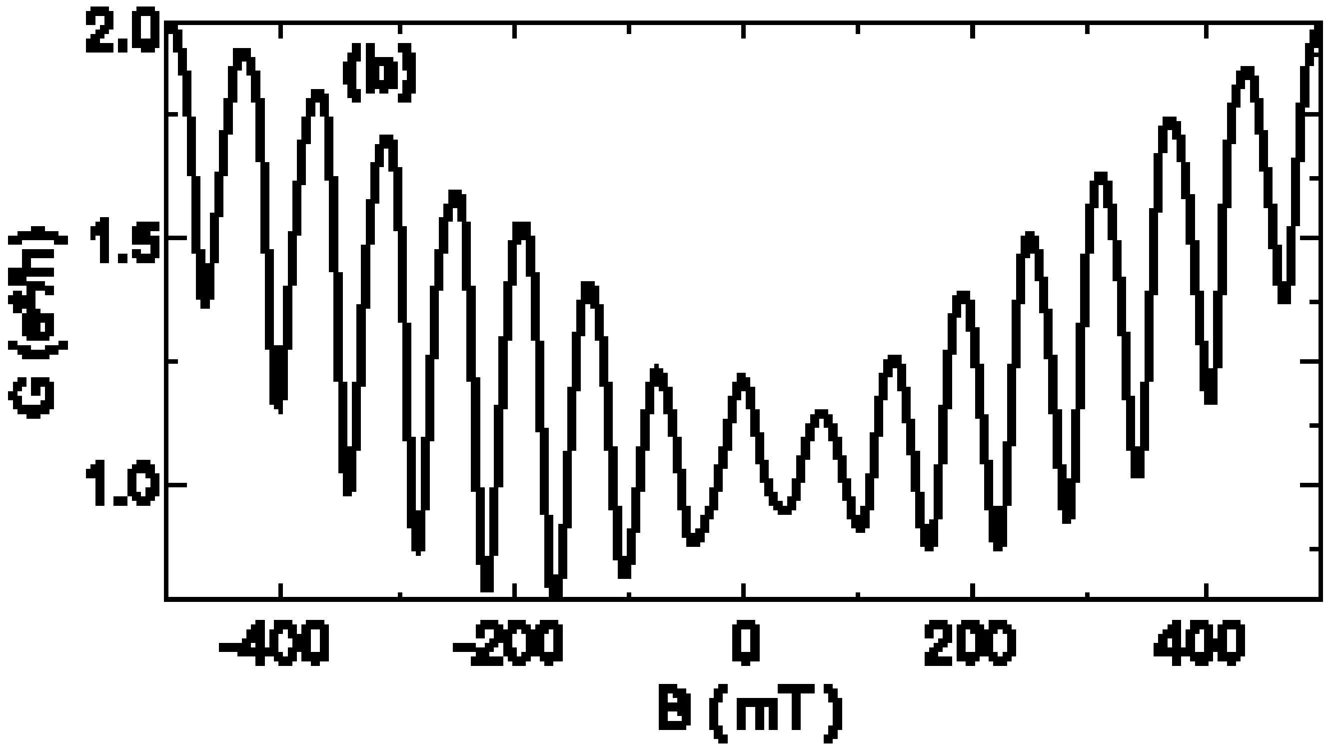
(a) En la figura debajo se muestra la conductancia experimental de un anillo construido en un gas de electrones de dos dimensiones en un semiconductor. Estime de la gráfica el período de éstas oscilaciones de Aharonov-Bohm. El eje horizontal es campo magnético en milésimas de Tesla. ¿Por cierto, porqué se esperaría que la conductancia G fuera simétrica al invertir B, tal y como aparece en la figura?

(b) Suponiendo que éstas oscilaciones son producidas por el efecto AB, la conductancia G se puede aproximar por:

$$G \approx G_0 \,|1 + \exp(i\Phi/\Phi_0)|^2 \,,$$

en done Φ=BA es el flujo a través del anillo de area A, y Φ_0=h/e es el cuanto de flujo magnético. Deduzca entonces el diámetro del anillo.

(c) Suponga que el anillo fuera "ancho" y que entonces tuviera mas de un canal transversal. Si el anillo es de 1 micra de radio interno y 2 micras de radio externo, estime el numero de canales permitidos para una longitud de Fermi de 50nm. De este número, estime que rango de períodos de oscilaciones AB se podrían ver en el experimento. ¿Se esperaría que las oscilaciones sobrevivieran? Explique brevemente su respuesta.



IFUAP-BUAP Mini-curso en Nanoestructuras

Parte II. – Toma 3

Sergio E. Ulloa – Ohio University

Department of Physics and Astronomy, **CMSS**,

and **Nanoscale and Quantum Phenomena Institute**

Ohio University, Athens, OH

Supported by
US DOE & NSF NIRT

Photons, excitons, spins, energy transfer & entaglement – all in QDs

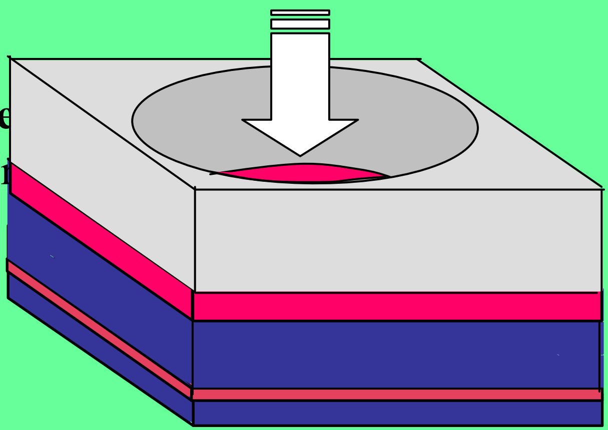
- Gammon/Steel – optical control of quantum dot state
- Imamoglou – single photon source from quantum dots
- Klimov – Förster coupling between quantum dots
- Zrenner – coherent control of quantum dot photodiode

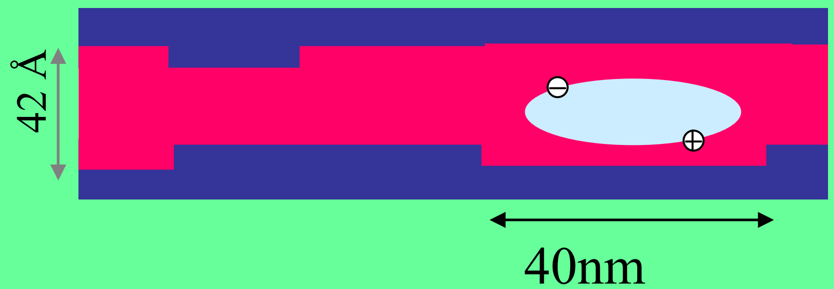
Gammon/Steel – optical control of quantum dot state

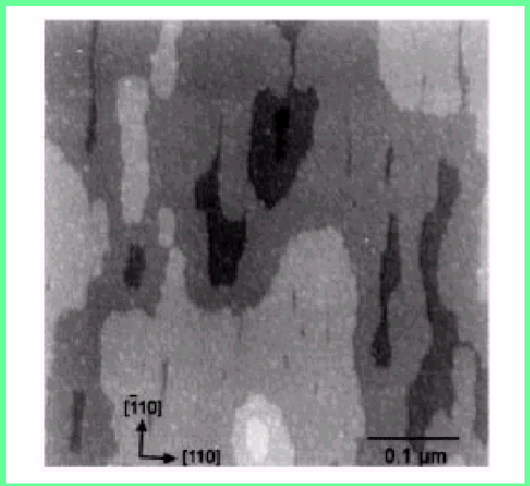
Gammon/Steel – optical control of quantum dot state

Bonadeo et al PRL 1998

Naturally Formed GaAs QDs



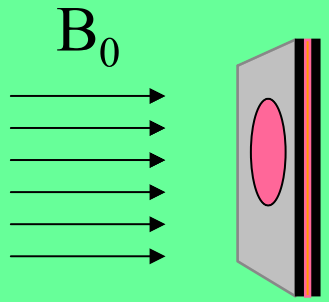




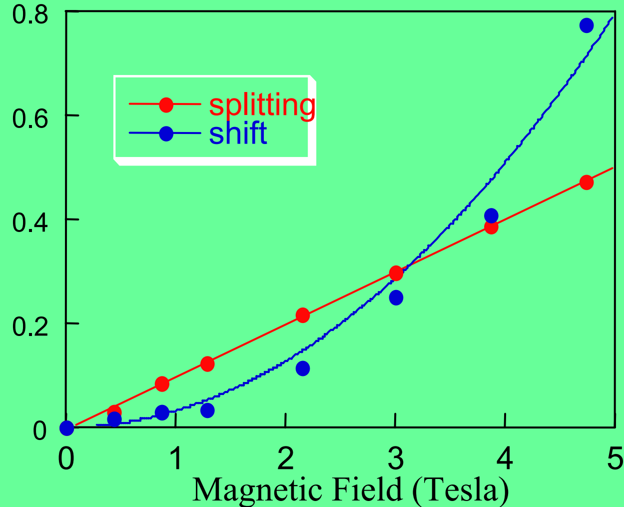
STM image showing the island formation and elongation.

Gammon *et al*., Phys. Rev. Lett.**76**, 3005 (1996)

Samples are provided by D.S. Katzer, D. Park, and E. S. Snow, NRL



QD Magneto-excitons * - g factor



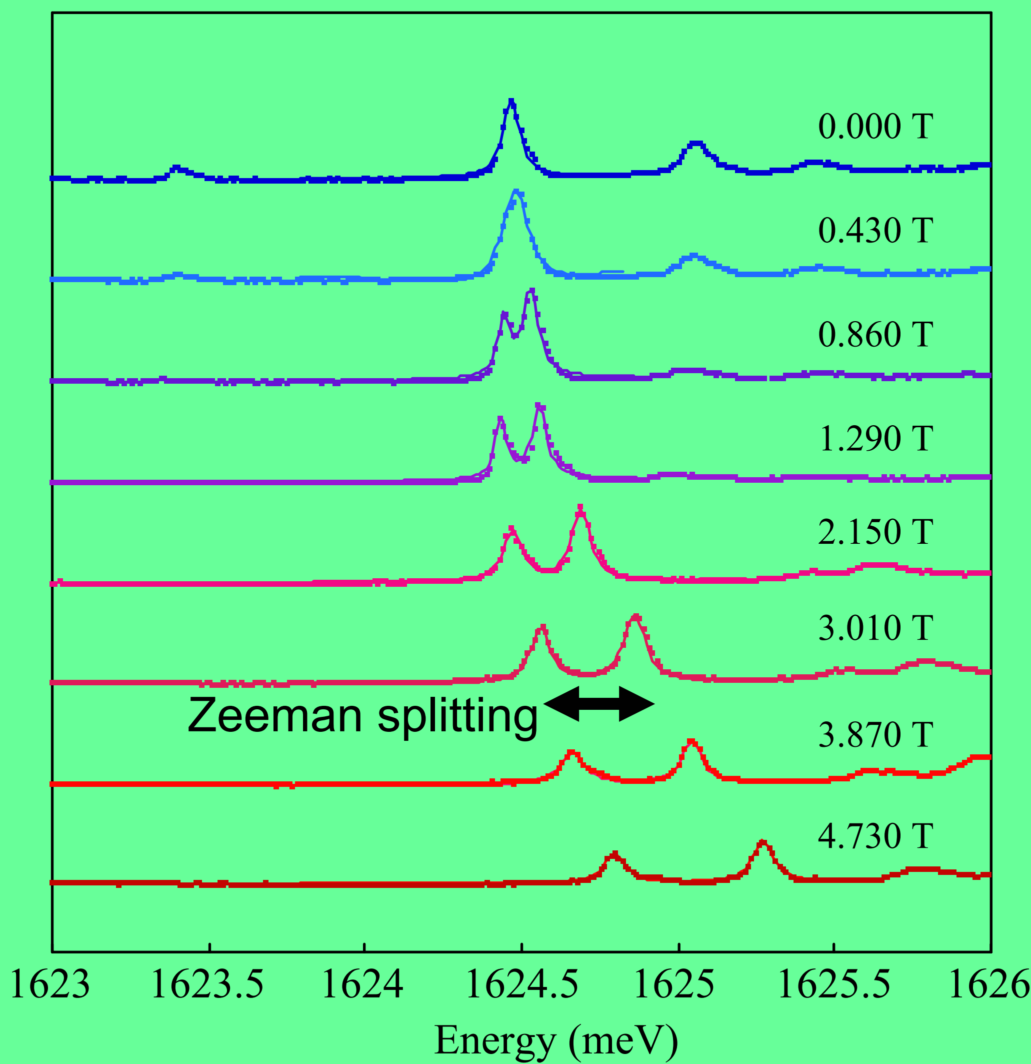
Weak field limit:

$$H_{int} = g^*_{ex}\mu_B B_0 S_z + \alpha B_0^2$$

$$g^*_{ex} = g^*_e + g^*_{hh}$$

$$g^*_{hh} = 6\kappa + 27q/2$$

Photoluminescence of a Single Exciton



* First reported by S. W. Brown *et al.*, Phys. Rev. B **54**, R17 339 (1996)

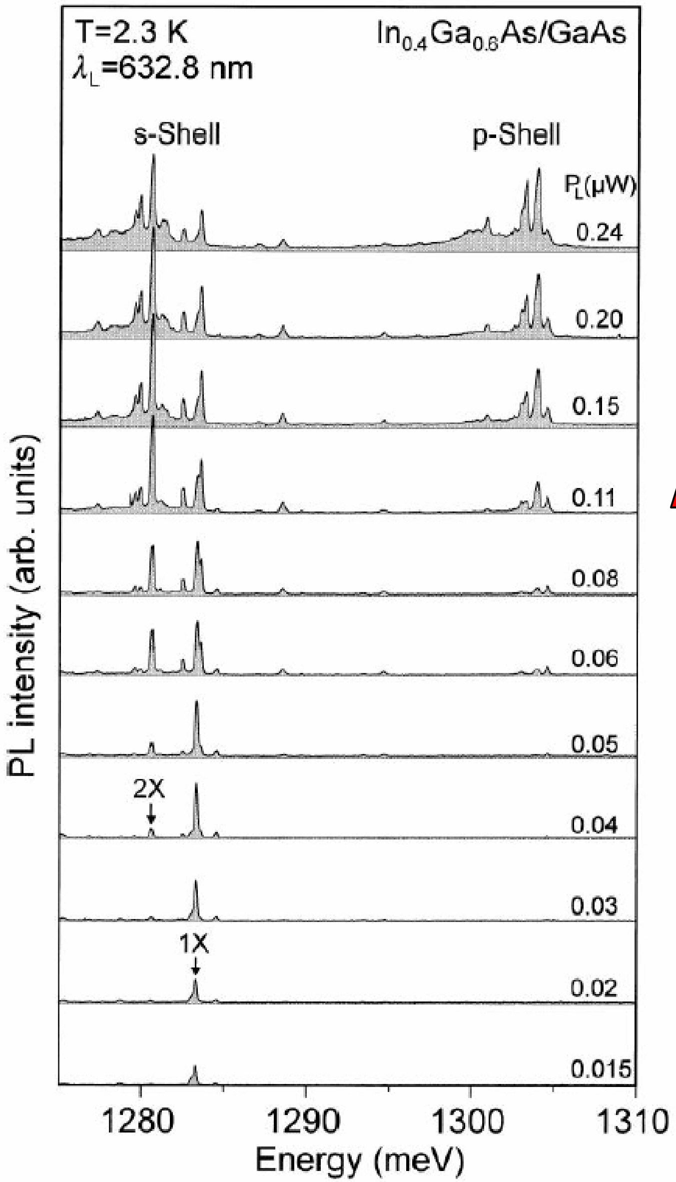
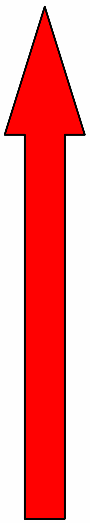


Fig. 1. Power dependent PL spectra from a single isolated quantum dot at zero magnetic field. Contributions from the s-shell and p-shell can be clearly distinguished. In the spectral region of the s-shell, the single exciton (1X) and biexciton lines (2X) are labelled.

Photoluminescence in SINGLE QD

- Excitons
- *Excited* excitons (s- and p-shell)
- Biexcitons, X^-, X^{--}, etc.



laser power

Findeis et al, Sol. State Comm, 2000

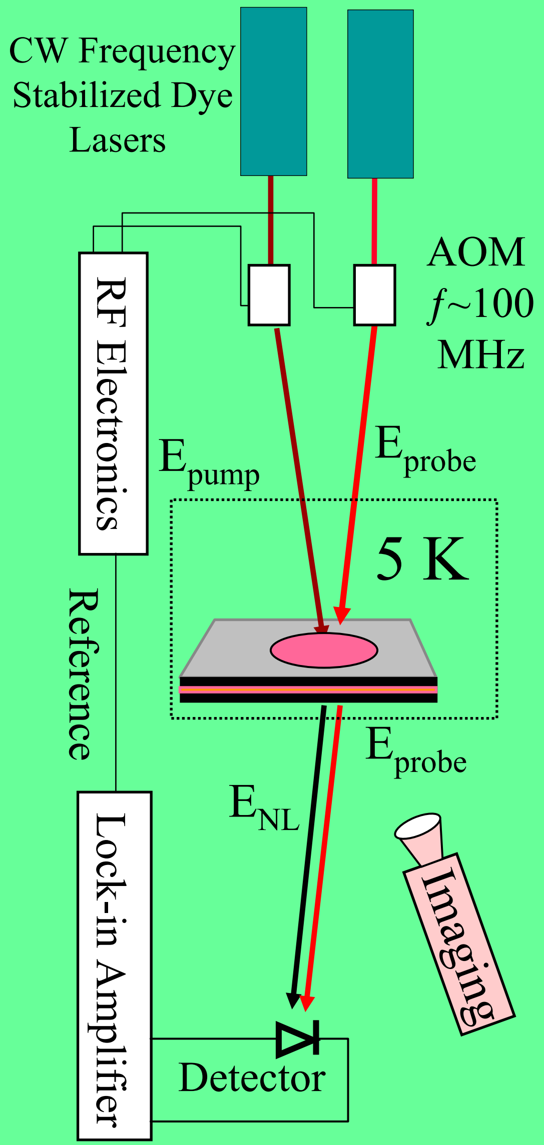
Experimental Techniques

- Linear Spectroscopy: Photoluminescence
  - Indirect probe of exciton resonances
  - Requires spectral diffusion of excited carriers

- Coherent Nonlinear Spectroscopy:
  - CW differential transmission
  - Resonant excitation
  - Probe coherent interaction in the system

$$E_{NL}\ (\varepsilon) \sim \chi^{(3)}\ (\varepsilon)\ E_{pump}\ E^*_{pump}\ E_{probe}$$

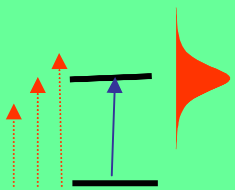
$$I_{signal} = \mathrm{Re}\ (\ E_{NL} E^*_{probe}\ )$$



D Steel et al U Michigan

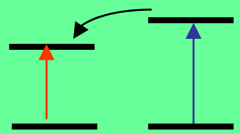
Non-degenerate Nonlinear Spectroscopy: Advantages

- Probe single excitonic state decay dynamics



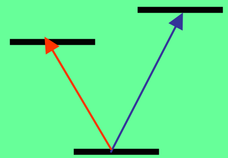
Measure both T_1 and T_2 *

- Probe coupling between different excitonic states



Probe inter-dot energy transfer and dot-dot coupling

Study excited states of excitons

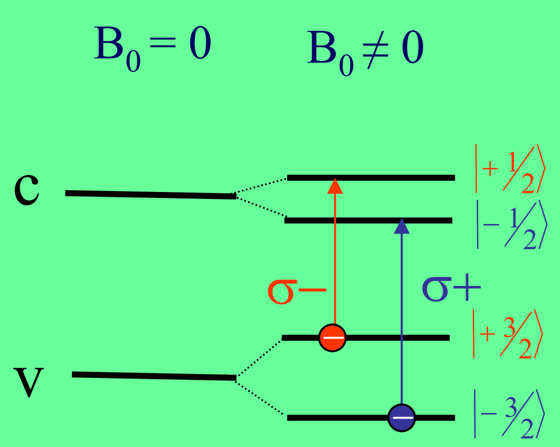


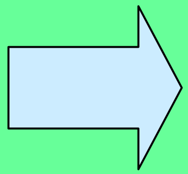
Probe multi-exciton correlation effects ;

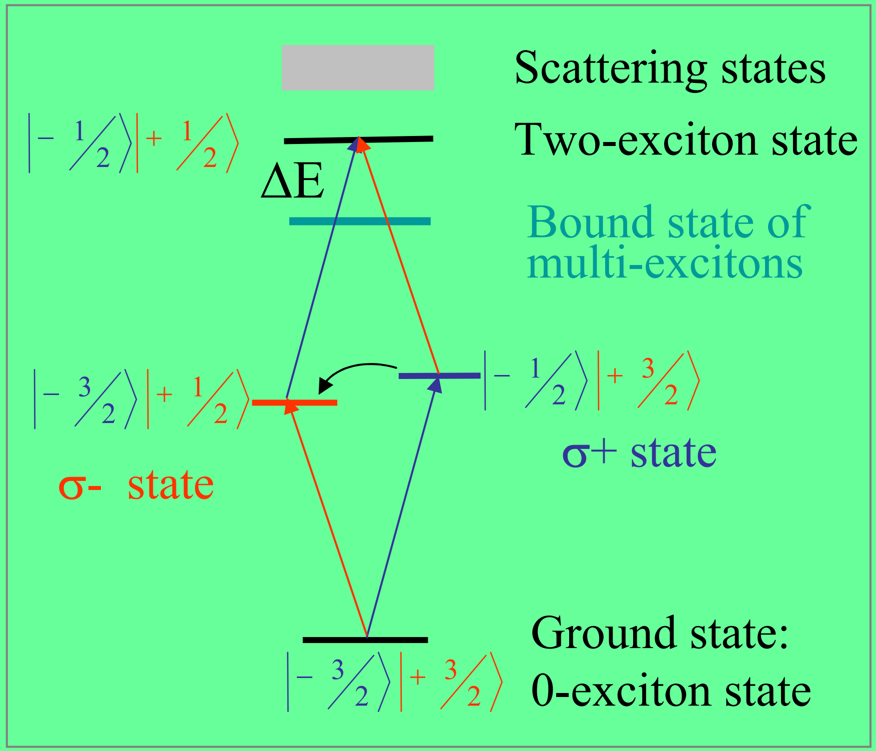
Study the coherent interaction between exciton doublet .

* **Nicolas H. Bonadeo *et al*., PRL, 81, 2759 (1998)**

hh Excitonic Level Diagram

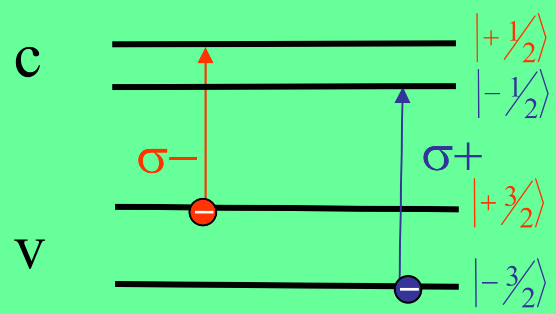






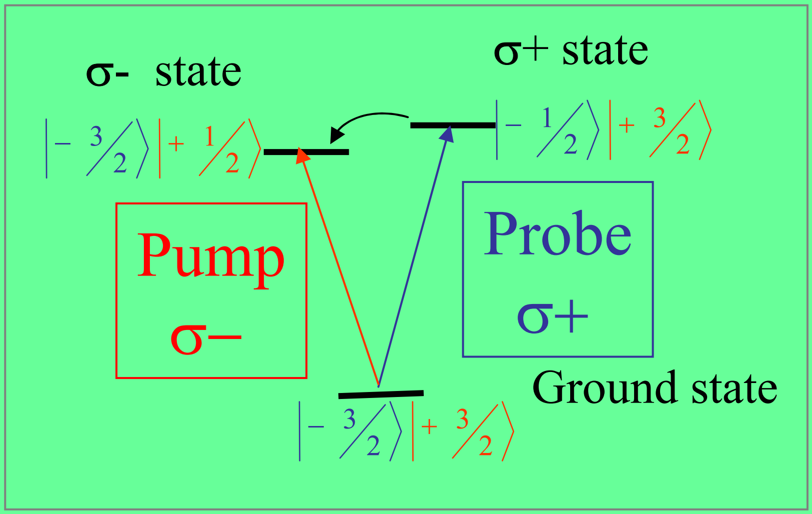
Non-degenerate experiments can excite both σ+ and σ- excitons by varying the frequency and the polarization of the excitation beams (pump and probe beam).

Exciting Two Electrons



Resonant and coherent excitation of two electrons

3-level diagram in 2-electron basis



Two contributions:

1. Incoherent:

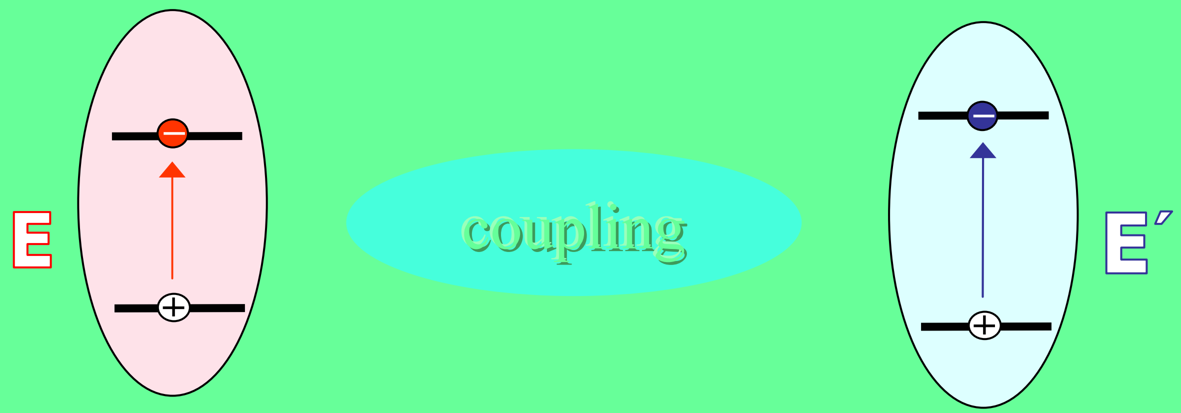
   Ground state depletion

2. Coherent:

   Zeeman coherence

   between σ- and σ+ state

Optically Entangling Two Systems: the Importance of Coupling



$$H_{total} = H_0 + H' \;+\; ?? \;+\; H_0 + H'$$

- Without coupling → Product state of the two subsystems.
- A strong coupling allows one system to see the excitation of the other.

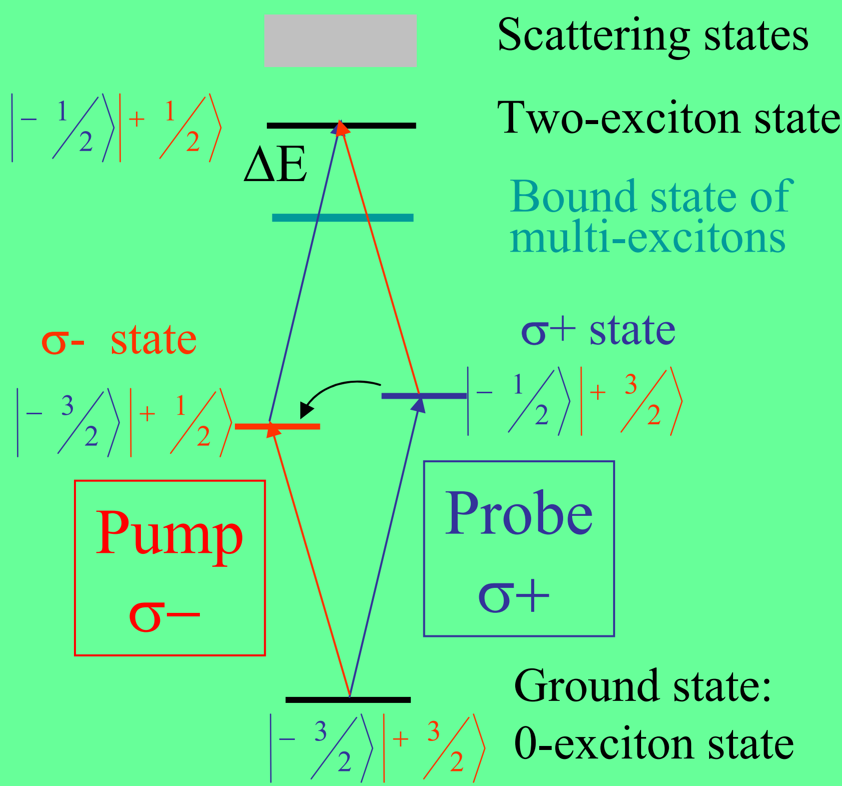
  Coulomb interaction between charged particles: trapped ions

  Magnetic dipole interaction: NMR systems

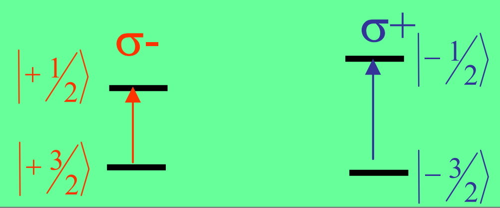
  Exciton-exciton Coulomb coupling: excitons
- Mutual coherence between E and E´ is essential .

Coulomb Correlation*:

Coulomb interaction between electron-hole pairs within single QD



Two-exciton state cancels the signal due to the symmetry in the level diagram.

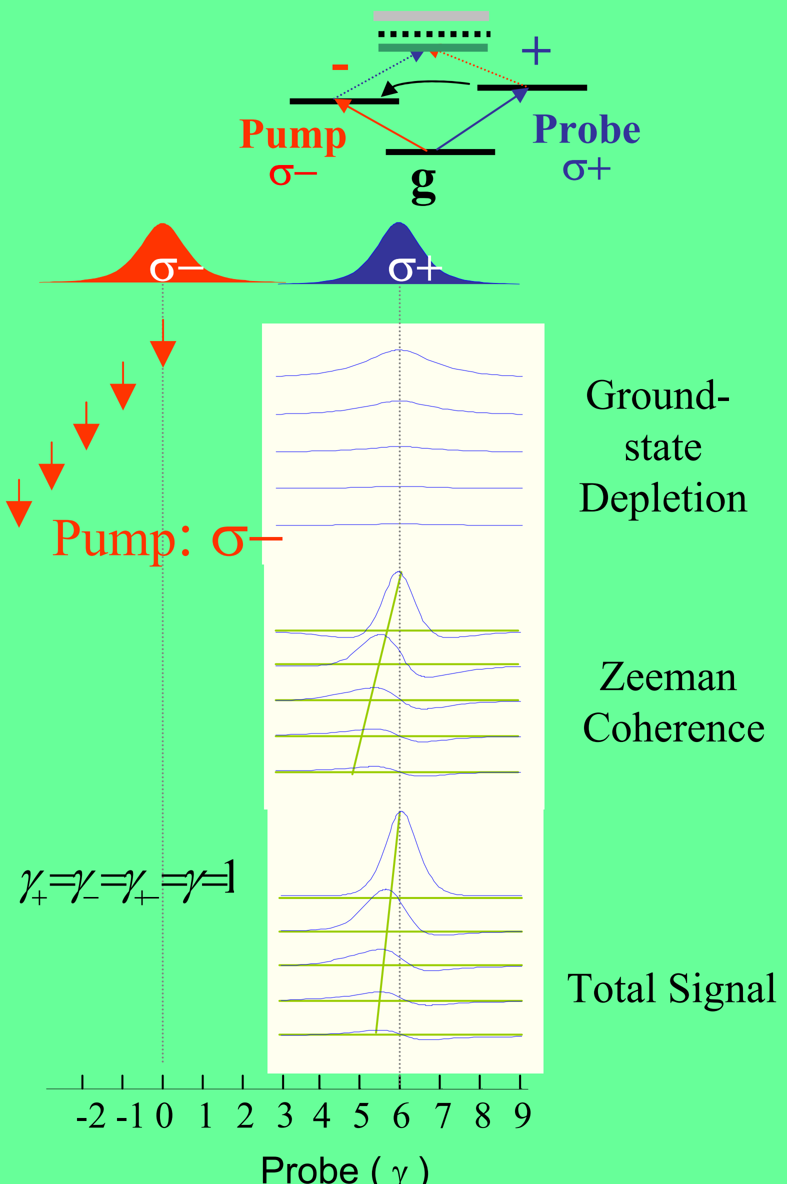


But

Two electrons are involved within a single QD. ⇨ Coulomb interaction between the two excitons is important.

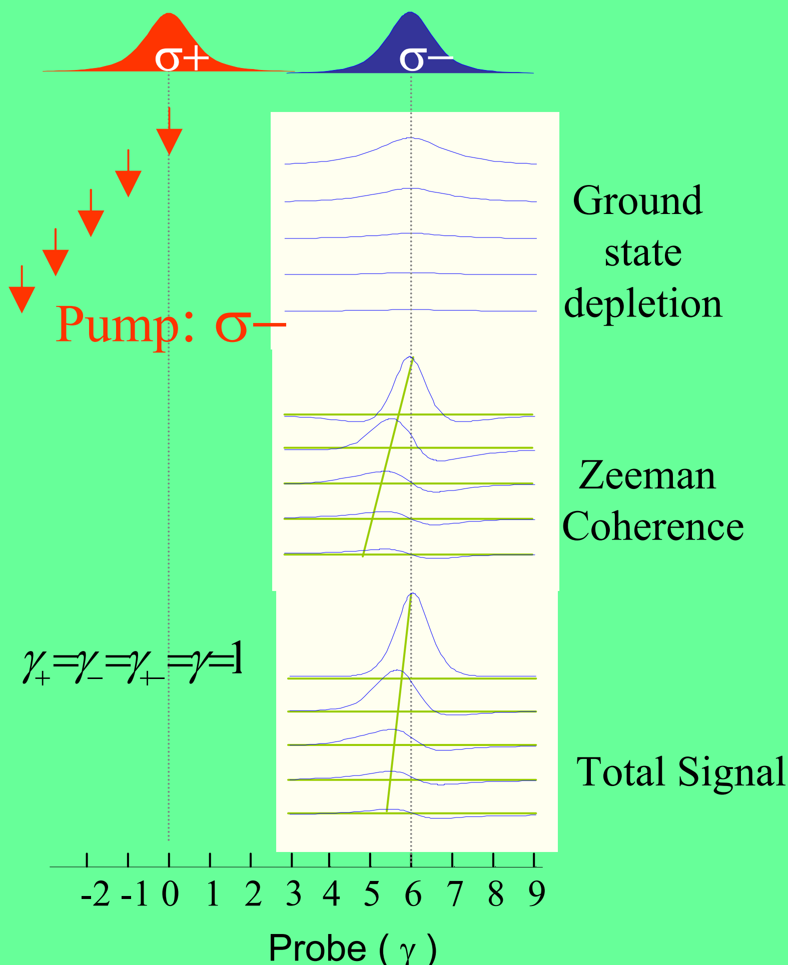
* Kner et al Phys. Rev. Lett. **78**, 1319 (1997)
Ostreich et al Phys. Rev. Lett. **74**, 4698 (1995)

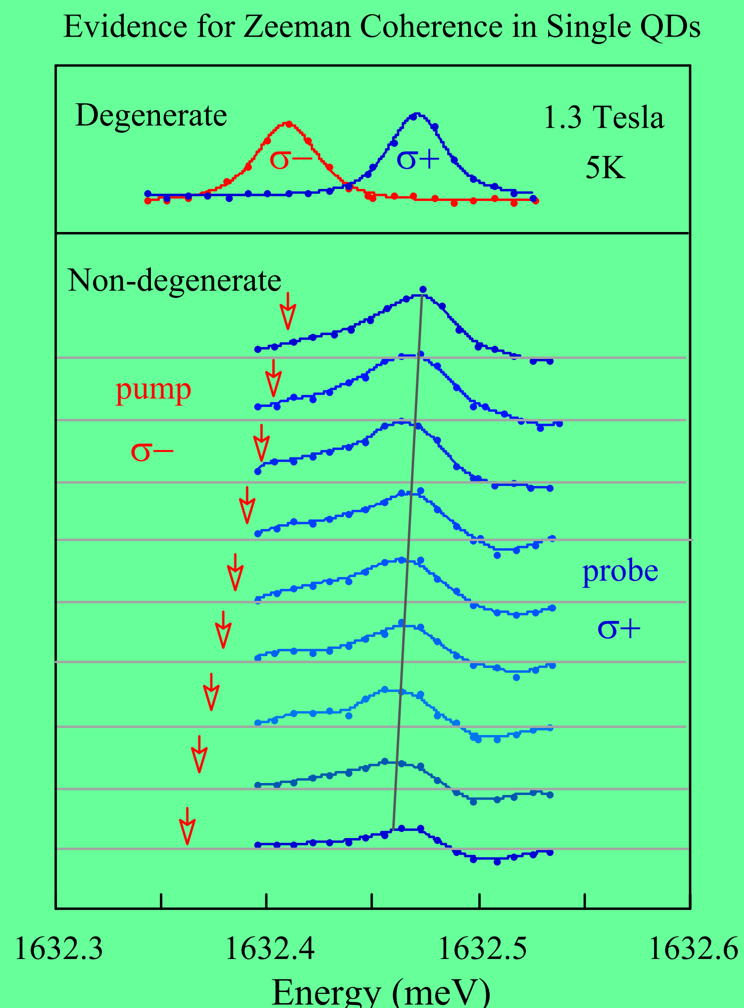
Turn on the Coulomb Correlation



Turn off the Coulomb Correlation

Experiment : Coulomb Correlation and Zeeman Coherence





Creation of two-electron entanglement

Ultimate Goal

- ***Combine:***

  Optical control of individual QDs

  Long spin lifetimes

  QD nanostructure engineering

- ***To produce:***

  Qubit register of QD spins

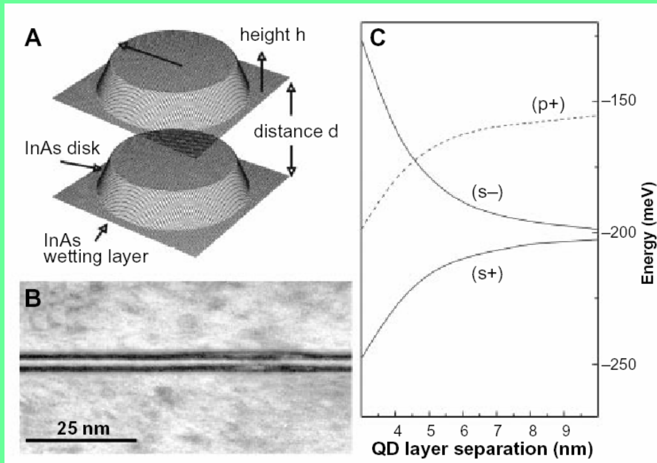
Coherence of a single QD can be controlled – Gammon et al have demonstrated this for excitons [Stievator et al., PRL (2001)]

Next: QNOT and other quantum gates

Spins have long coherence times: dephasing times T2* = 5ns-300ns

[Dzhioev et al., PRL (2002)]

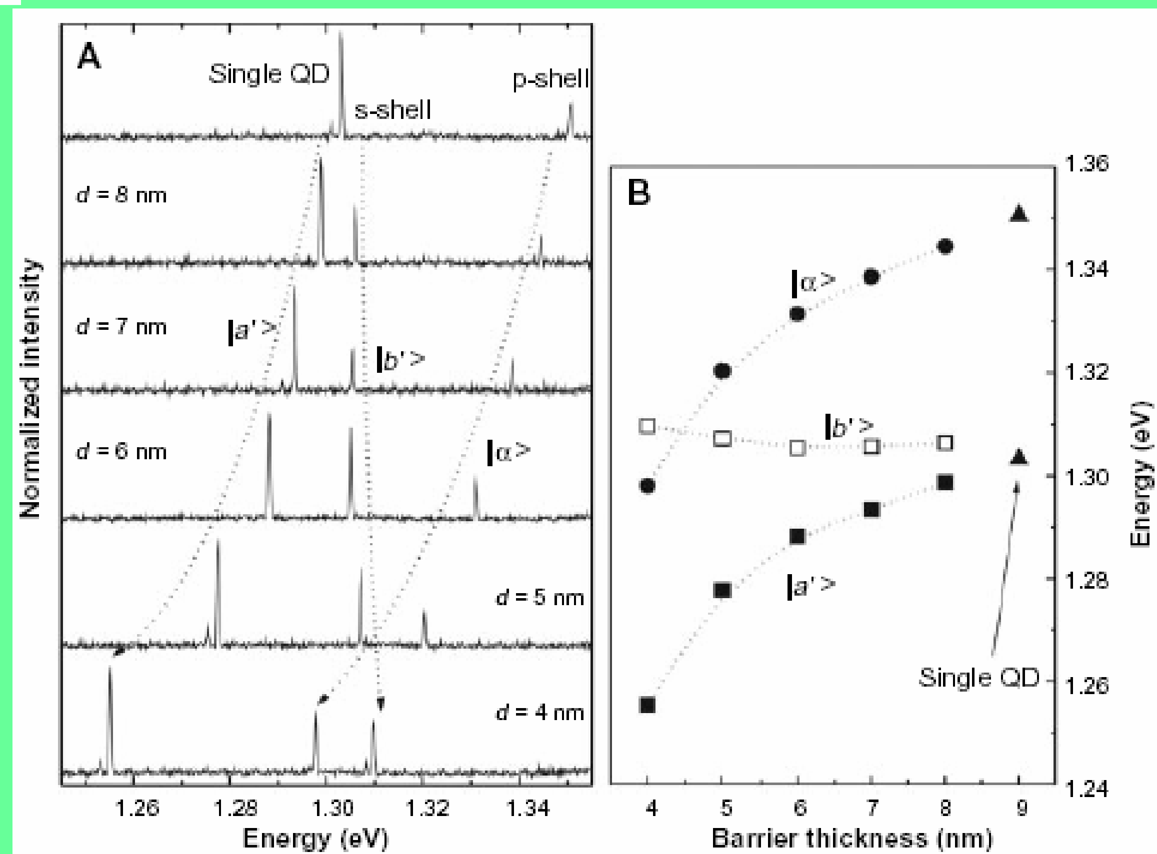
Next: explore in single QDs; optical read-out and initialization schemes



Coupling and Entangling of Quantum States in Quantum Dot Molecules

M. Bayer,[1*] P. Hawrylak,[2*] K. Hinzer,[2,3] S. Fafard,[2]
M. Korkusinski,[2,3] Z. R. Wasilewski,[2] O. Stern,[1] A. Forchel[1]

Science 2001



Imamoglou – single photon source from QDs

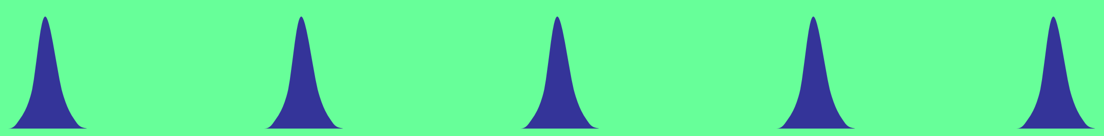
Imamoglou – single photon source from QDs

Michler et al., Science **290**, 2282 (2000)

- It is difficult to isolate a single photon, or fix the number of photons in a pulse
- Fluctuations in photon number mask the quantum features of light
- A stable train of laser pulses have Poissonian (photon number) statistics

→ It is desirable to have *single photon* sources:

single photon turnstile



Complete regulation of photon generation

Single Photon Sources

Quantum Cryptography: secure key distribution by single photon pulses

Quantum Computation: single photons + linear optical elements
E. Knill, R. Laflamme, and G.J. Milburn, Nature **409**, 46 (2001)

Available sources:

- Highly attenuated laser pulse $\Rightarrow$ Poisson fluctuations
- Parametric down conversion $\Rightarrow$ Random generation of single photons

Possible solution: Deterministic (triggered) single photon emission:
Single Photon Turnstile Device
A. Imamoglu, Y. Yamamoto, Phys. Rev. Lett. **72**, 210 (1994)

Experiments:

Coulomb blockade of electron/hole tunneling in a mesoscopic pn-junction:

J. Kim et al. Nature **397**, 500 (1999)

Single Molecule at room temperature:

B. Lounis and W.E. Moerner, Nature **407**, 491 (2000)

Single InAs Quantum Dot in a microcavity:

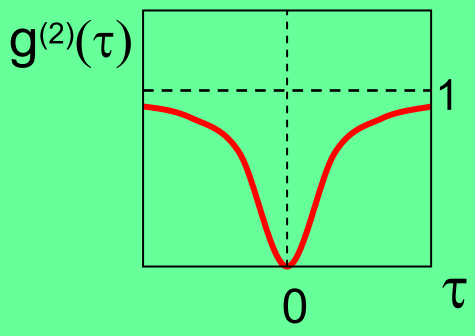
P. Michler et al., Science **290**, 2282 (2000)

Signature of a triggered single-photon source

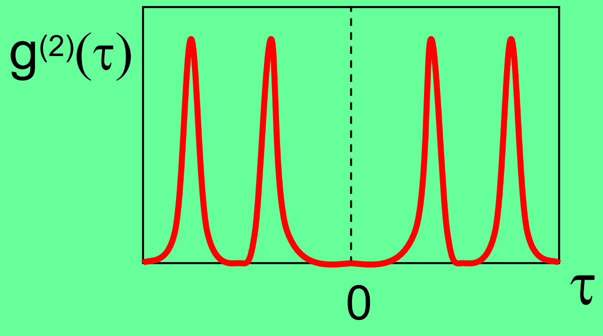
- Intensity (photon) correlation function:

$$g^{(2)}(\tau) = \frac{\langle : I(t) I(t+\tau) : \rangle}{\langle I(t) \rangle^2}$$

- Single quantum emitter (I.e. an atom) driven by a cw laser field exhibits photon antibunching.

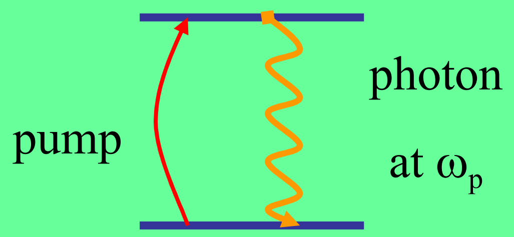


- Triggered single photon source: absence of a peak at τ=0 indicates that none of the pulses contain more than 1 photon.



Photon antibunching

- Intensity correlation ($g^{(2)}(\tau)$) of light generated by a single two-level (anharmonic) emitter.



- Assume that at τ=0 a photon is detected:

  - We know that the system is necessarily in the ground state |g>

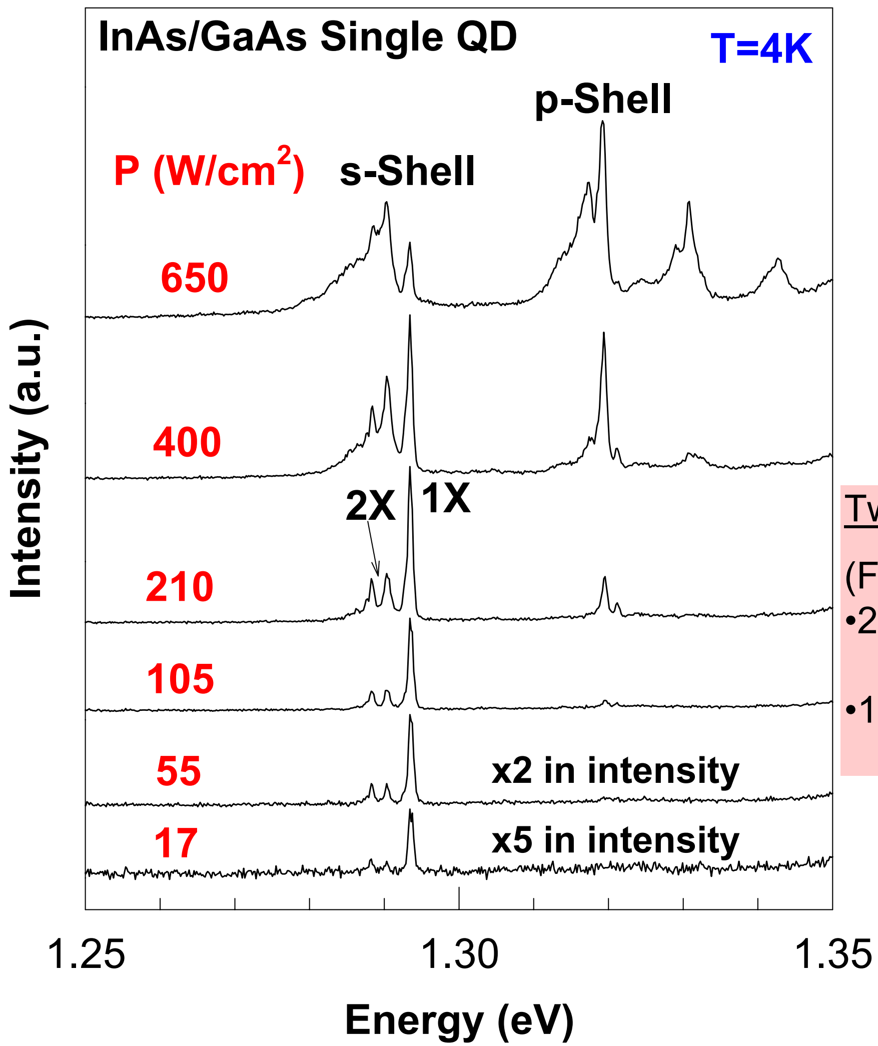
  - Emission of another photon at $\tau=0+\varepsilon$ is impossible.

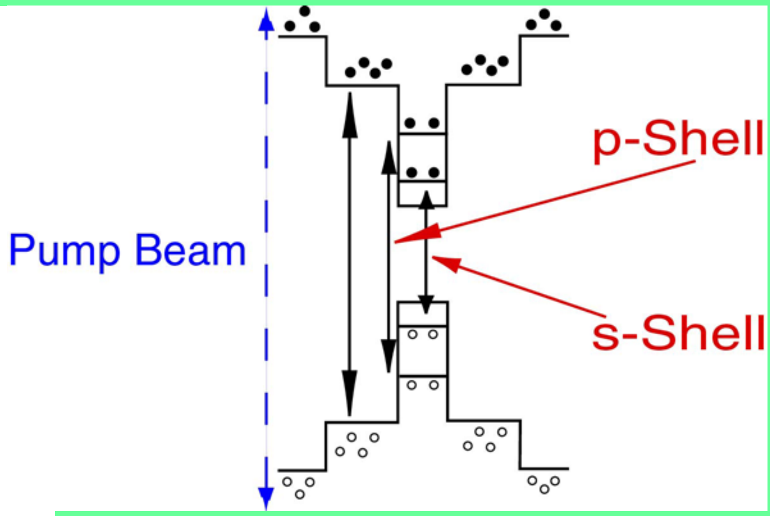
  $\Rightarrow$ Photon antibunching: $g^{(2)}(0) = 0$. (nonclassical light)

- $g^{(2)}(\tau)$ recovers its steady-state value in a timescale given by the spontaneous emission time.

- If three are two or more 2-level emitters, detection of a photon at τ=0 can not ensure that the system is in the ground state ($g^{(2)}(0)$ >0.5).

Single InAs Quantum Dots





Two main wavelengths emitted from each shell

(For s-Shell)

- 2X recombination while there is already an e-h pair in the s-shell (biexciton)
- 1X recombination while there is no other e-h pair in the s-shell (exciton)

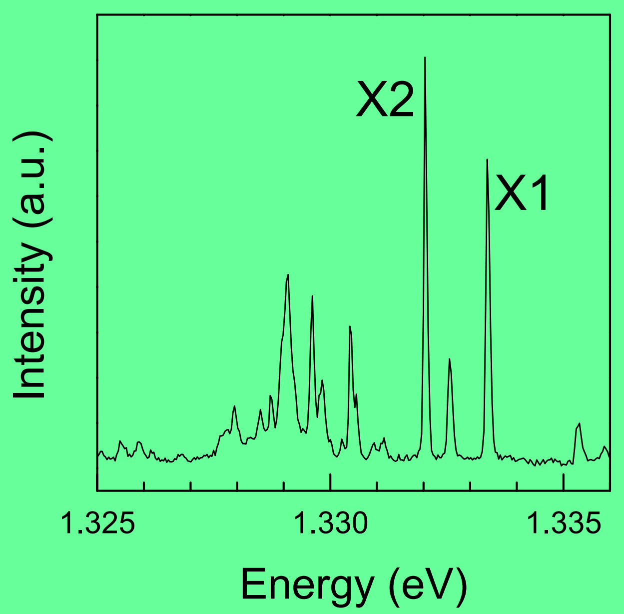
Due to ***carrier-carrier interaction***

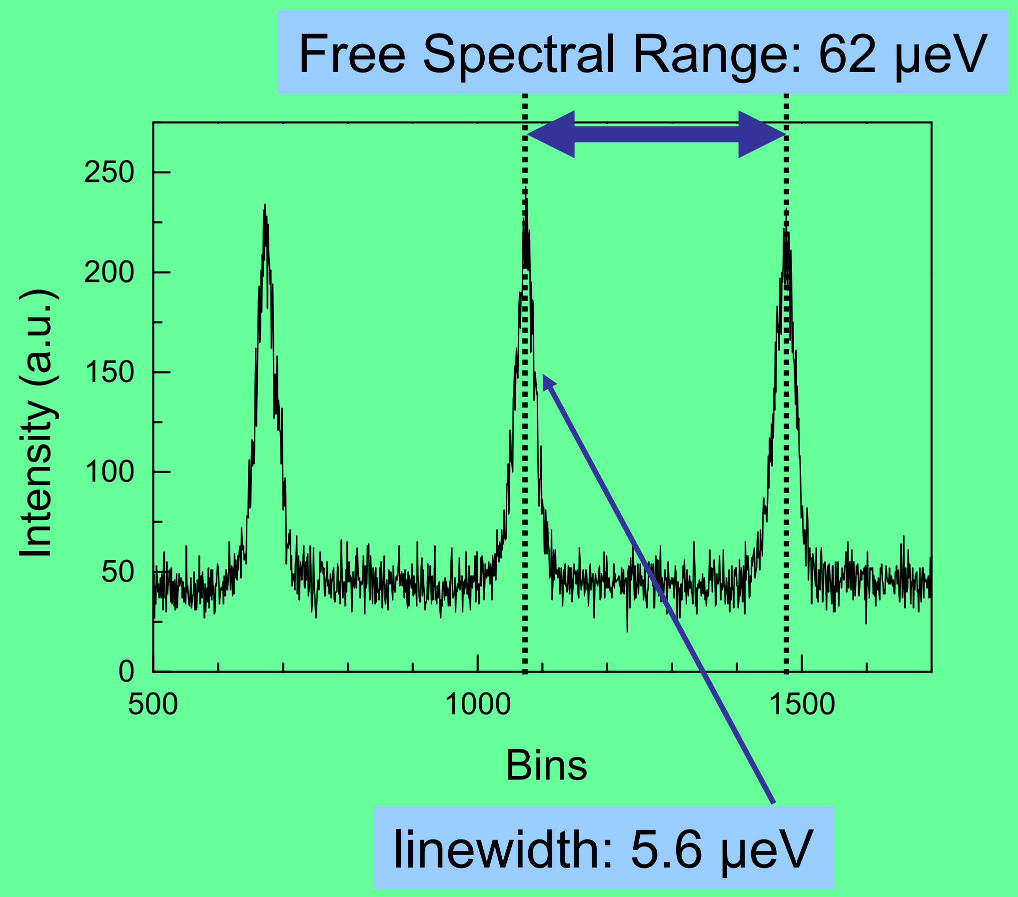
Typically $h\nu_{1X} = h\nu_{2X}$ + **2-3meV**

Unique wavelengths for 1X and 2X transitions

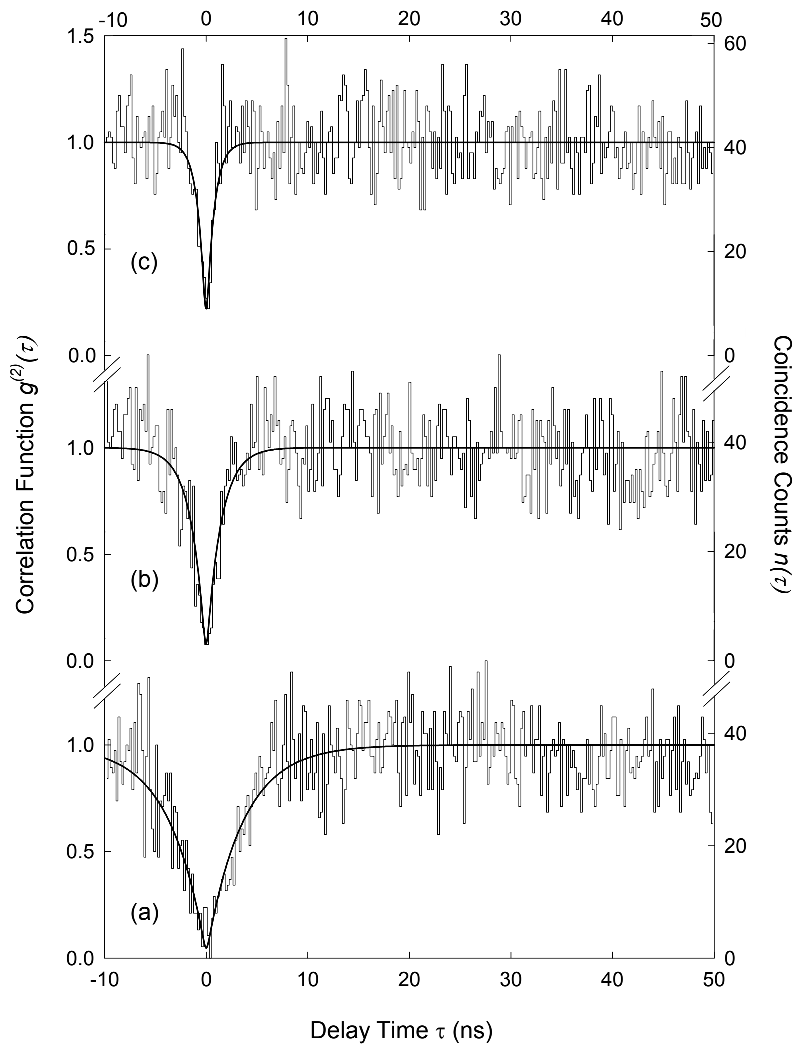
Exciton linewidth measured by a scanning Fabry-Perot

Under non-resonant pulsed excitation





Photon antibunching from a Single Quantum Dot

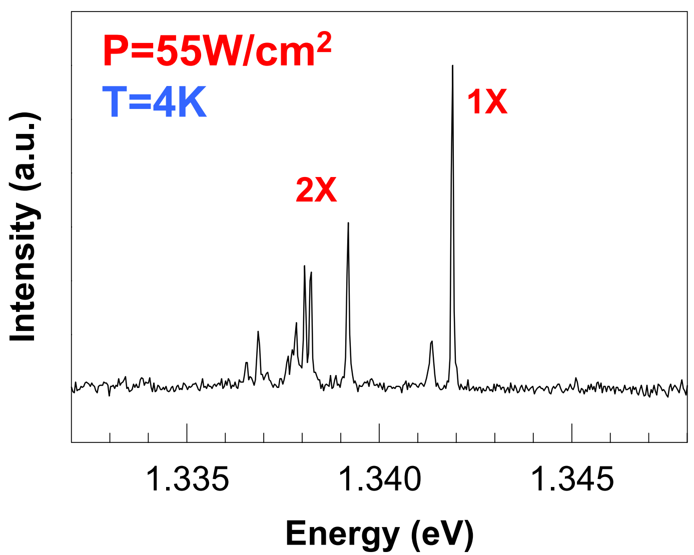


<u>105 W/cm²</u>
$g^2(0)$=0.1
τ = 750 ps

<u>55 W/cm²</u>
$g^2(0)$=0.0
τ = 1.4 ns

<u>15 W/cm²</u>
$g^2(0)$=0.0
τ = 3.6 ns

PL spectrum



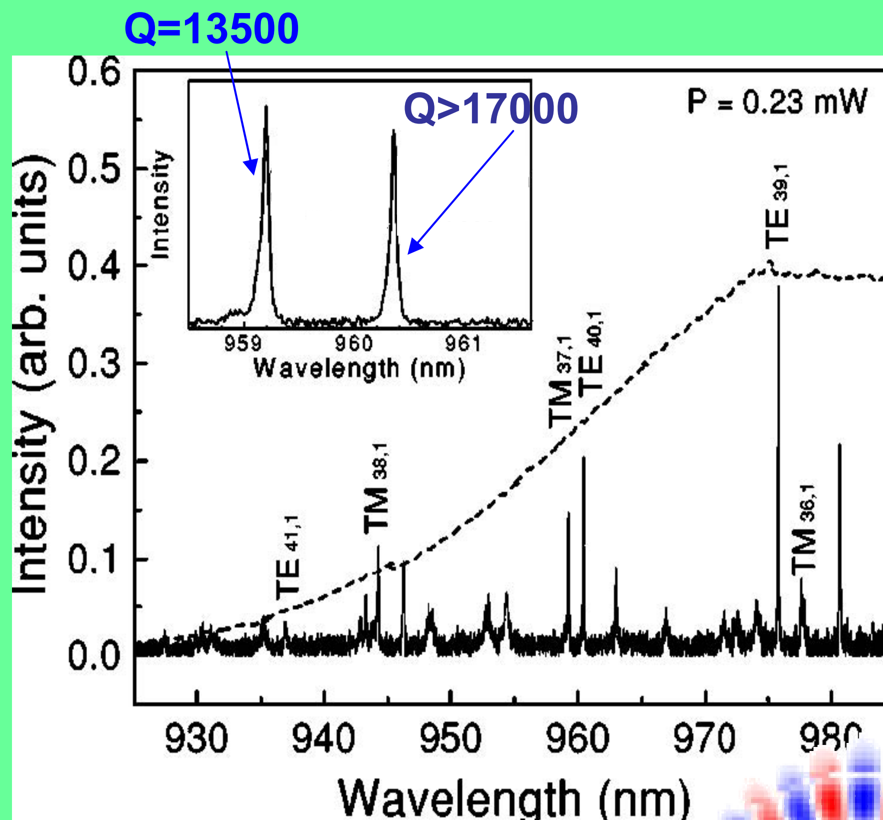
⇨ **proof of atom-like behavior**

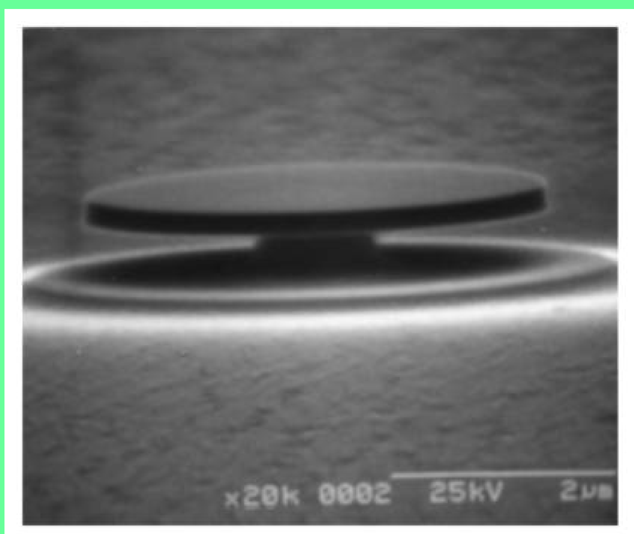
A single quantum dot excited with a short-pulse laser can provide single-photon pulses on demand

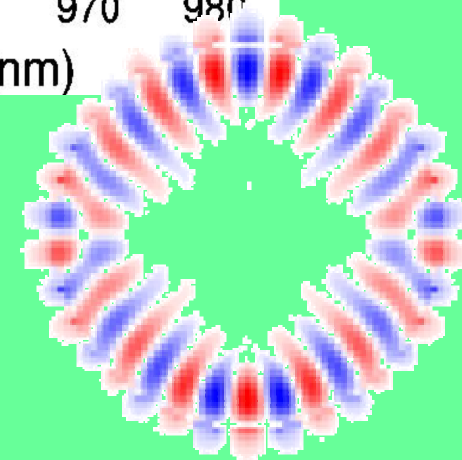
BUT

How about embedding quantum dots in a microcavity to increase collection efficiency and fast emission?

Microdisk Cavities







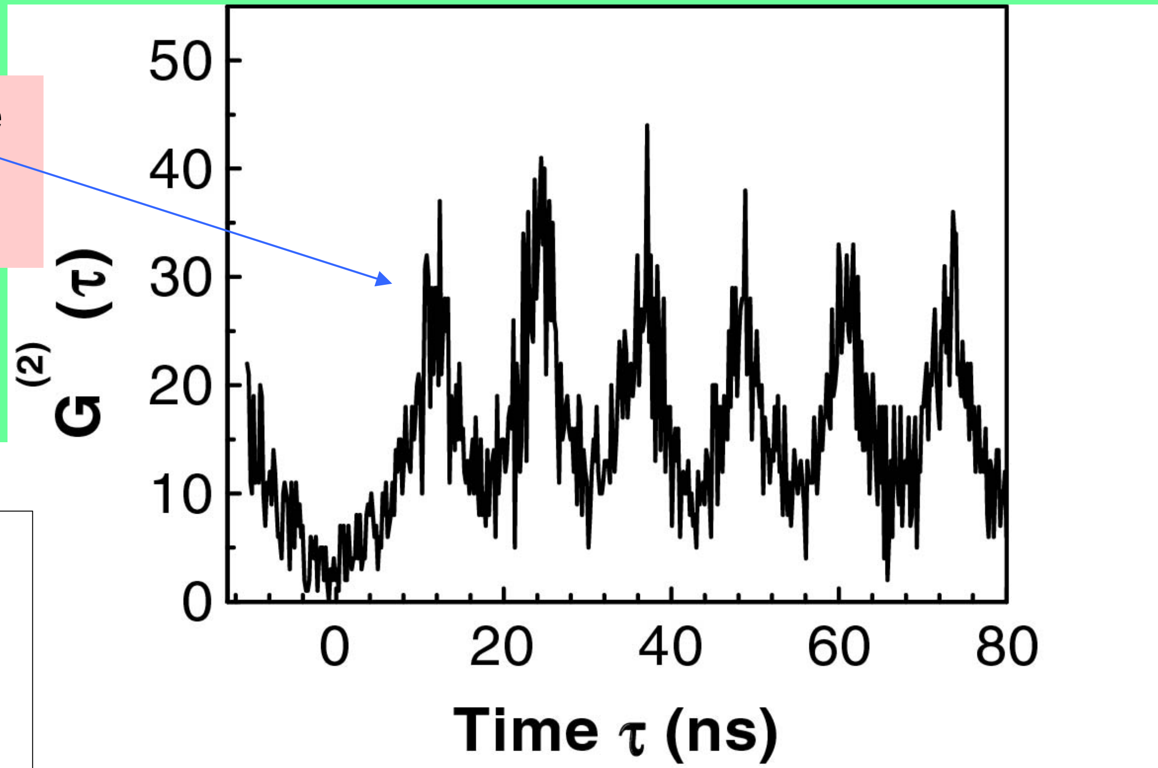
No roughness on the sidewall up to 1nm !
Q>18000 for 4.5μm diameter microdisk
Q=9000 for 2μm diameter microdisk

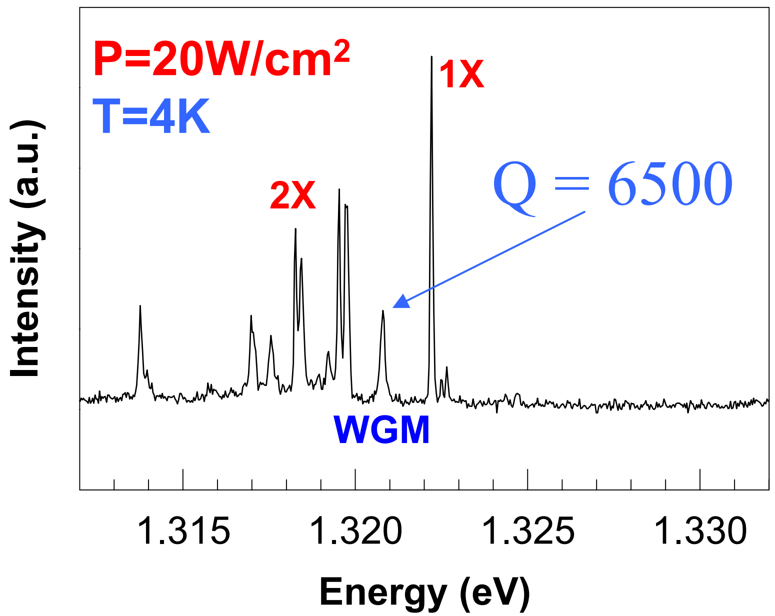
Fundamental whispering gallery modes cover a ring with width ~ $\lambda/2n$ on the microdisk

Michler et al., Appl. Phys. Lett. **77**, 184 (2000)

A single quantum dot in a microdisk

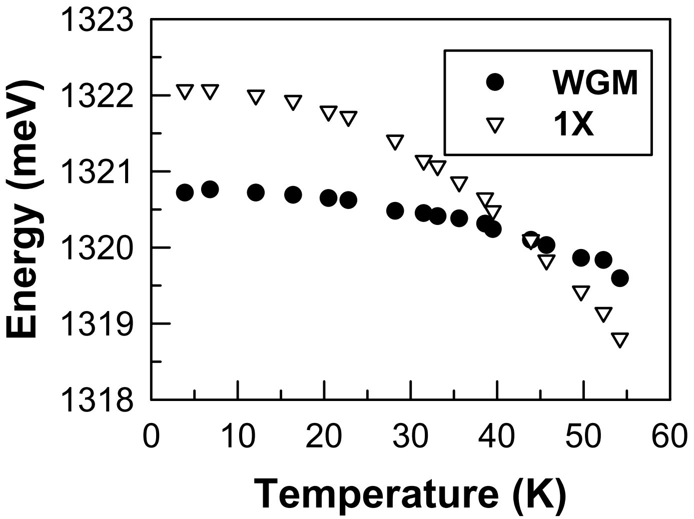
Larger width of the peaks due to *larger lifetime* of the quantum dot

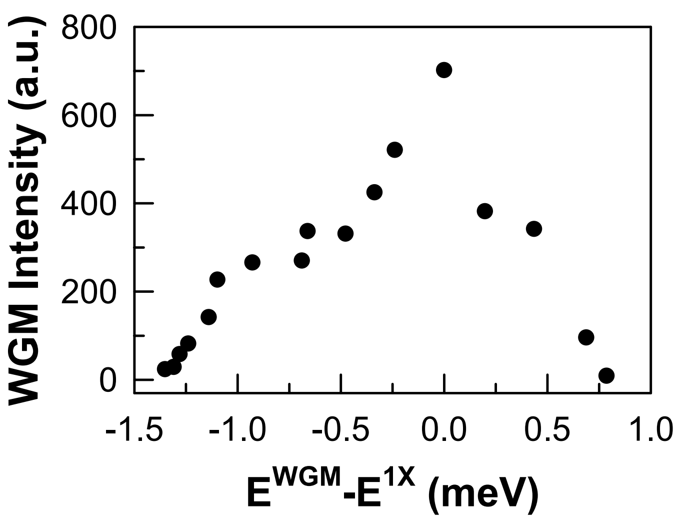




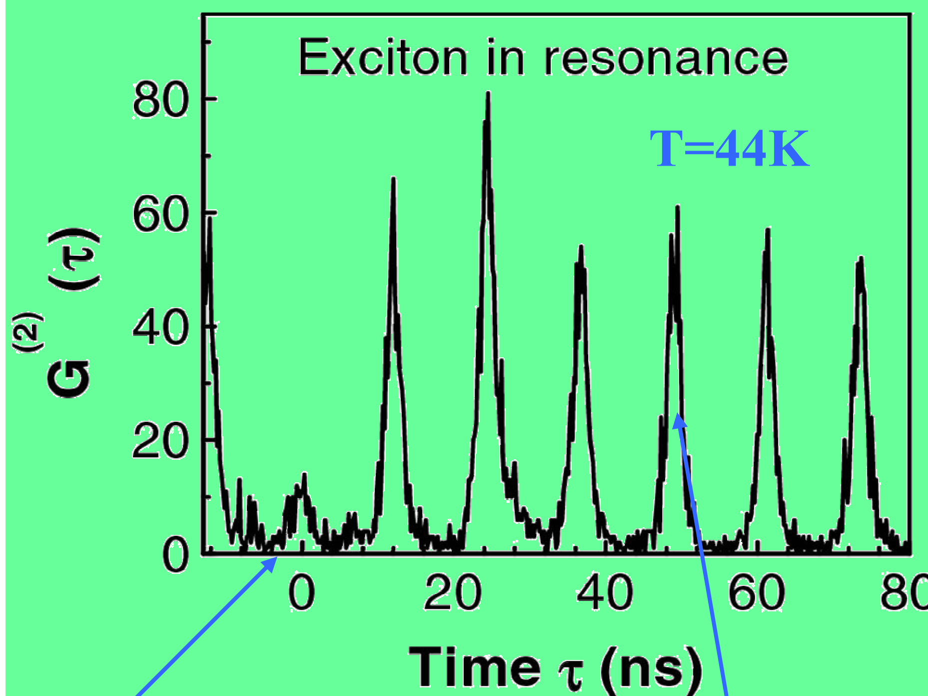
Pump power *well above* saturation level

Tuning the quantum dot into resonance with a Cavity Mode





Cavity coupling can provide **better collection**



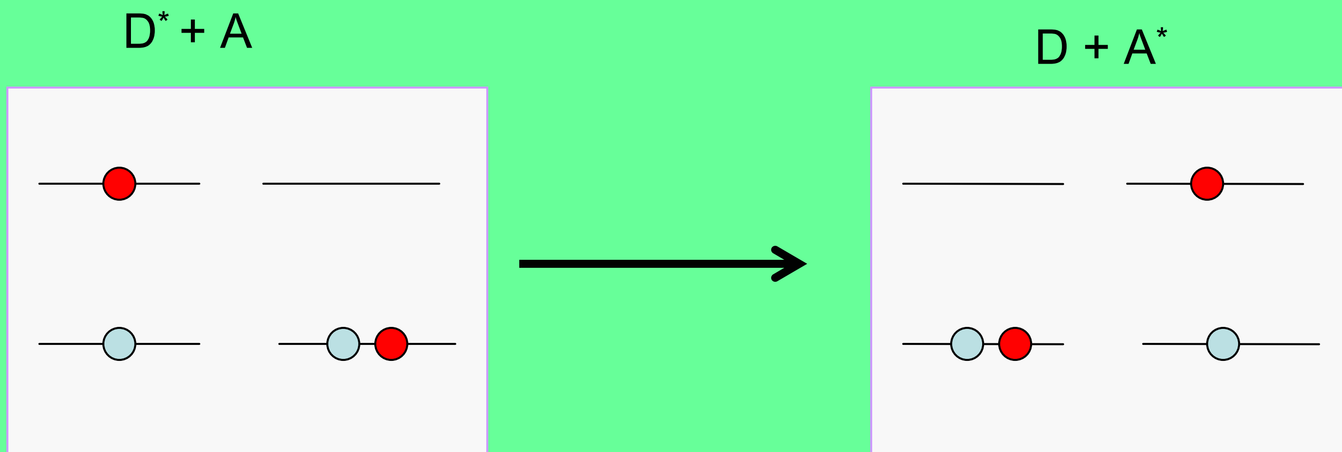
- Small peak appears at τ=0:
- **Purcell effect**: reduction of emission time

Klimov – Förster coupling between QDs

Klimov – Förster coupling between QDs

Crooker et al PRL 2002

<u>Non-Radiative Energy Transfer Mechanism</u>



Coulomb-driven interaction

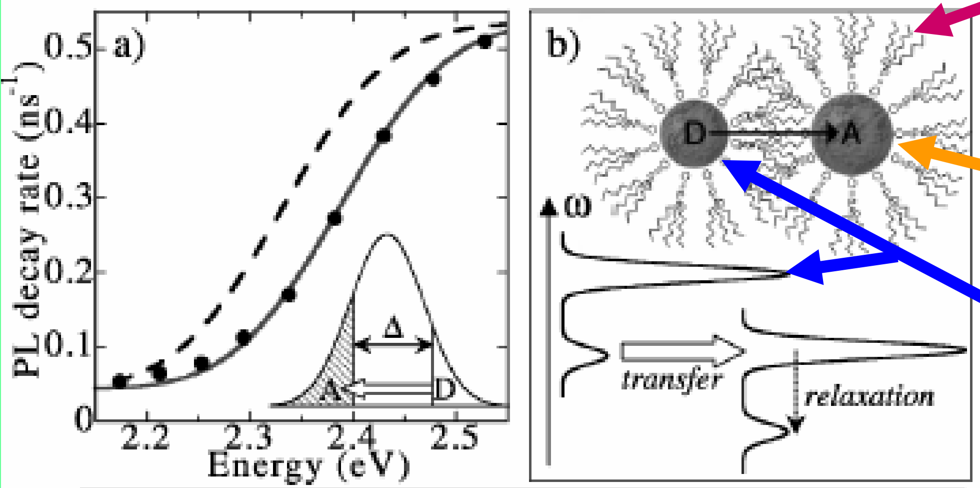
Dipole-dipole interaction (Förster 1946)
Higher multipoles interaction (Förster – Dexter)

Exchange-driven interaction (Dexter)

triocytlphosphine oxide (TOPO) ~11A

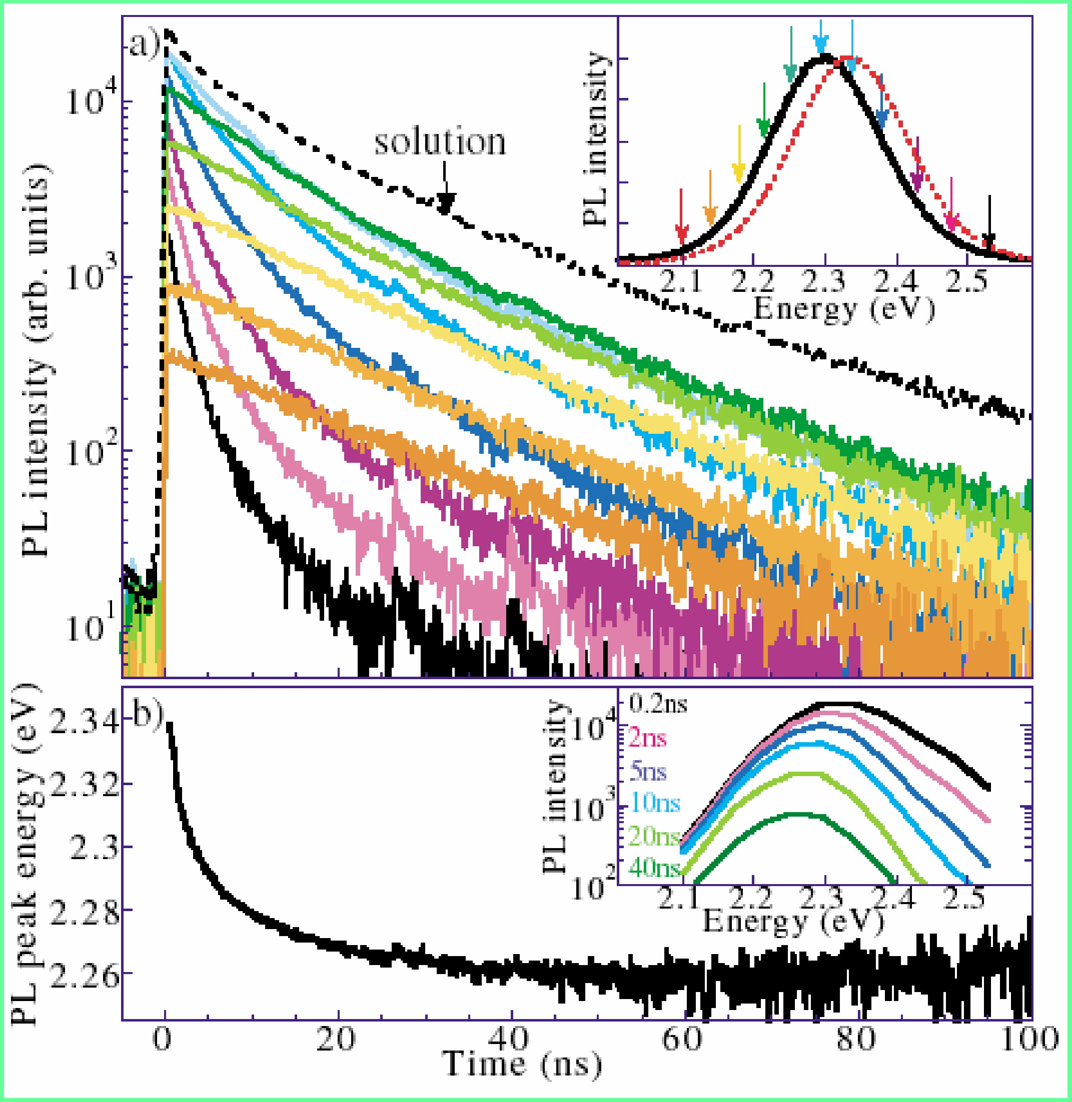
A acceptor: larger dot

D donor: smaller dot



c)	resonant NQDs	LH2 [Ref.12]
Dipole μ_A (μ_D)	25 (4.4) Debye	8.2 (8.2) D
Distance R_{DA}	54 A	18 A
Coupling J	2.4 cm^{-1}	26 cm^{-1}
Overlap integral Θ	0.004 cm	0.0004 cm
Estimated rate Γ_{et}	(38 ps)$^{-1}$	(3 ps)$^{-1}$

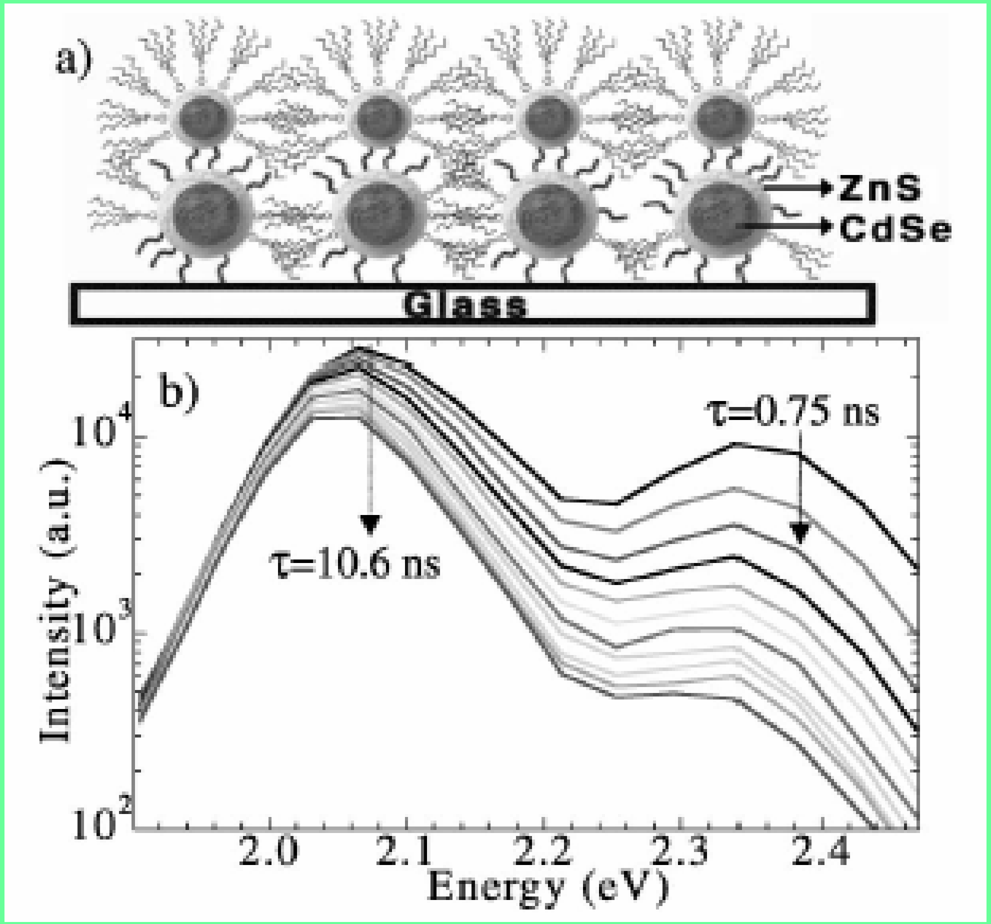
NQDs have *better* characteristics than biological light harvesting compounds, eg LH2



(a) PL decays from a dense film of monodisperse R=12.4A/9A CdSe/ZnS NQDs at the energies specified in the inset. Inset: cw PL spectra from film (solid) and original solution (dashed).

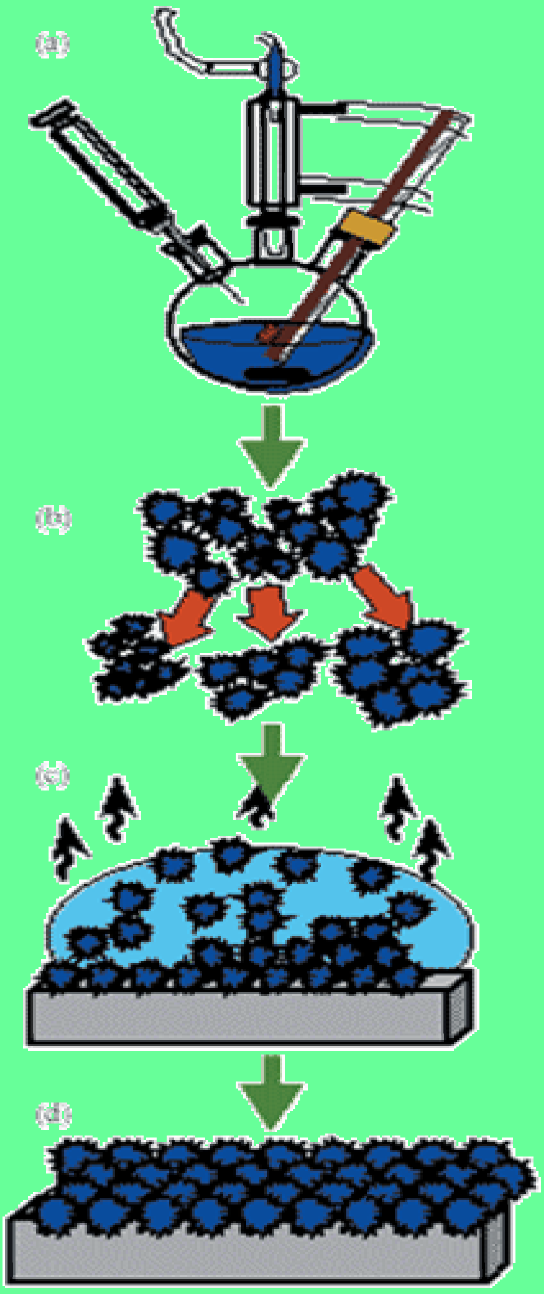
(b) Dynamic redshift of the peak emission.
Inset: PL spectra at the specified times.

Crooker et al PRL 2002



(a) Schematic of NQD energy-gradient bilayer for light harvesting—13 A dots on 20.5 A dots.

(b) ‘‘Instantaneous’’ PL spectra at 500 ps intervals (from 0 to 5 ns), showing rapid collapse of emission from 13 A dots.


(a)
(b)
(c)
(d)

Our Goal

Study the excitation energy transfer in quantum-dot arrays using an appropriate model Hamiltonian

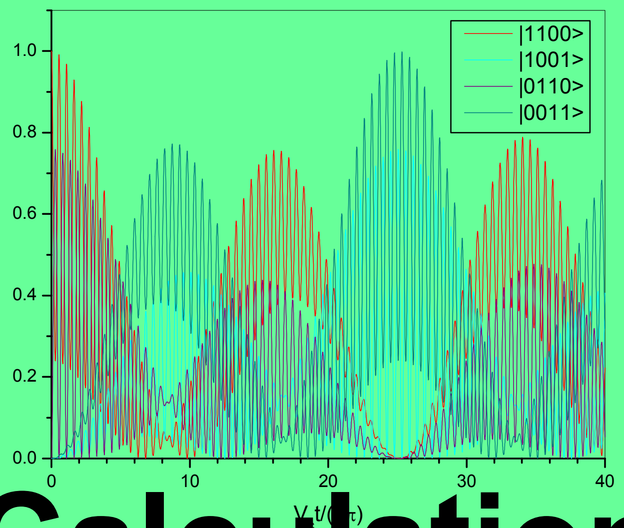
$$H = \sum_{i,j}^{N} (T_e c_i^\dagger c_j + T_h d_i^\dagger d_j) + \sum_{i}^{N} U c_i^\dagger c_i d_i^\dagger d_i + \sum_{i,j=NN}^{N} U_{NN} c_i^\dagger c_i d_j^\dagger d_j + \boxed{\sum_{i \neq j}^{N} V_s c_i^\dagger d_i^\dagger d_j c_j}$$

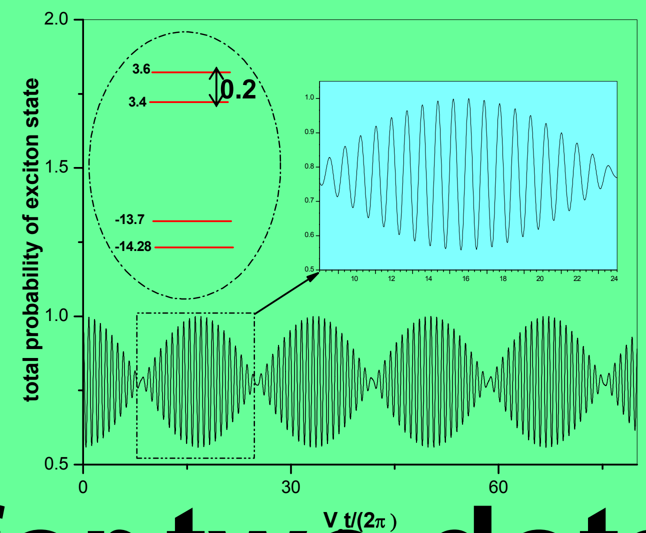
Calculations for two dots

Period of small oscillation ~ 1.3 ~ $\frac{1}{2T_e/V_c}$

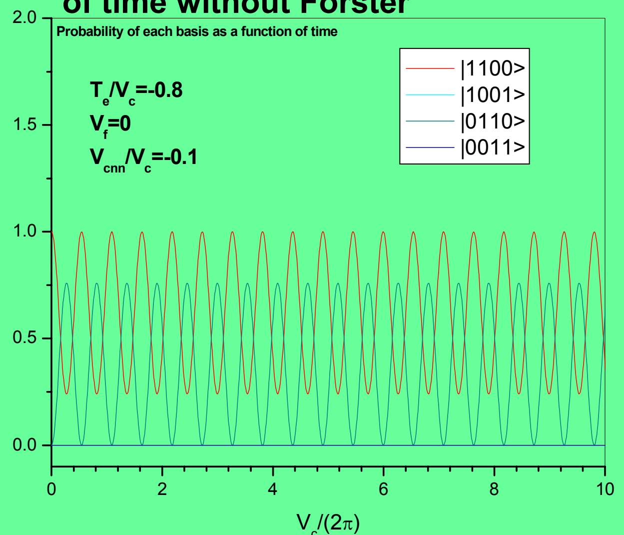
Period of large oscillation ~ 15 ~ $\frac{1}{2V_f/V_c}$

Probability of each basis state as a function of time

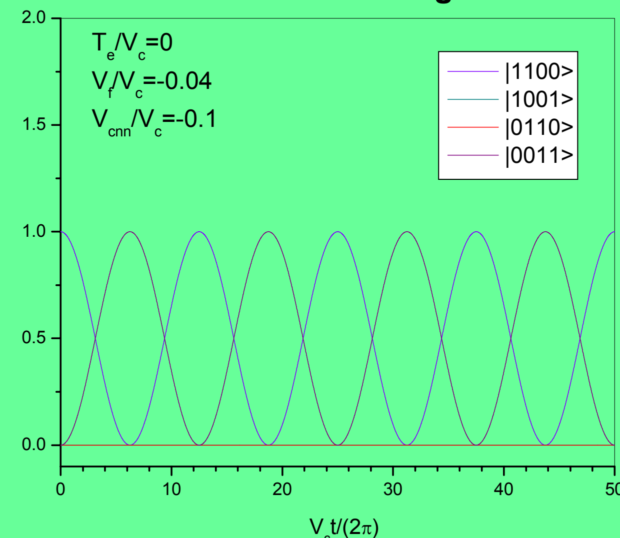


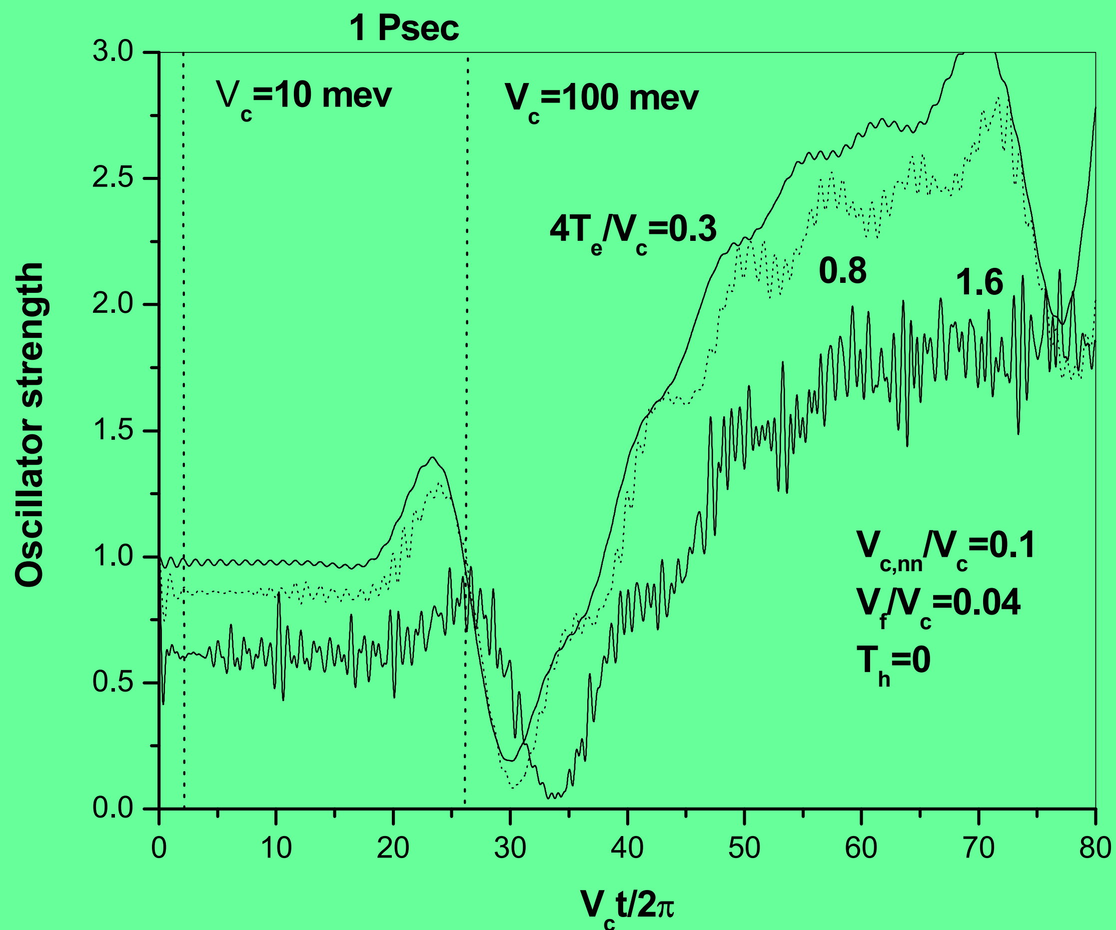


Probability of each basis as a function of time without Förster



Probability of each basis as a function of time without tunneling

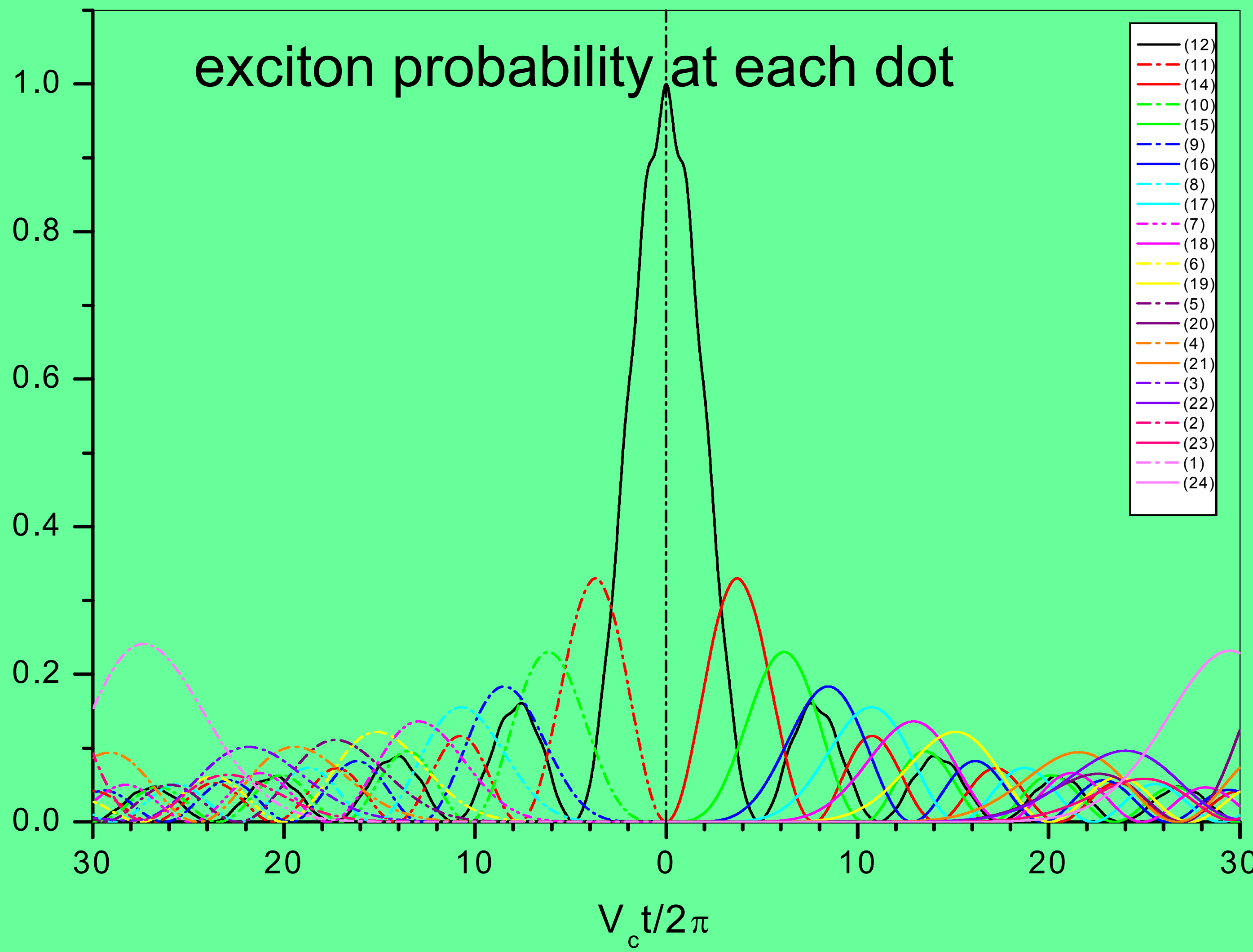


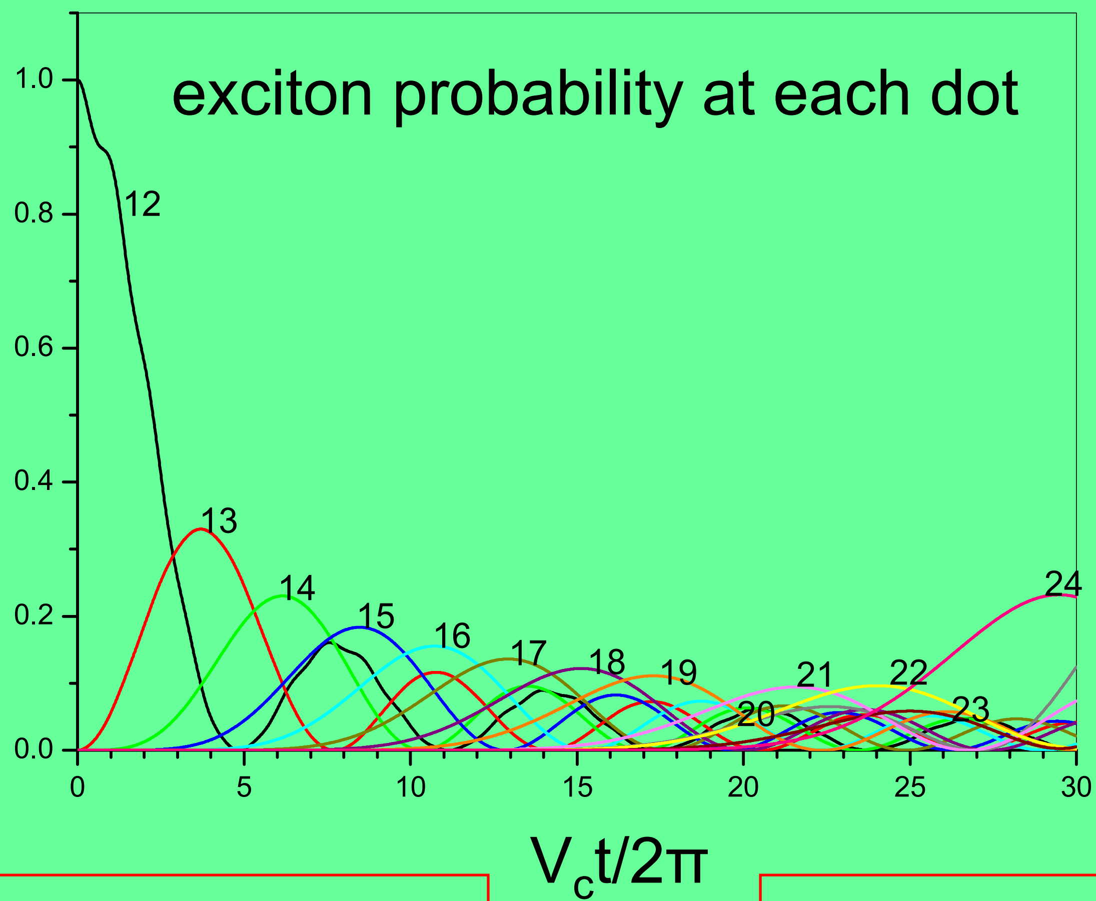


Time evolution of the oscillator strength of an exciton initially localized at dot 12 in a 24 dot chain

G. W. Bryant, Physica B 314, 15 (2002).

The “movie” of the 24 dots





Förster rate for polymer

$$\bar{t}_{n\rightarrow n+1} = \frac{h}{8|U|} = \frac{2\pi\hbar/V_c}{8|U|/V_c} \approx 3.1\frac{2\pi\hbar}{V_c}$$

From graph/calculation

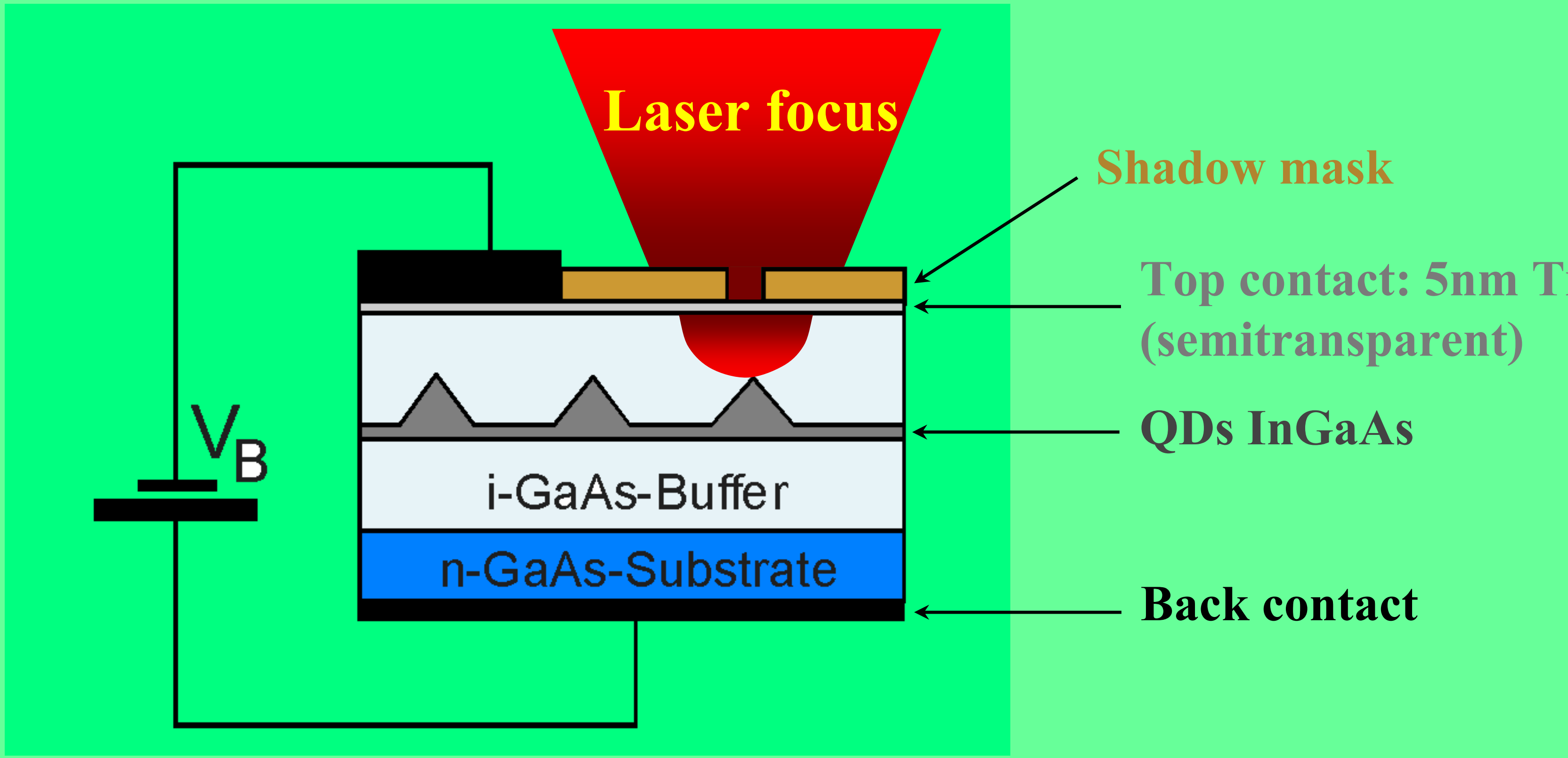
$$\bar{t} \approx 2.2\frac{2\pi\hbar}{V_c}$$

efficient interdot transfer rate

Zrenner – coherent control of quantum dot photodiodes

Zrenner – coherent control of quantum dot photodiodes

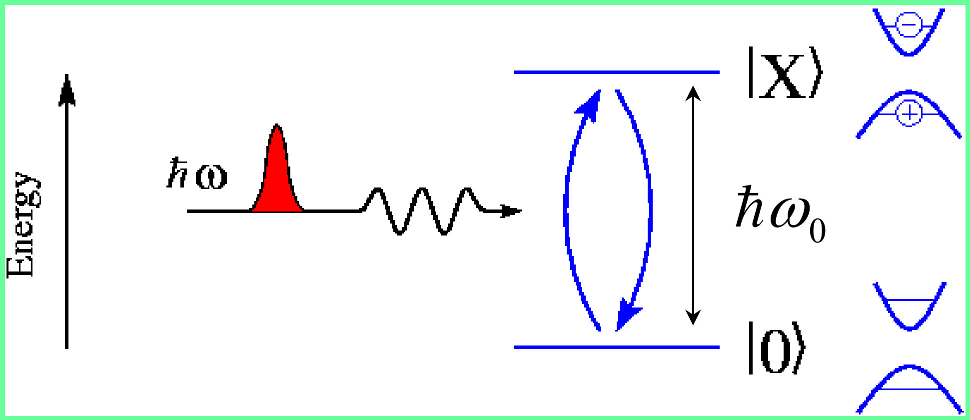
Zrenner et al Nature 2002



Possibles measurements

- ★ **Photolumenescence (PL) spectrum (for $\tau_{rad} < \tau_{tunnel}$)**
- ★ **Photocurrent (PC) spectrum (for $\tau_{rad} > \tau_{tunnel}$)**

Rabi Oscillation in a two level system



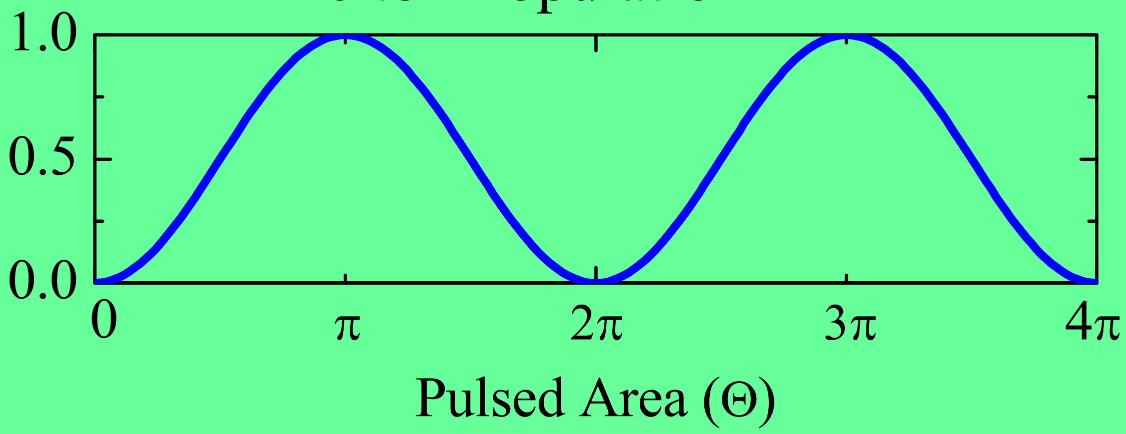
$$H = \frac{\hbar\omega_0}{2}\sigma_z + \frac{\hbar\Omega(t)}{2}\cos(\omega t)\sigma_x$$

$$\Omega(t) = \frac{\vec{\mu}\cdot\vec{\varepsilon}(t)}{\hbar}$$

For $\omega = \omega_0$ and using RWA

$$P_{0\to X} = \sin^2\left(\frac{\Theta}{2}\right)$$

$$\Theta = \int_{-\infty}^{\tau} \Omega(t)\,dt$$



Time scales for the device

$\boxed{\tau_{\mathrm{rad}} \simeq 1\ \mathrm{ns}}$ $\boxed{\tau_{\mathrm{tunnel}} \simeq 3\ \mathrm{ps} \rightarrow \infty}$ **Tuned by Vb**

Photoluminescence $\tau_{\mathrm{rad}} < \tau_{\mathrm{tunnel}}$

NeHe laser

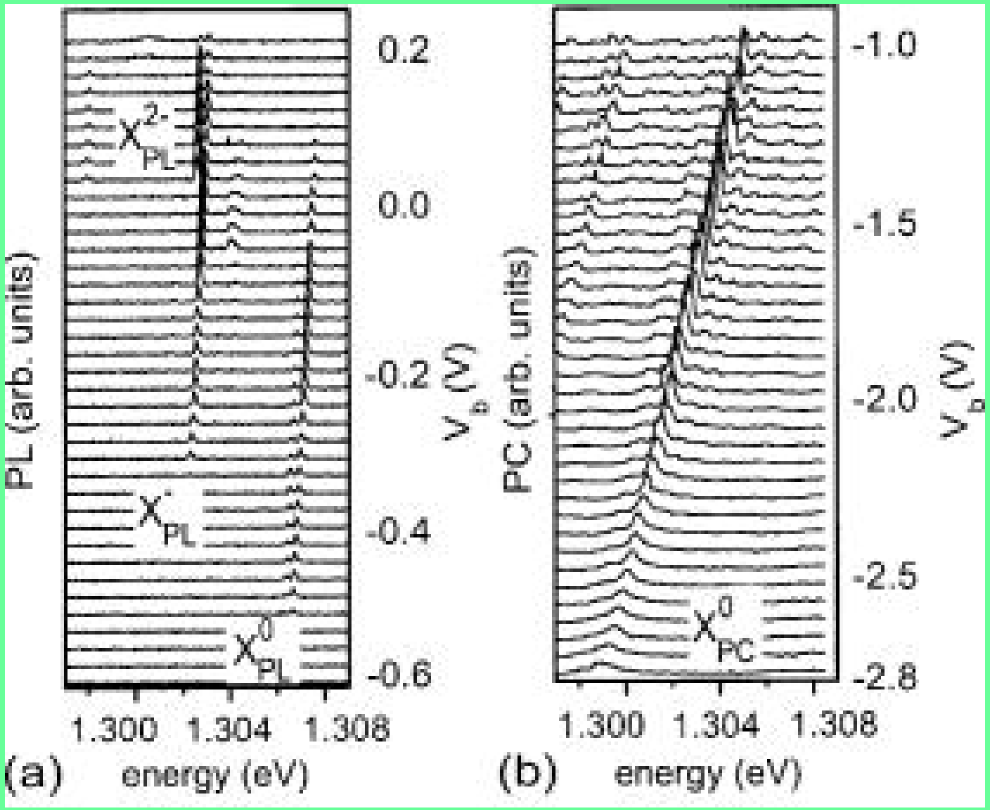
Photocurrent $\tau_{\mathrm{rad}} > \tau_{\mathrm{tunnel}}$

Ti:sapphire laser

$\tau_{\mathrm{pulse}} \simeq 1\ \mathrm{ps}$

$f_{\mathrm{pulse}} \simeq 82\ \mathrm{MHz}$

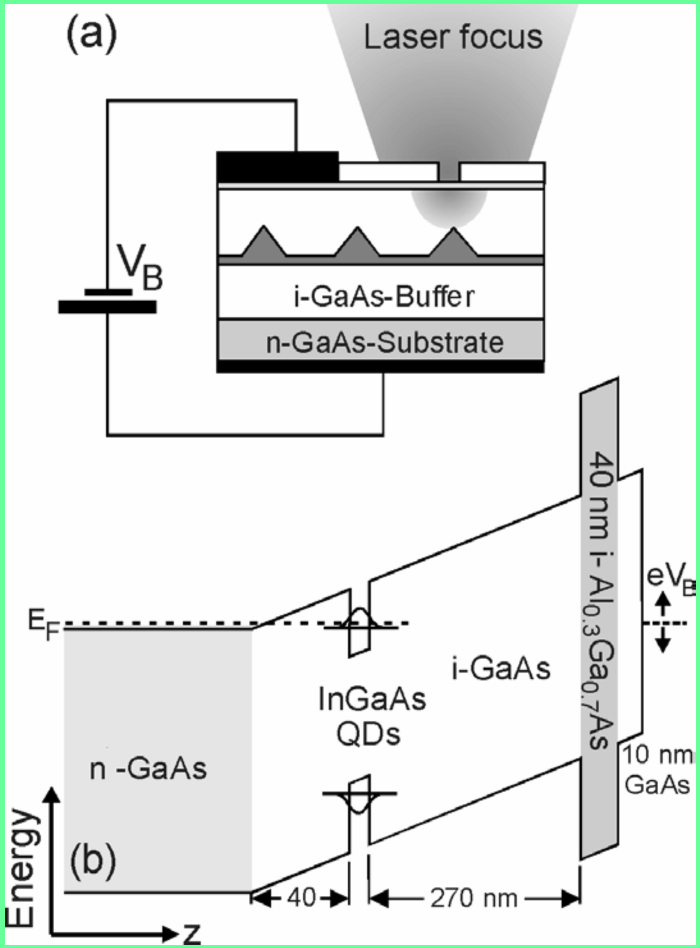
$\tau_{\mathrm{rad}} < \tau_{\mathrm{tunnel}}$ $\tau_{\mathrm{rad}} > \tau_{\mathrm{tunnel}}$

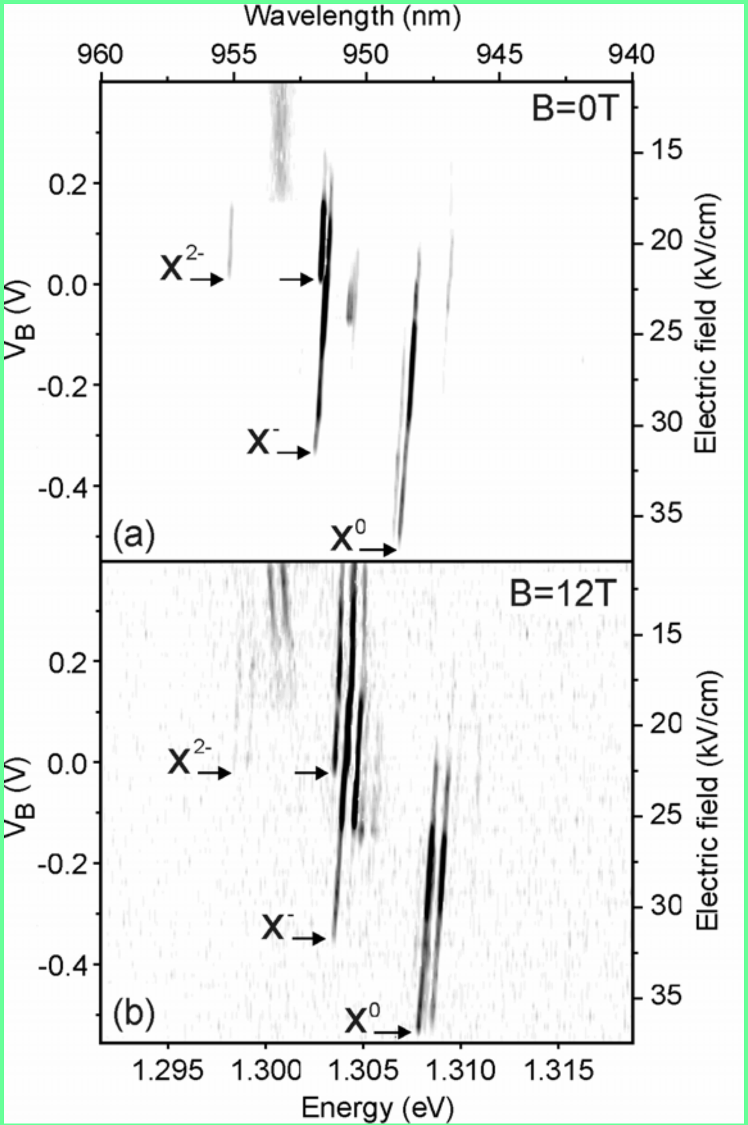


F. Findeis *et al.* Appl. Phys. Lett. **78**, 2958 (2001)

Artificial ion (charged exciton)

$\tau_{rad} < \tau_{tunnel}$

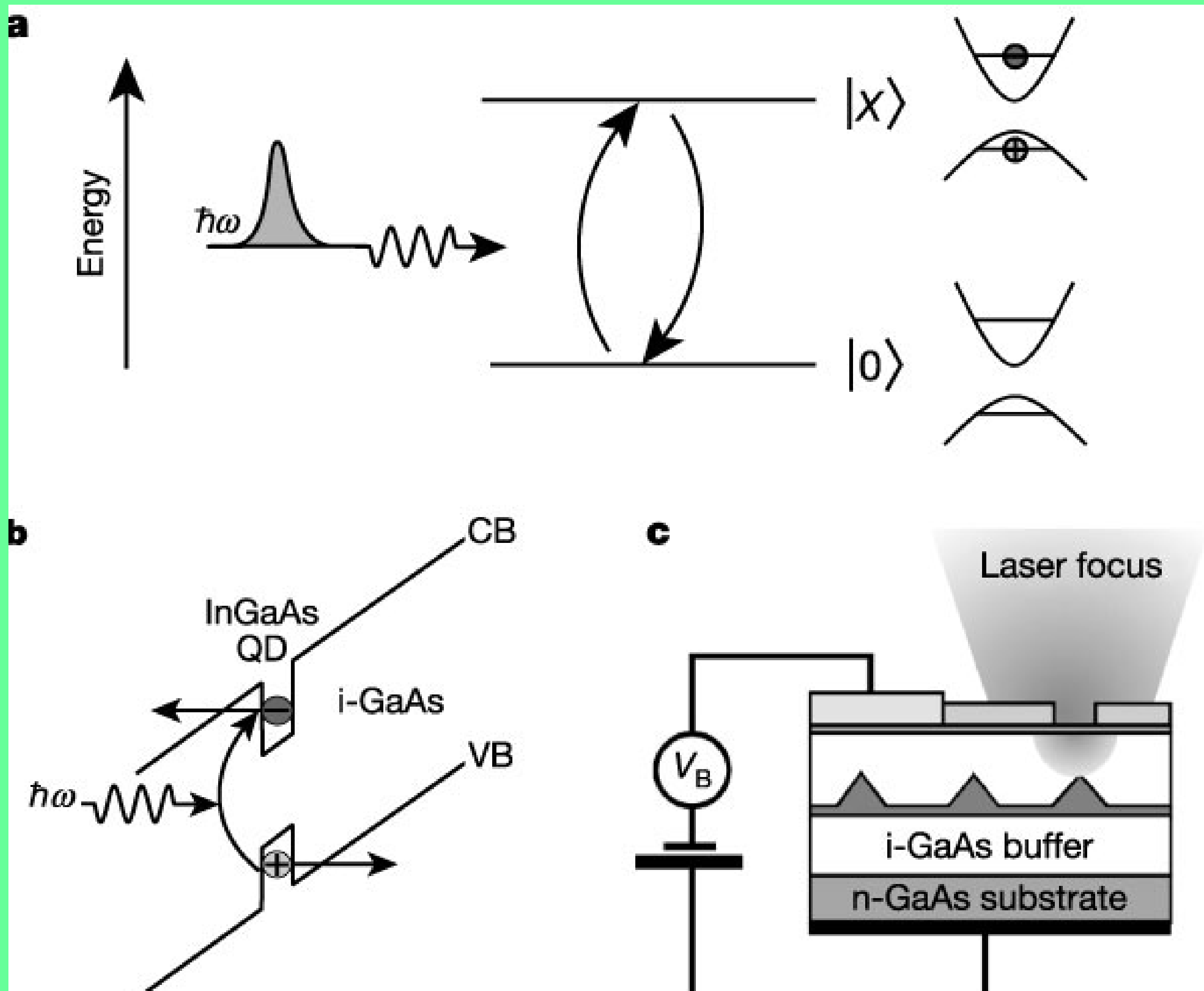




F. Findeis *et al.* Phys. Rev. B **63**, 121309 (2001)

How does the Zrenner device work?

$\tau_{rad} > \tau_{tunnel}$

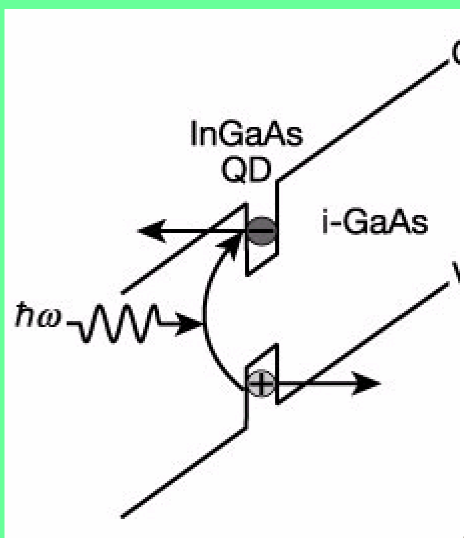


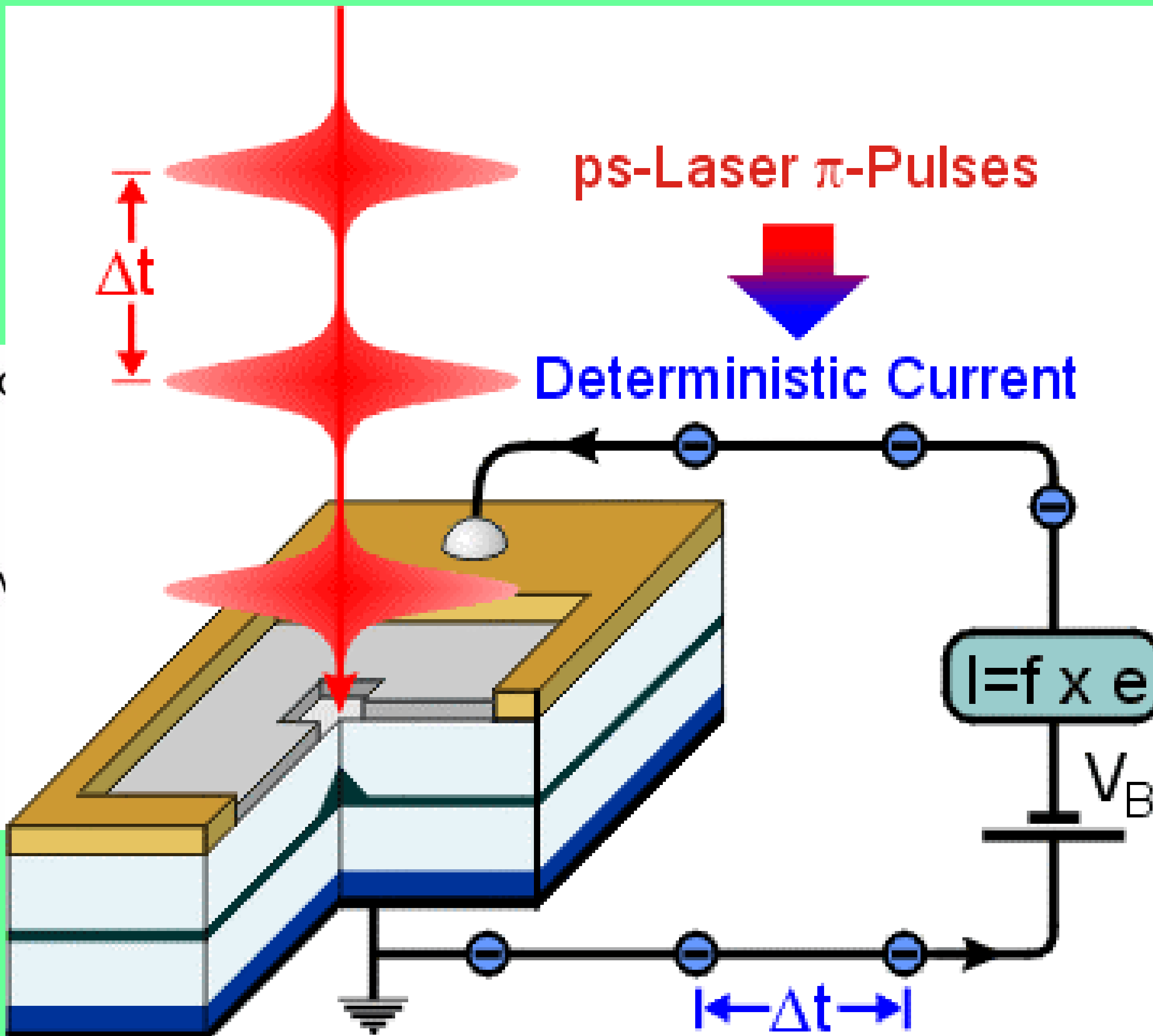
A. Zrenner *et al.* Nature **418**, 612 (2002)

How does the Zrenner device work?

$$\tau_{\text{pulse}} \simeq 1\ \text{ps}$$

$$f_{\text{pulse}} \simeq 82\ \text{MHz}$$

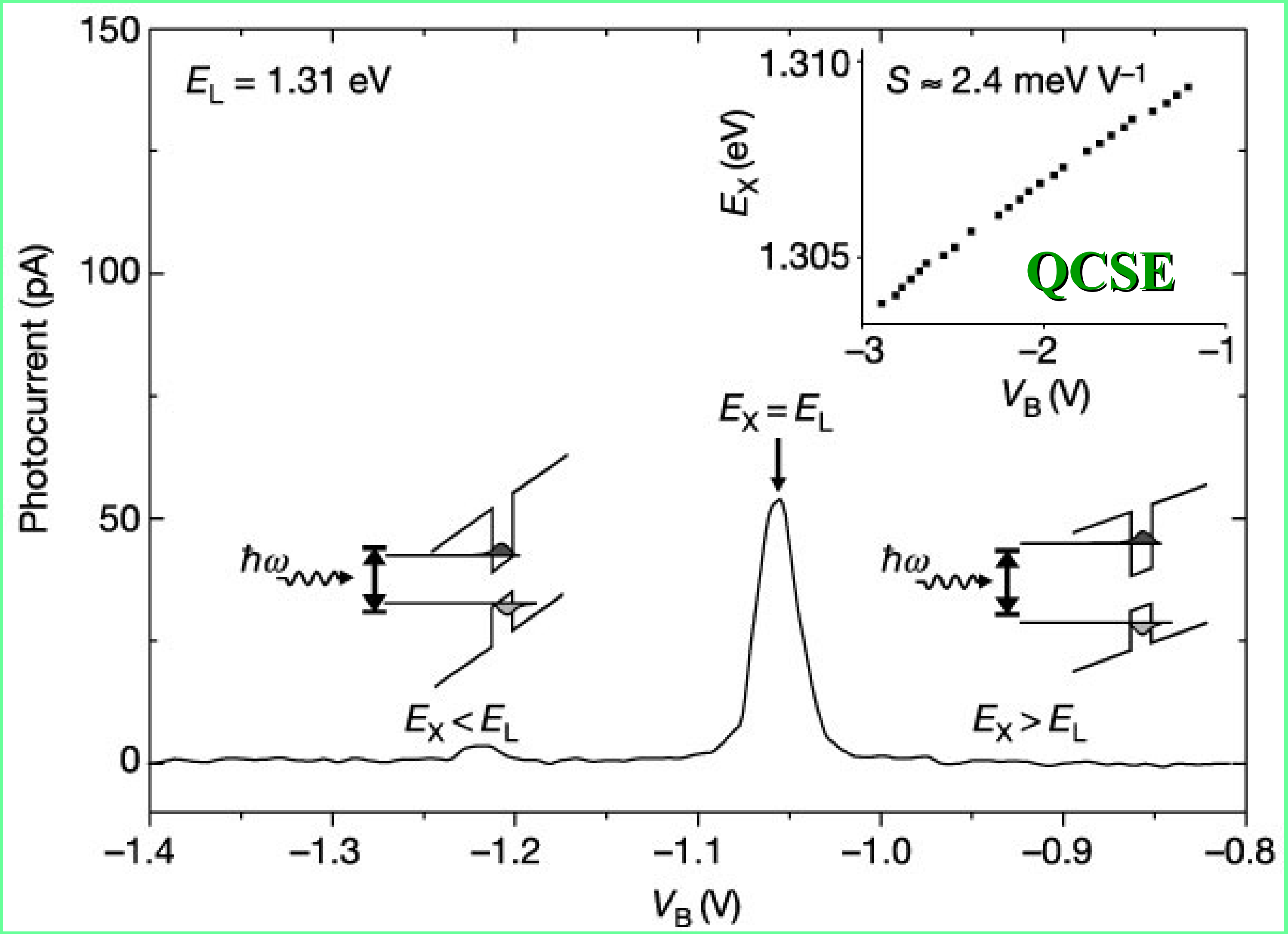




$$I = e\rho_{XX} f_{\text{pulse}}$$

$$\rho_{XX} = \sin^2\left(\frac{\Theta}{2}\right)$$

Mesoscopic optical spectrum analyser



Rabi Oscillation in the photocurrent