Walking with coffee: when and why coffee spills

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February 20-24, 2012

Motivation and introduction

"It is common everyday knowledge to each of us that any small container filled with liquid must be moved or carried very carefully to avoid spills."

- H. Norman Abramson (NASA SP-106, 1966)

Spilling coffee when walking … a problem for a wide audience

- Analysis of *when* and *why* coffee spills is a confluence of many subject including: (i) biomechanics, (ii) sloshing engineering, and (iii) dynamical systems.
- The motions of the human body, while seemingly regular, are quite complex and coupled to a container/liquid.
- As we have all walked with coffee (or liquid filled containers), we have all experienced this problem – it is a familiar problem.

Introduction: Sloshing vs. Biomechanics

Biomechanics of walking

Large body of literature (primarily medical) dedicated to the study of human walking:

- Motion of body CM, energy expenditure, efficiency of process.
- Quantifying gait patterns related to gender, age, health, etc.
- Motion of appendages (arms and legs) in natural positions.
- Regularity of walking over long periods of time.

Sloshing and engineering

Large body of handbooks, technical documents, and journal articles *(NASA SP-106, NASA SP – 8301, Ibrahim, R.A. (2005)...)* concerned with

- Large liquid filled structures (tanks).
- Structures subjected to considerable accelerations/vibrations.
- Forces and torques due to sloshing liquid.
- Suppression of sloshing necessary for vehicle control.

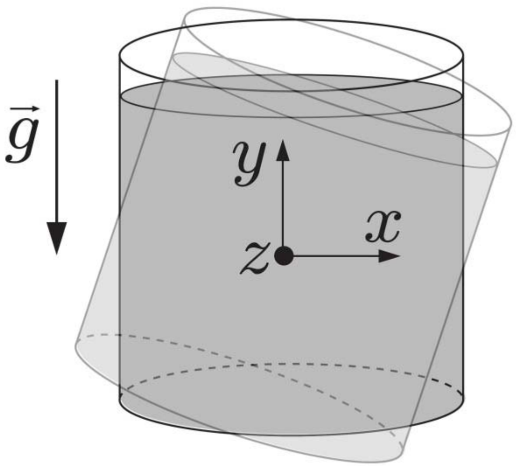
Motivation and introduction: Sloshing vs. Biomechanics

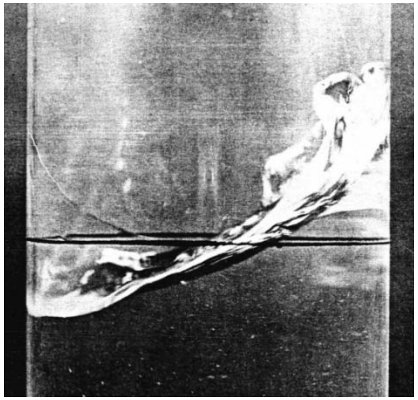
Sloshing in cylindrical containers...

(a) **Lateral sloshing**: response to *translational* or *pitching* motions (antisymmetric modes)

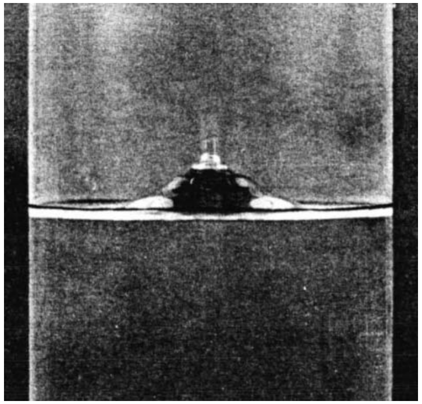
(b) **Vertical sloshing:** response to vertical motions normal to equilibrium surface (symmetric modes)

(c) **Rotational sloshing:** swirling motion of liquid





Lateral – "Back-Forth"

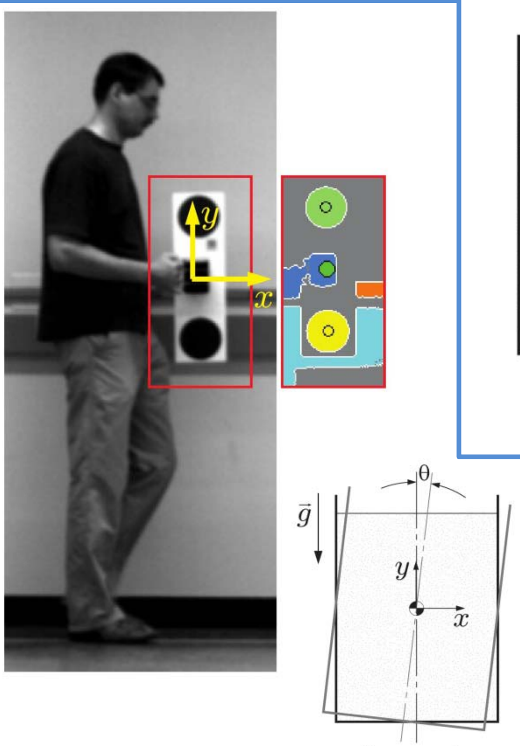


Vertical

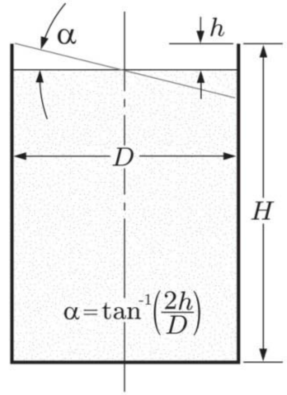
Rotational

(Abramson et. al, NASA SP-106)

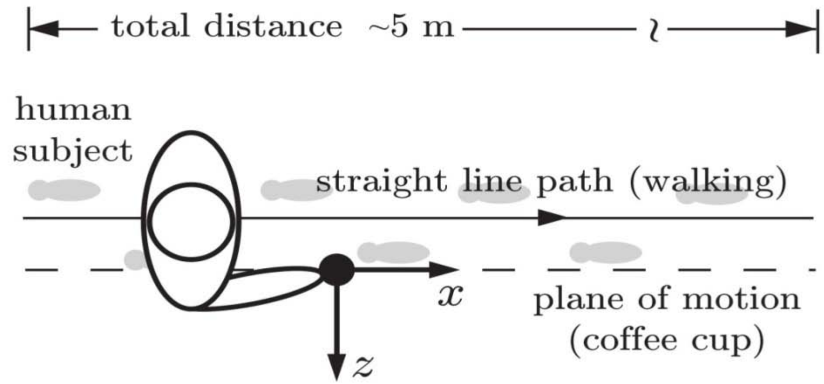
Experimental setup: bits and pieces

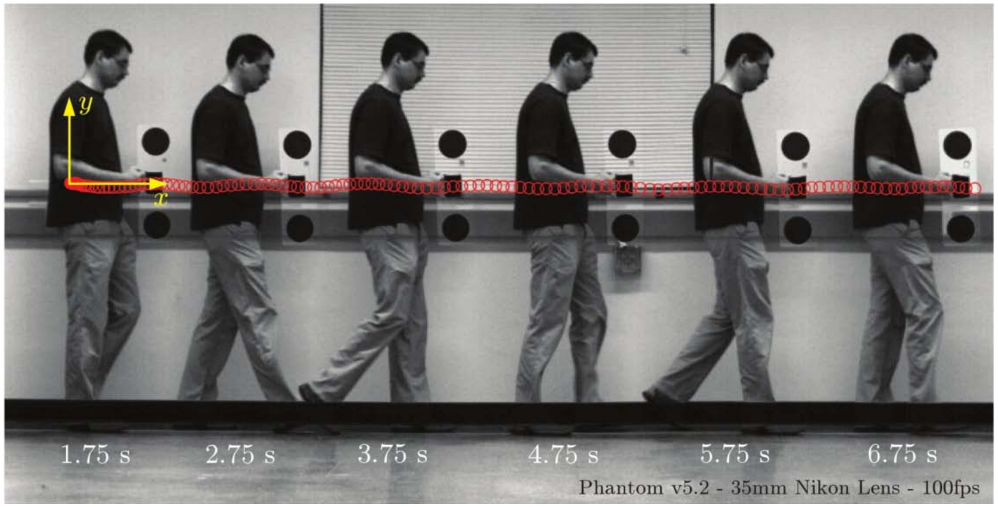


coffee mug coordinates



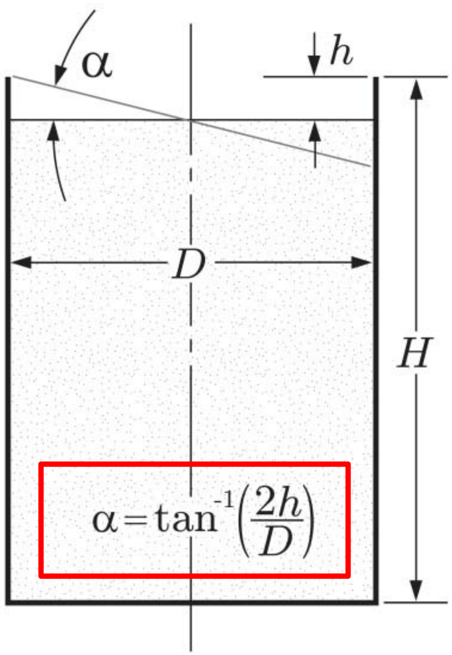
coffee mug geometry





- Coffee mug geometry
- Filming walking
- Image analysis

Experimental setup: coffee mug geometry



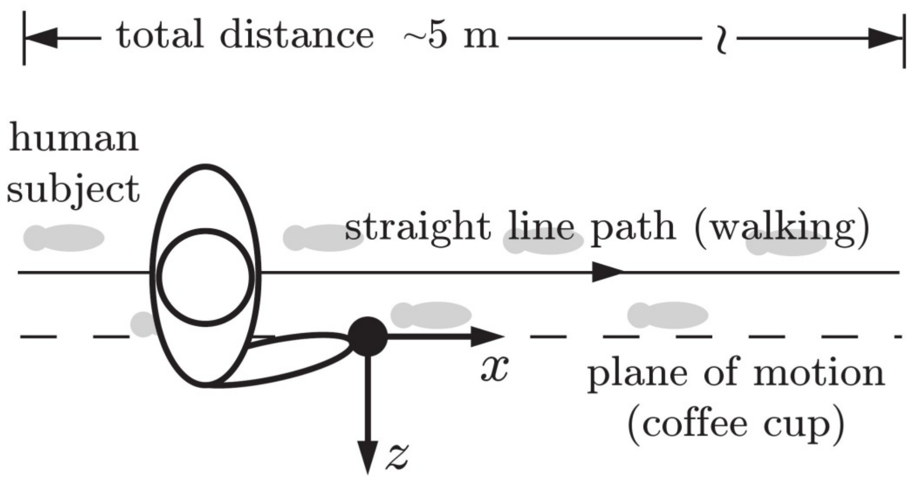
coffee mug geometry

- Coffee mug manufactured from cylindrical beaker
- Dimensions comparable to regular mug ($D \sim 7.1$ cm, $H \sim 10.3$ cm)
- The level of coffee h in the mug was varied but kept near full (normal filling).
  coffee level parameter α is used to characterize the critical filled level based on fluid statics.
  $\sim 8^\circ < \alpha < \sim 16^\circ$ ($h \sim 5 - 10$ mm)
- Properties of coffee similar to water

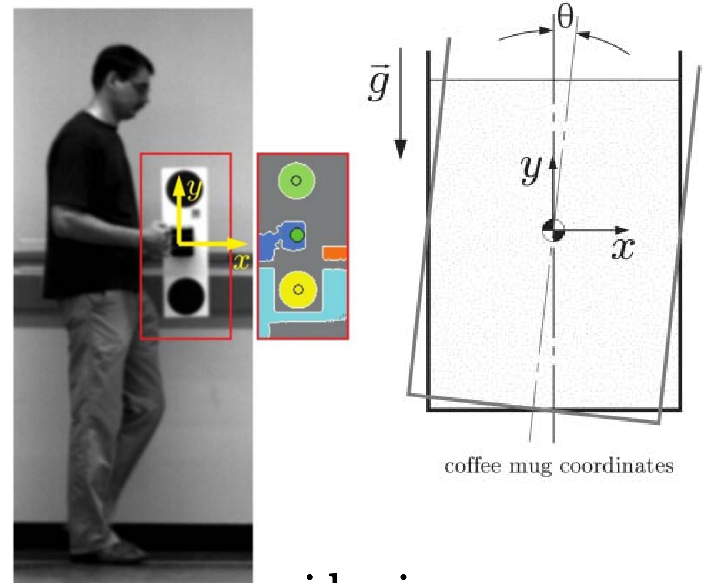
$$Bo = \frac{\rho g R^2}{\sigma} \sim 10^2 \qquad Re = \frac{\rho \sqrt{gR}\, R}{\mu} \sim 10^4$$

(neglect surface tension) (neglect viscosity)

Experimental setup: capturing mug motion



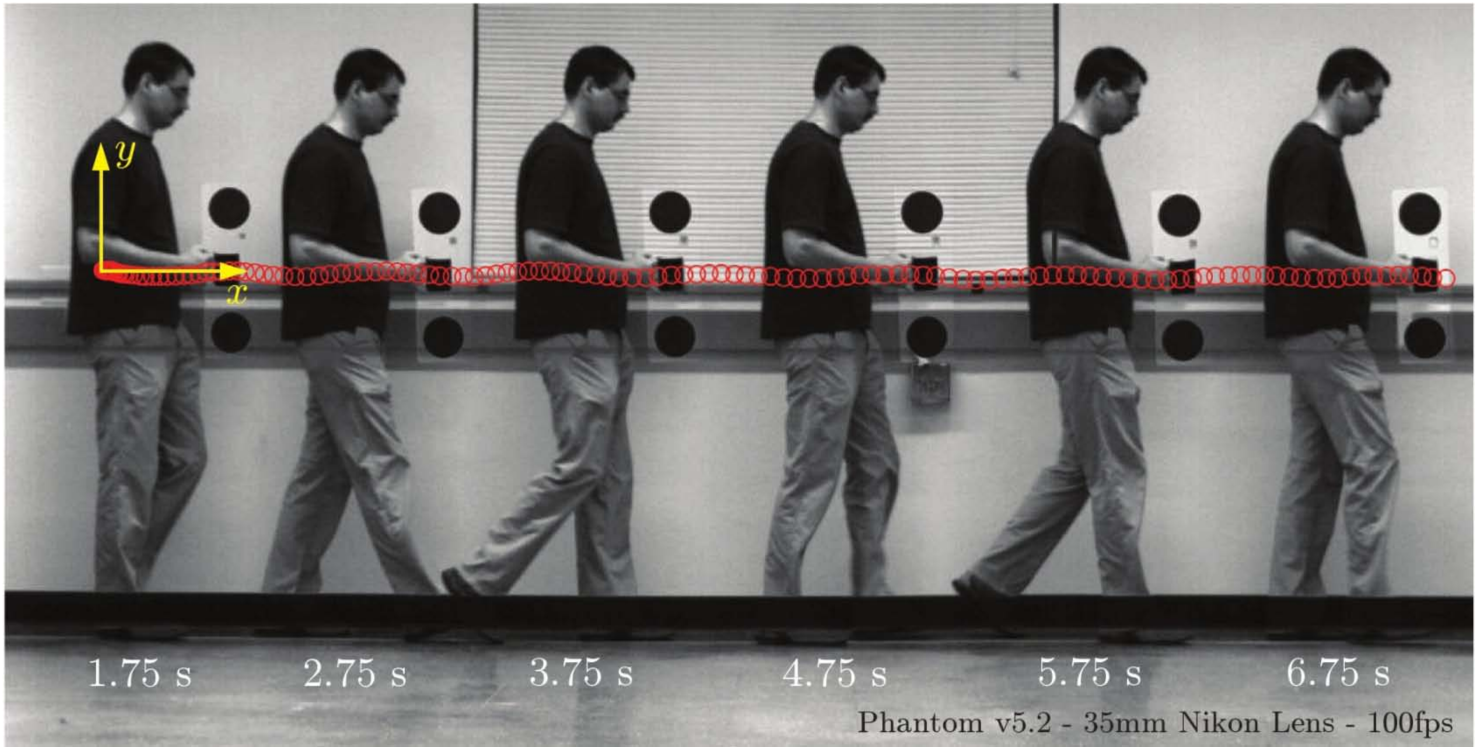
top view



side view

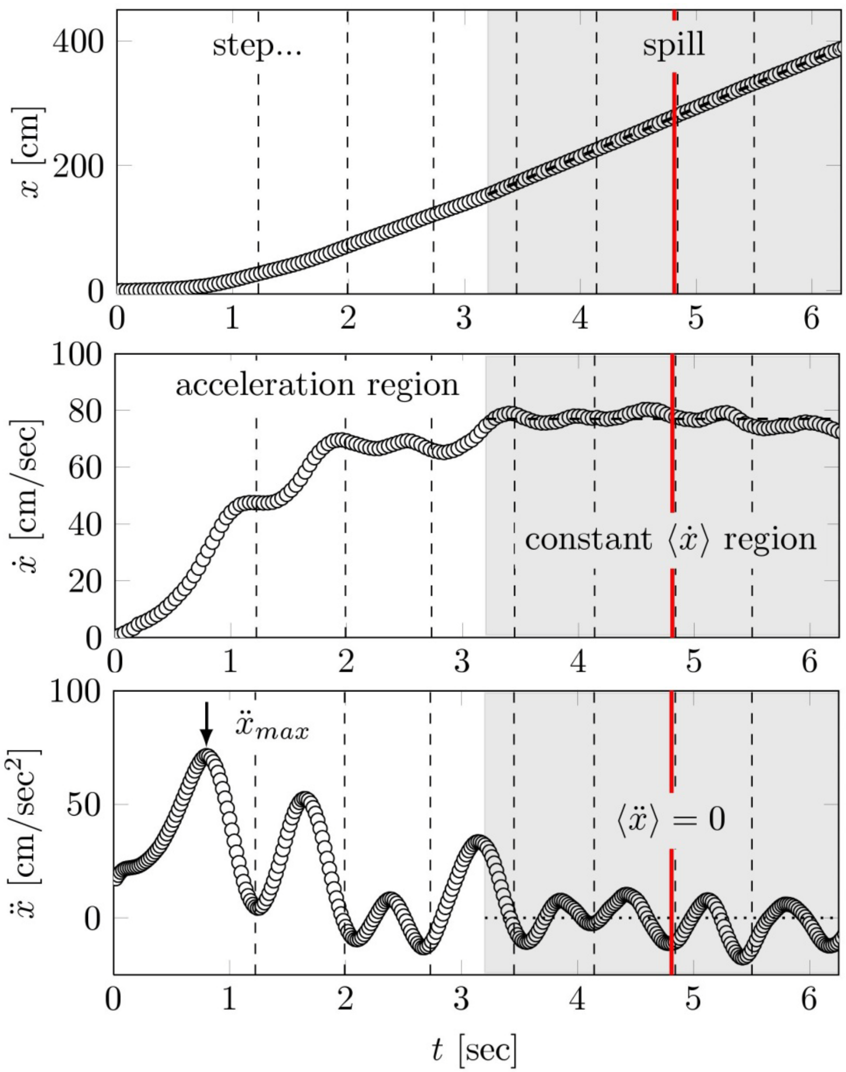
- Subject was filmed at 100 fps walking on ground instead of treadmill (more natural start).
    - along a straight line path
    - motion analyzed in the xy - plane
- Extent of walking distance limited by desired resolution ($\sim$ 3 mm).
- MATLAB code (using image analysis toolbox):
    - finds markers
    - determines mug coordinates
- Manual determination of spill (using LED signal)

Experimental setup: movie files



- Subject recorded walking at different average walking speeds, $\langle \dot{x} \rangle$.
- Acceleration, $\ddot{x}$, naturally "chosen" by subject.
- Various values of the coffee level paramter α tested.
- Subject was asked to walk in two regimes: (1) **"focused"** (trying not to spill) and (2) **"unfocused"** (not concerned with spilling).

Results from image analysis: coffee mug kinematics (x direction)

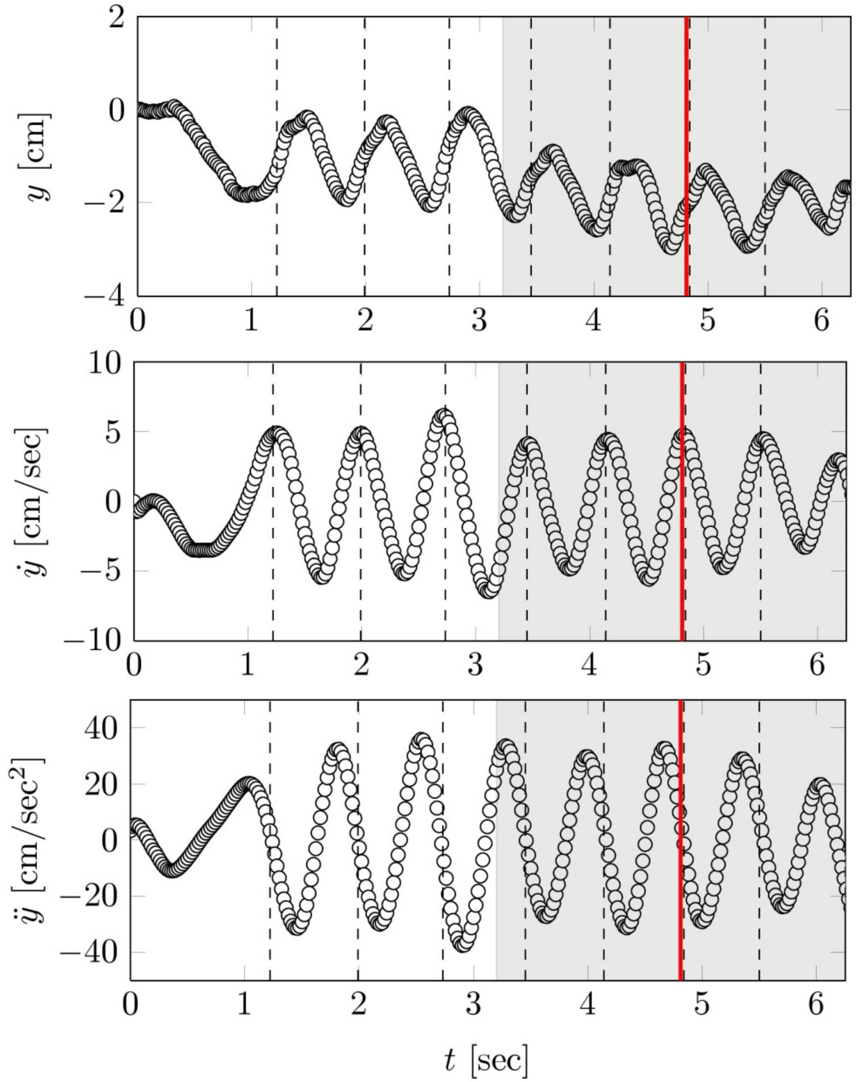


Typical output of image analysis

Note the following:

- Total walking distance of ~ 4 - 5 m filmed in each experiment.
- Initial acceleration region followed by a region of constant walking speed, $\langle \dot{x} \rangle$.
- Average walking speeds are in the range of 0.5 – 2.5 m/s.
- Maximum acceleration, $\ddot{x}_{max}$ realized early on during the walk.
- Oscillatory motion superimposed on average trajectory of human subject... **evidence of back-and-forth excitation that the cup experiences.**

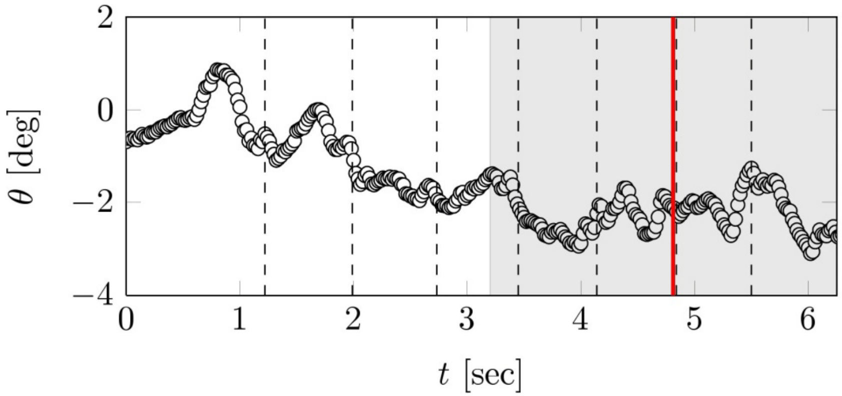
Results from image analysis: coffee mug kinematics (y direction)



Typical output of image analysis

- Mug elevation essentially constant during experiment.
- Distinct oscillations of the mug in y direction (few cm in magnitude)
- y - velocity component, $\dot{y}$, small compared to average walking speed $\langle \dot{x} \rangle$.
- y - accelerations small compared to gravity ($\sim 5\%$ of $\vec{g}$).
- Maximum y - velocity, $\dot{y}_{max}$, and zero y - acceleration appear synchronized with steps.
- **Evidence of vertical excitation that the cup will experience.**

Results from image analysis: coffee mug kinematics (θ direction)



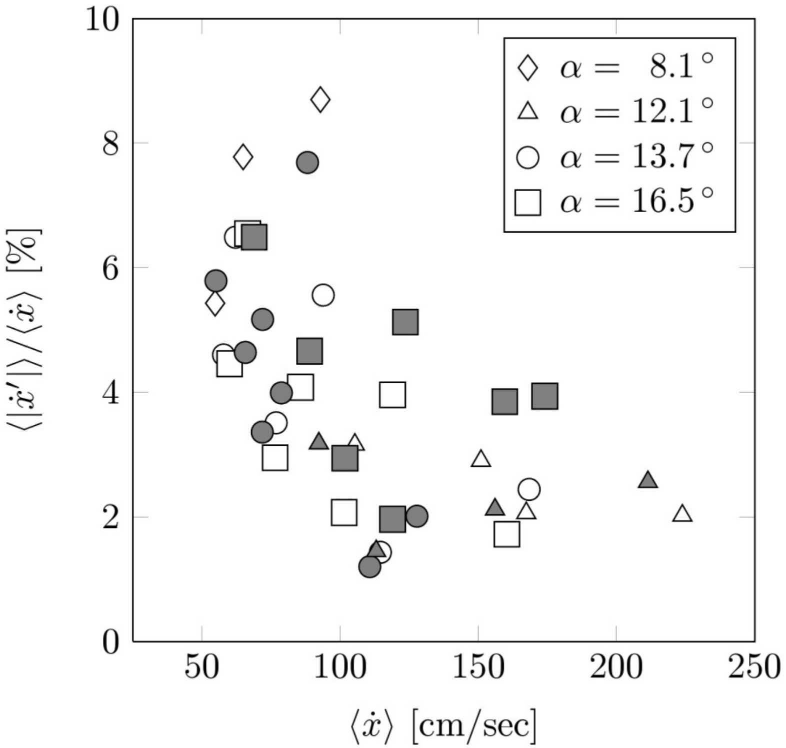
Typical output of image analysis

- Although wrist is flexible, one can anticipate that the human subject will not allow the mug to pitch substantially (small θ values)
- Pitching motion captured is really a combination of arm swing and flexible wrist.

Image analysis of coffee mug motion during walking...

- Demonstrates walking motion produces *back-and-forth*, *vertical*, and *pitching* oscillations.

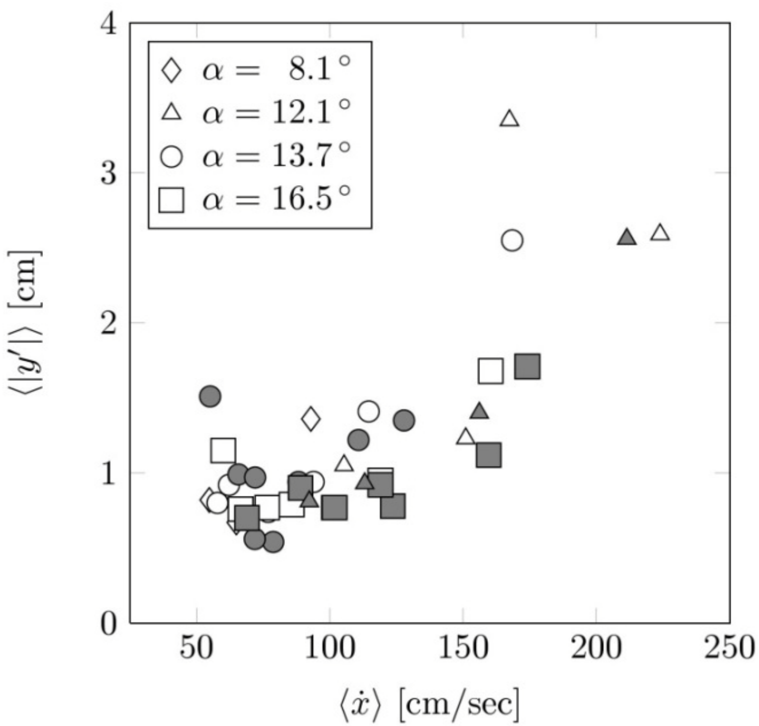
Results from data analysis: back-and-forth excitation

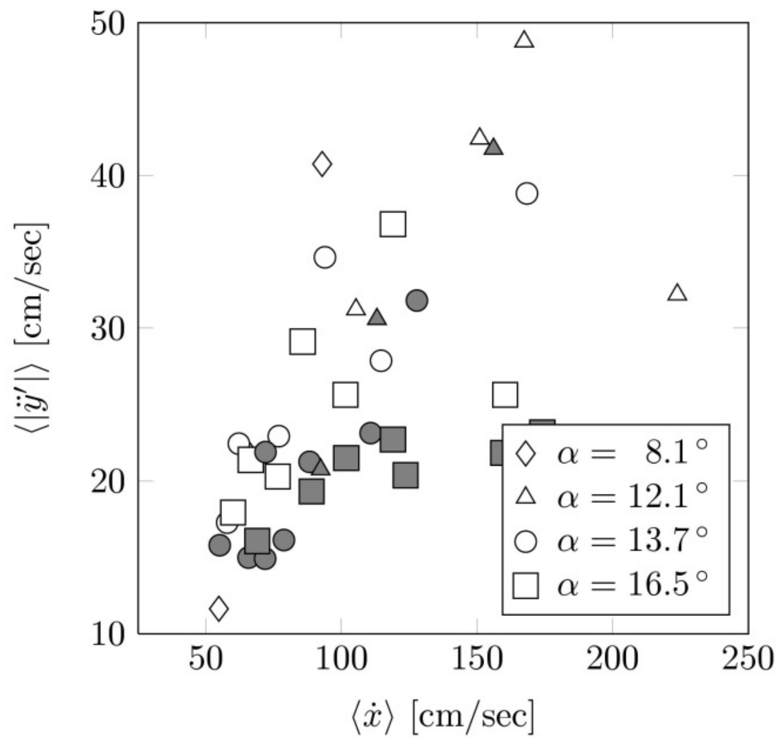


Standard deviation of the velocity about the average decreases with increasing walking speed.

Due to the oscillatory nature of the x – direction motion, back-and-forth forcing of the coffee mug exists, but it is apparent that the magnitude of the oscillations change with walking speed.

Results from data analysis: vertical excitation

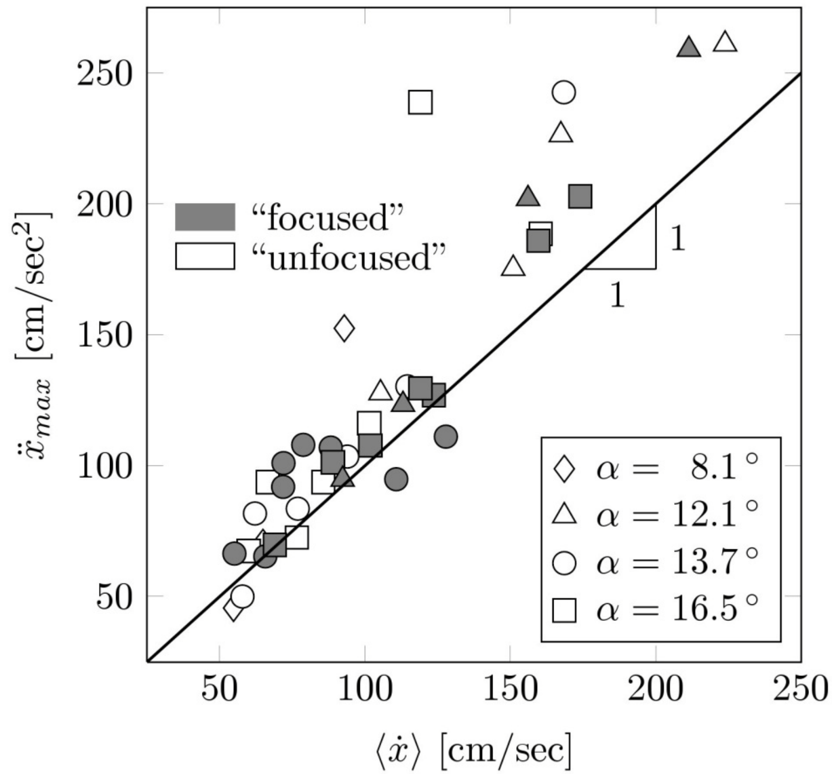




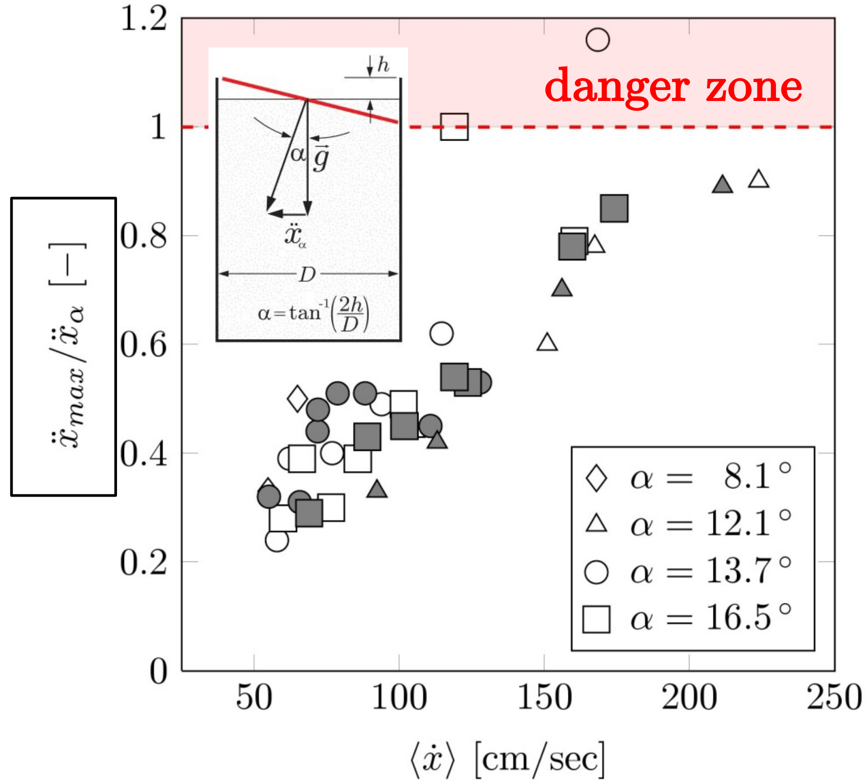
Magnitude of y - motion, y - velocity, and y – acceleration increase with walking speed (although average y, y - velocity, and y - acceleration are essentially zero).

Results from data analysis: comparing all data

What can we observe about walking...

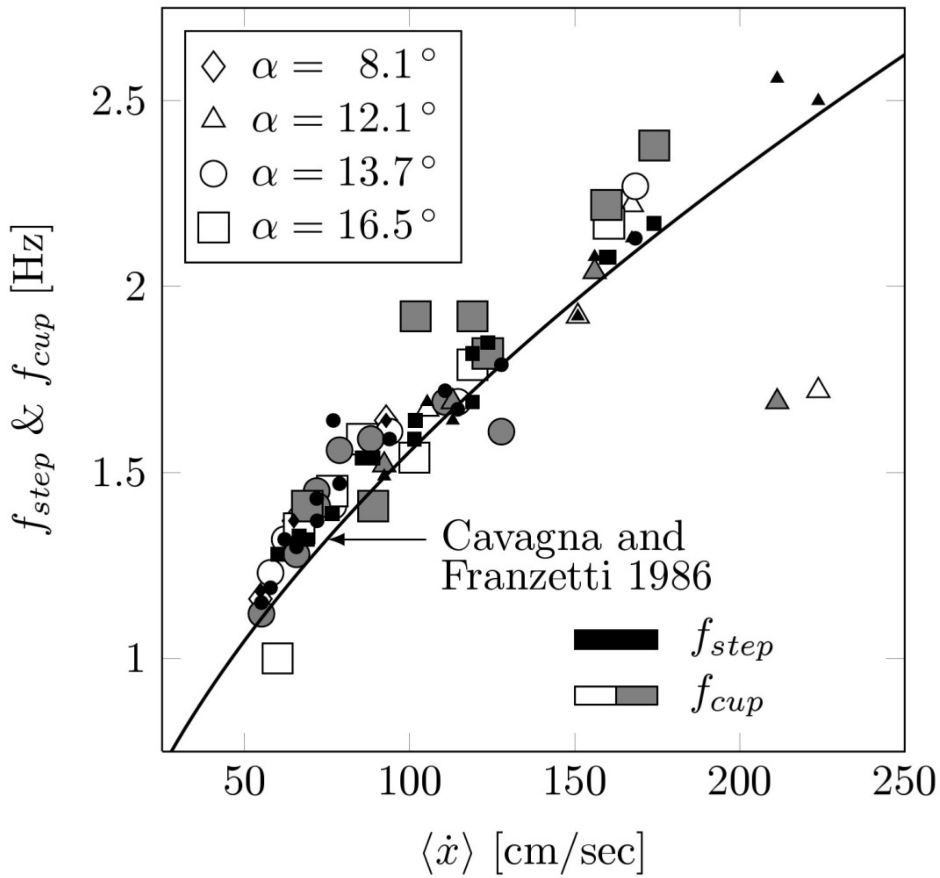


Horizontal acceleration (suggested by $\ddot{x}_{max}$) chosen by human subject increases with average walking speed.



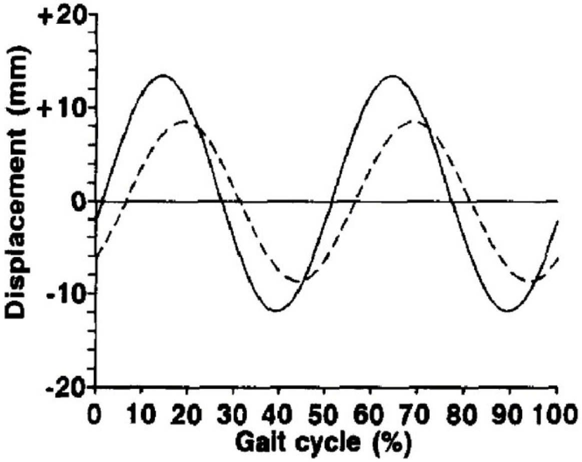
Maximum accelerations compared to those needed to initiate nearly immediate spilling (based on α)

Results from data analysis: step frequency

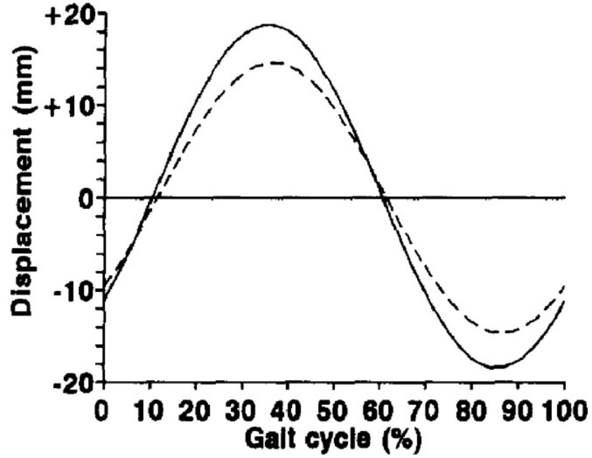


- Coffee mug is excited at a frequency f_{cup} nearly identical to that of the step frequency f_{step} ($\sim 2\%$ difference).
- Mug excitation frequency increases with walking speed.
- **Most importantly, the excitation frequency is in the neighborhood of 2 Hz.**

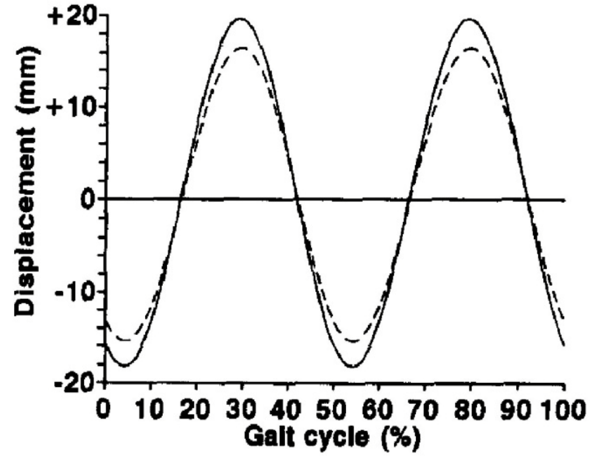
Comparison of components of body motion (biomechanics)



Back-and-Forth



Lateral



Vertical

M. W. Whittle *Human Movement Science* 16 pg. 347 (1997)

Results from data analysis: sloshing natural oscillations frequency

Cylindrical vessel

Antisymmetric mode
(natural frequency)

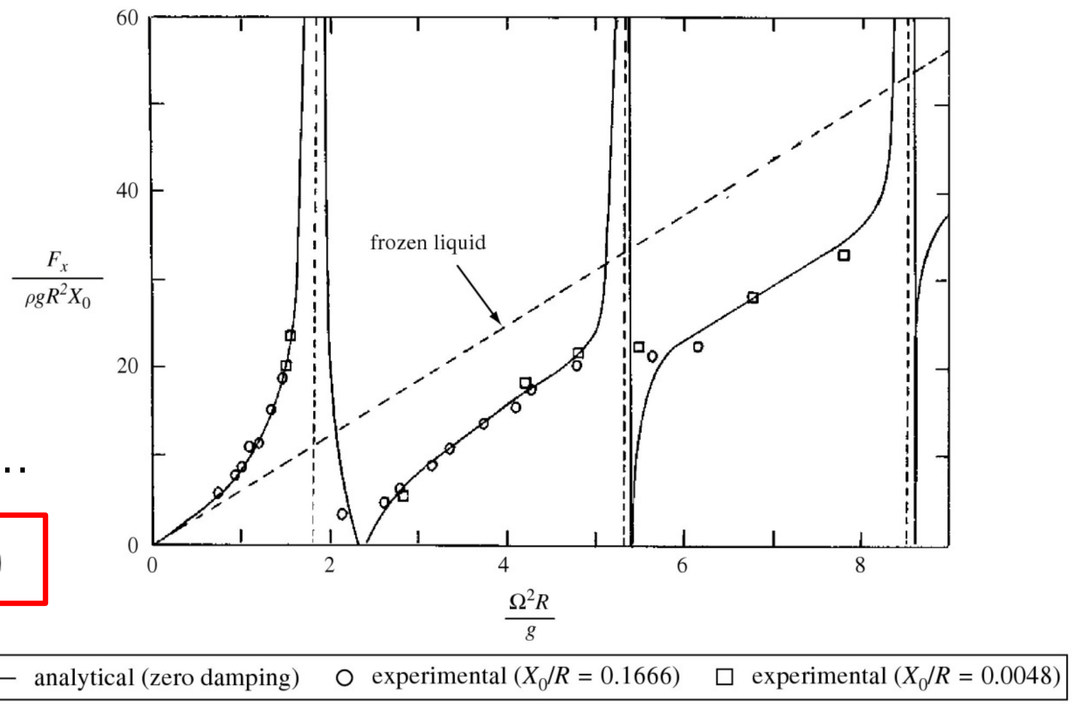
$$\omega^2 = \frac{g\epsilon}{R} tanh\left(\epsilon \frac{H}{R}\right)$$

$$\epsilon = 1.841$$

Calculated for our coffee mug ...

$$\boxed{\nu = 3.65\ Hz\ (\omega = 23\ \text{rad/sec})}$$

approximately the same frequency for all coffee mugs!

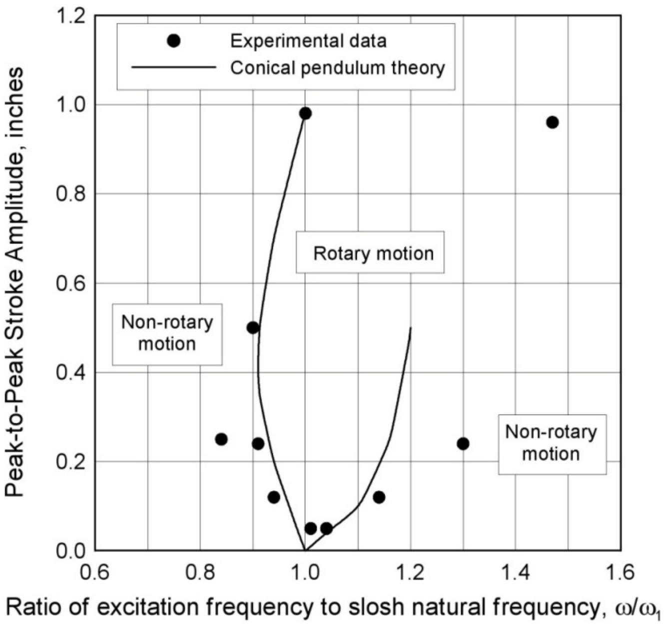


R. A. Ibrahim *Liquid Sloshing Dynamics* (2005)

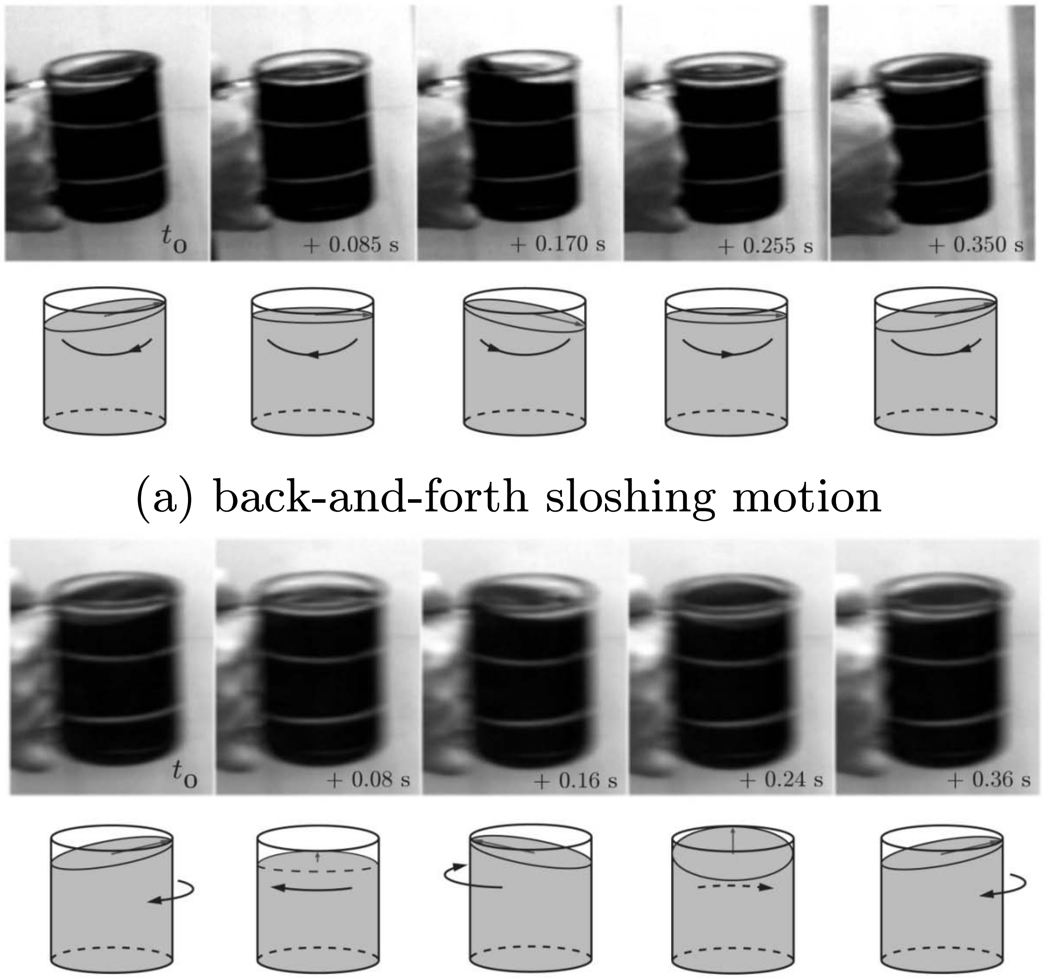
Conclusion: back-and-forth motion should excite the antisymmetric sloshing mode

Results from data analysis: liquid motion and observation of swirl

Closer look suggests that back-and-forth *and* swirling liquid motions can be excited by walking at typical speeds.



(Dodge (2005))



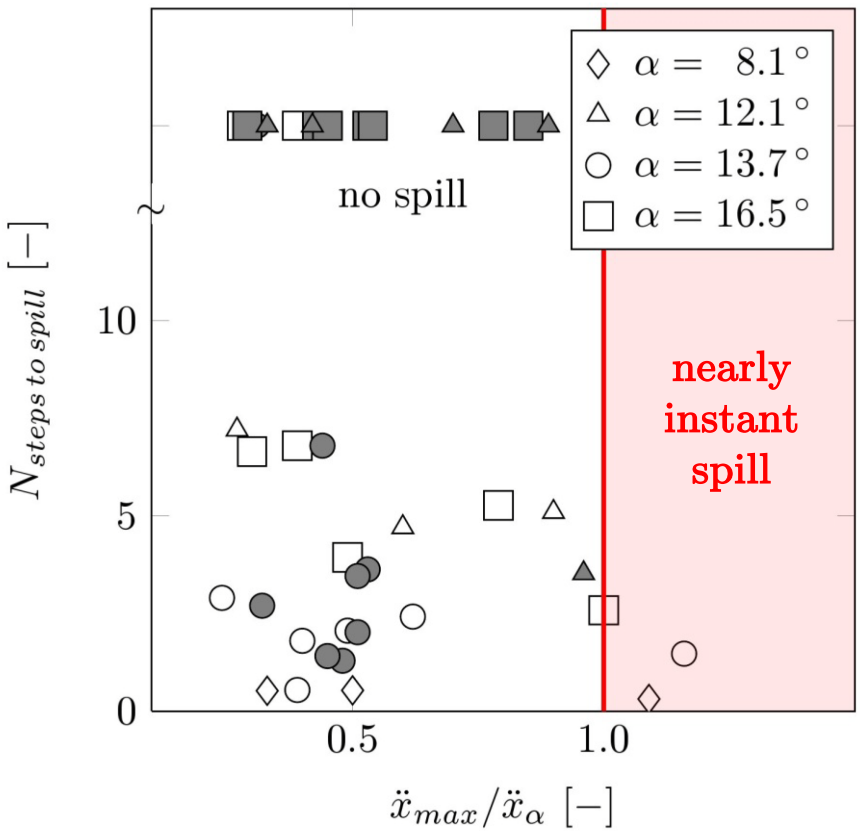
(a) back-and-forth sloshing motion

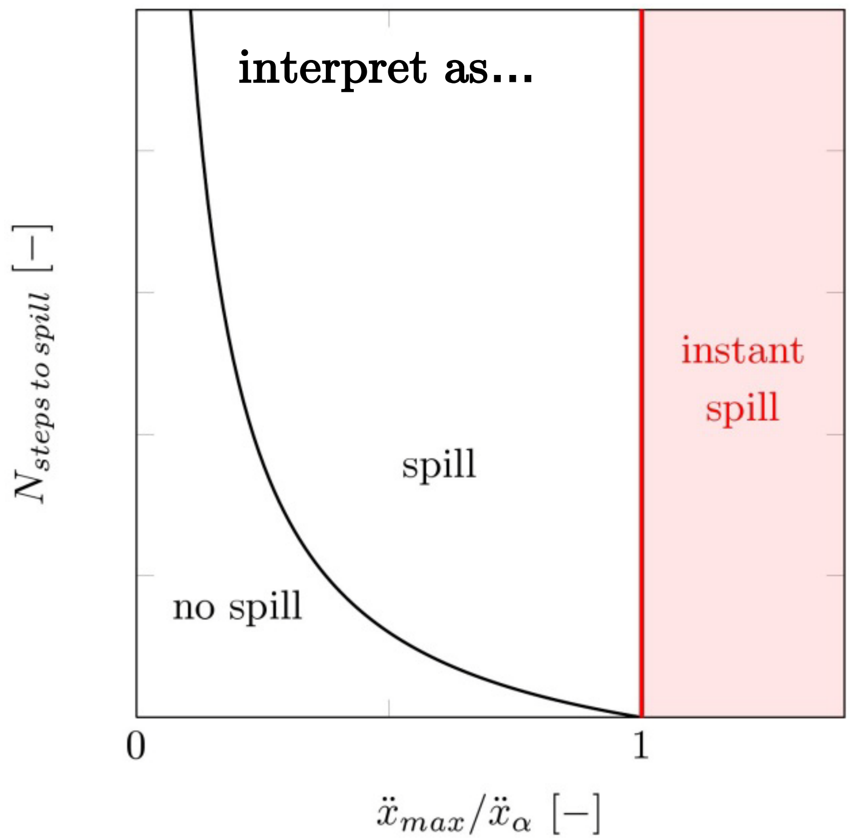
(b) rotational sloshing motion

Question: why is rotational motion generated?

Results from data analysis: how long does it take to spill?

Time to spill is extracted from all walking movies

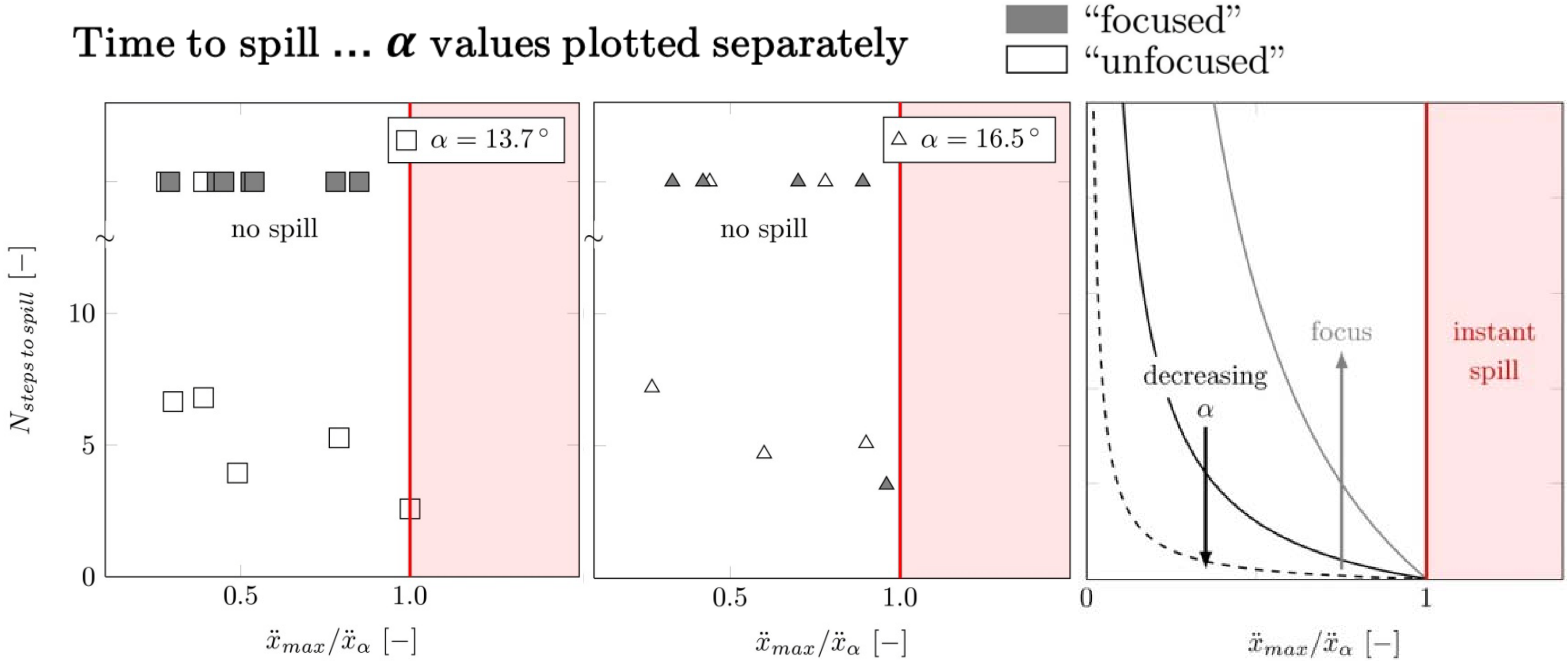




- Time to spill transformed into N (number of steps before spilling)
- All data plotted with respect to $\ddot{x}_{max}/\ddot{x}_\alpha$ ($\ddot{x}_{max}$ provides initial displacement)

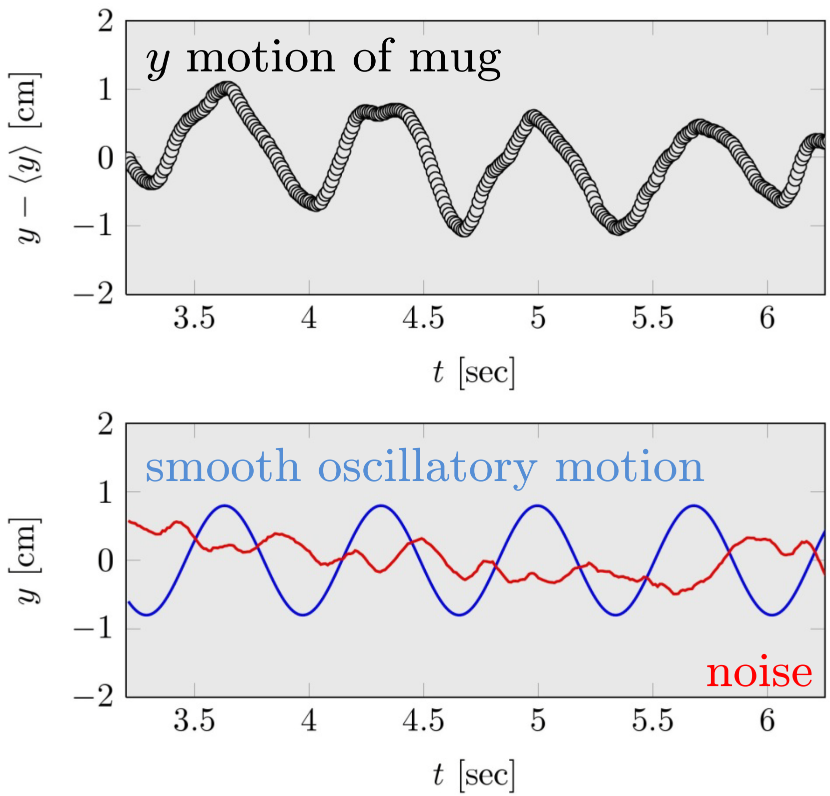
Results from data analysis: how long does it take to spill?

Time to spill … α values plotted separately



- N decreases with increasing $\ddot{x}_{max}/\ddot{x}_\alpha$(and thus walking speed)
- If $\ddot{x}_{max}/\ddot{x}_\alpha > 1$, very short spill time is observed.
- Small α (high coffee levels) exhibit fewer N before spilling.
- "Focusing" tends to increase N before spilling.

A more complex picture: dynamical systems point of view



Decomposition of motion

Effect of noise:

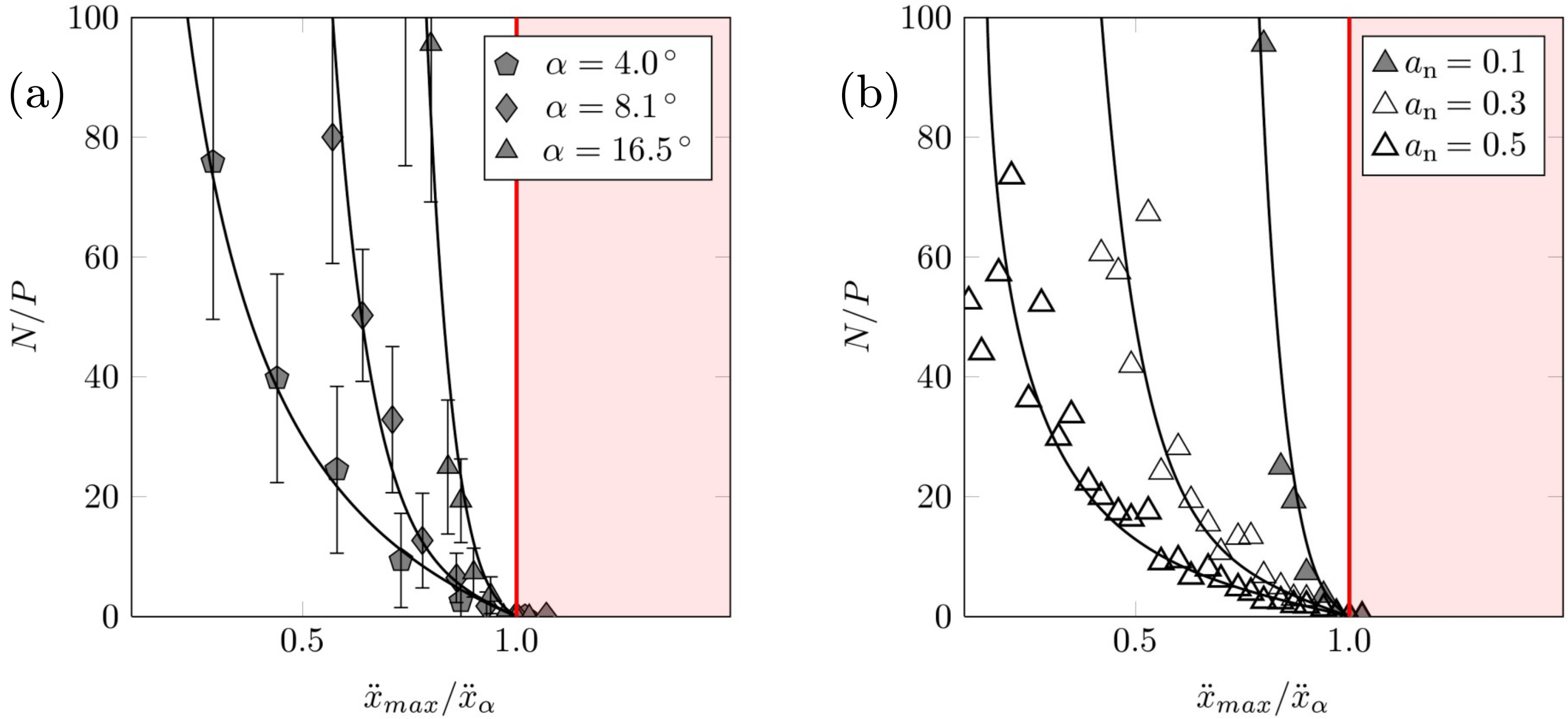
• Looking more closely at the mug motion during constant acceleration reveals that it is composed of noise and smooth oscillations.

• Comparing "focused" and "unfocused" regimes suggests that noise substantially intensifies the spilling and noise intensity is lower in the focused regime.

Unsteady effects:

• Since walking is usually non-uniform, the frequency of step changes and thus when excitation approaches the natural frequency one can expect resonance to occur much earlier (*jump frequency phenomenon*)

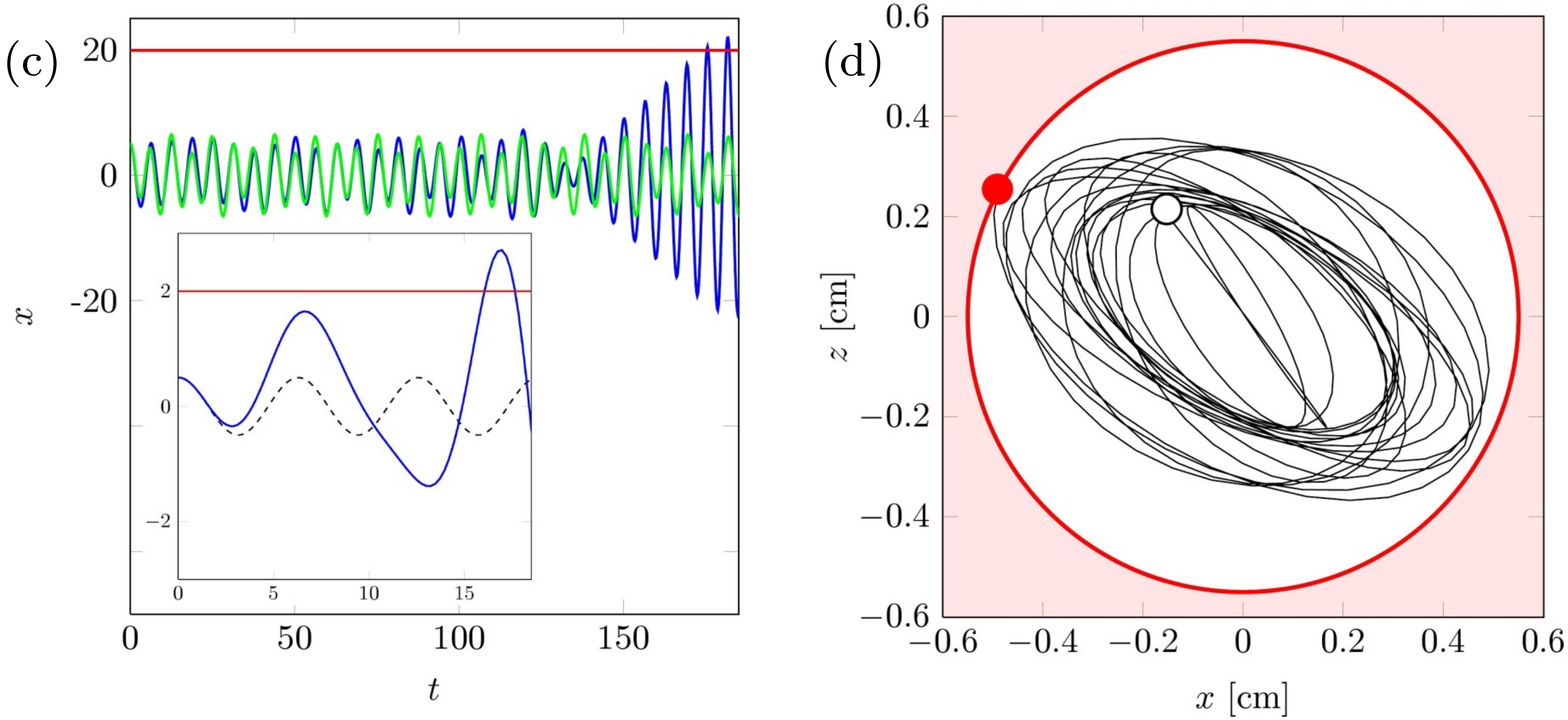
Modeling of spilling as an equivalent spherical pendulum: steps to spill



• Figure (a) shows steps to spill N/P versus the normalized initial deflection of the interface for three different coffee levels. Each data point shown in the figure corresponds to the average number of steps N of all model runs that yield a spill divided by the fraction of runs P that result in a spill.

• Figure (b) reflects the influence of the noise amplitude an on the number of steps to spill for a fixed angle.

Modeling of spilling as an equivalent spherical pendulum: noise effects



- Figure (c) shows the effect of time-dependent (chirped) forcing frequency on coffee spilling over short time (inset) and longer times.
- Figure (d) shows example trajectory of spherical pendulum projected onto the x-y plane. A “critical spill radius” is denoted by the solid red line.

Conclusions: how do we spill coffee? *

(a) Either by accelerating too much for a given coffee level (fluid statics)

(a) Or, through more complicated dynamical phenomena:

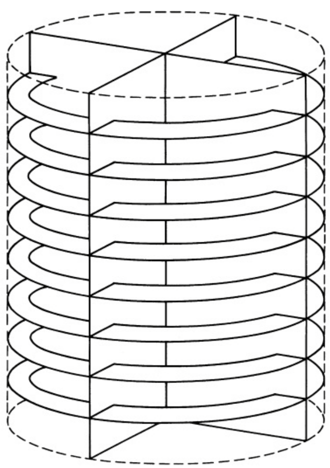
- Initial acceleration sets an initial sloshing amplitude, which is analogous to the main antisymmetric mode of sloshing.
- This initial perturbation is amplified by the back-and-forth and pitching excitations since their frequency is close to the natural one because of the choice of normal mug dimensions.
- Vertical motion also does not lead to resonance as it is a subharmonic excitation (Faraday phenomena).
- Noise has higher frequency, which makes the antisymmetric mode unstable thus generating a swirl.
- Time to spill depends on "focused"/"unfocused" regime and increases with decreasing maximum acceleration (walking speed).

*Scientific American **305**, 22 (2011)

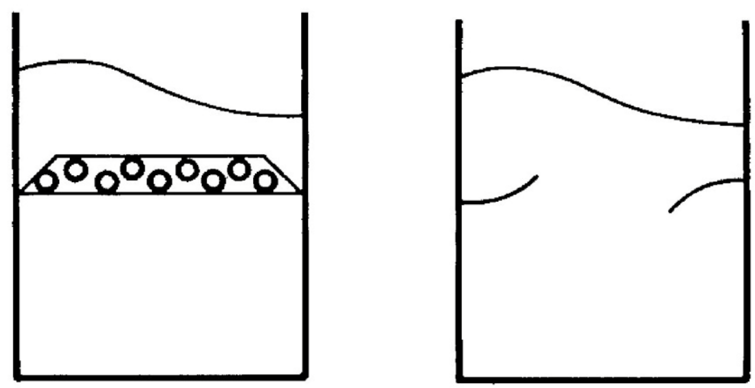
Conclusions: how can we prevent spilling?

Lessons learned from sloshing dynamics may suggest strategies to control spilling, e.g. via using

- a flexible container to act as a sloshing absorber in suppressing liquid oscillations.
- a series of concentric rings (baffles) arranged around the inner wall of a container.



(*Abramson, NASA SP-8031 (1969)*)



Truncated cone (perforated)

Flexible baffle

(*Langner SRI Tech Report 7 (1963)*)