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Dynamic elastic properties of magneto-rheological slurries

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Abstract

We study the propagation of elastic perturbations in magneto-rheological slurries of iron particles dispersed in glycerine. The complexity of these systems is revealed in the fibrillar structure acquired under the application of a magnetic field. Recently, it has been reported the observation of two different low frequency modes of propagation. One of these modes has been associated to the propagation of the perturbation through the fluid medium. The other one has been qualitatively explained as the propagation of the elastic perturbation through the suspended particles. This second mode appears when a magnetic field is applied to the slurry. The propagation speed for both modes depends on the field intensity and on the properties of the magnetic particles. Theoretically, we analyze these modes and calculate the sound velocity. We obtain a quantitative good agreement with the experimental results. © 2001 Elsevier Science B.V. All rights reserved.

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Magneto-rheological (MR) fluids are composed by small magnetic particles suspended in inert fluids; oils are commonly used for these support media. One of the distinctive characteristics of MR fluids is that noticeable changes in its mechanical properties can be induced by means of the application of external magnetic fields. There has been in the last few years an increasing interest on several physical properties of rheological fluids [1]. In particular, recently it has been reported an experimental study on the sound propagation in a MR slurry of iron particles suspended in glycerine [2].

It is well known that in a colloidal suspension only one longitudinal mode can exist when an elastic excitation propagates in it with wavelength much larger than

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the characteristic size of the suspended particles. When the wavelength of the elastic excitation is of the order of the size of the particles, two longitudinal modes have been observed in these kind of systems [3]. Thus, it was quite unexpected the observation by Nahmad-Molinary et al. [2] of two distinct modes propagating in the low frequency regime in a MR slurry. Their interpretation is that one of these correspond to the propagation of an elastic mode through the fluid phase, and the other one, through the suspended particles, arranged in perfect chain structures. Furthermore, they found that the second mode is slower than the first one and appears only while the external magnetic field remains applied. The speed of propagation of both elastic modes depends on the intensity of the applied magnetic field. In their experiment the particle size distribution was in the range 3–25 μm.

In this work, we focus our attention in both of these modes and calculate their speed of propagation by the following procedure. We made use of a simple effective media approximation, which consists in calculating the sound speed in the composite medium v, in terms of an effective density ρ and an effective elastic modulus β, by means of the expressions [4]

$$v = (\beta/\rho)^{1/2}, \quad \rho = \phi\rho_s + (1-\phi)\rho_f, \quad \frac{1}{\beta} = \frac{\phi}{\beta_s} + (1-\phi)\frac{1}{\beta_f}\,. \tag{1}$$

Here ϕ is the volume fraction of the metallic particles; s and f stand for solid and fluid, respectively. In calculating the sound speed, we assume that in the first mode, the elastic perturbation propagates through almost clear glycerine channels. Provided the perturbation wavelength is much larger than the characteristic size of the iron particles, one can assume that the effect of the iron spheres in this dilute dispersion is only of inertial nature. The elastic properties of glycerine will not be affected by the presence of a few non-interacting iron spheres. Then, neglecting dissipation effects, the only one average which must be considered in Eq. (1) is over the density ρ. This scheme should be more suitable if a high magnetic field is present. This is so because under that condition, more iron particles will be sticked to the fibrillar structure, and consequently fewer of them will be in the glycerine channels. The second mode has been observed at even larger wavelength and only appears in the presence of a magnetic field. The fibrillar structure acquired by the suspension upon the application of a magnetic field, is the medium on which this mode propagates. Only under very specific conditions of volume fraction and applied magnetic field the particles form regular chains with cross section about the average diameter of them [5]. Moreover, it has been observed that a more general trend is the formation of clusters which pile up in the cell along the field direction from one extreme to the other [6]. The size and geometry of the clusters are predominantly determined by the morphology and magnetic dipole of the particles, which additionally to other physical factors like applied magnetic field and temperature, determine its fractal nature, as found by Morimoto and Maekawa [1]. As we shall show below the size and shape of the clusters also influence the elastic properties of the MR suspension. In order to calculate the sound speed for the second mode, we consider the following physical picture. We assume that, in concordance with the fractal nature of some of the physical characteristics of the system, one may conceive the system as composed by two coexisting dispersions. One of them is a dilute dispersion of iron particles with sizes distributed in the range 3–25 μm suspended in glycerine. The first mode propagates in this medium as we discussed above. The other dispersion is formed by the clusters suspended in glycerine. The second mode propagates in this medium. It has been found that the cluster average size originated by a dispersion of particles interacting through dipolar potentials is given by a relation of the form $R(n,\lambda) = \lambda^{\alpha} f(n/\lambda)$, where n is the number of particles in a given cluster, α is a constant of the order of the unit (for 3D systems), and the parameter λ is the ratio of the magnetic energy per particle to the thermal energy. Thus, in this picture, to calculate the sound speed for the second mode we may proceed as we did for the first one, however, we need to know the effective density and the elastic modulus of the clusters dispersed in glycerine. The evaluation of the effective density is direct, but the evaluation of the elastic modulus requires some discussion. The dipolar force between two spheres whose centers are separated a distance r is given by

$$F = -(3\mu_0/2\pi)(m^2/r^4)\,. \tag{2}$$

From here, by dividing by the cross section area of the chain formed by the clusters, the longitudinal stress is obtained. Then, taking the derivative with respect to the longitudinal strain i.e., $r(\partial/\partial r)$, one obtains an elastic modulus $\beta = \frac{2}{3}\mu_0 M^2$, where M is the magnetization.

The behavior of the sound speed as a function of the volume fraction can be seen for both modes in Fig. 1a. The first mode speed is represented by the pair of lower curves and the dots are the experimental results, the dashed line is obtained by using Eq. (1) taking the average value for the density as well as for the elastic modulus. This approach was used in Ref. [2] to explain their measurements. The continuous line is obtained averaging only the density values, taking into account that for dilute

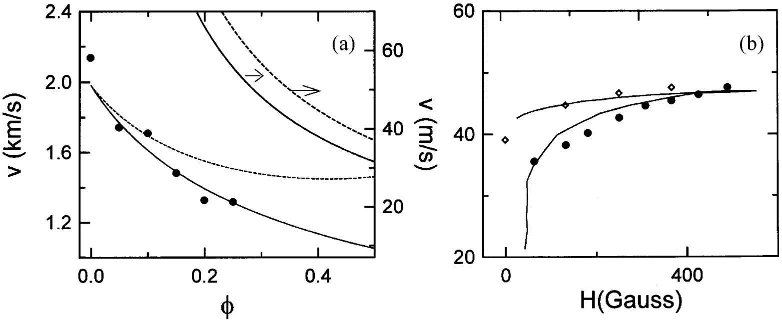

Fig. 1. (a) Sound speed as a function of volume fraction ϕ. First mode, lower pair of curves and experimental results (dots). Second mode, upper pair, with the scale on the right side. (b) Sound speed vs magnetic field intensity. Experimental results for increasing (dots) and decreasing (squares) field values. See text.

suspension one can neglect the effects of suspended particles on the elastic properties of the composite medium. It is clear that this approximation leads to a better agreement with the experimental results.

The upper pair of curves correspond to the second mode speed. In this case its scale appears on the right side. The dashed line is obtained from Eq. (1), by calculating ρ and β from the corresponding values for the clusters and the glycerine. Here we consider the saturation value of the magnetization and an applied magnetic field of 500 G, as it was assumed in Ref. [2]. The continuous curve is calculated by the same procedure for a more realistic value of the magnetization at this field, namely $M=0.97M_s$. Fig. 1b shows our calculations for the second mode speed as a function of the applied magnetic field (continuous line). Dots are the experimental results for increasing values of magnetic field and the squares correspond to decreasing values of magnetic field, as reported in Ref. [2]. For this calculation we have made use of a typical iron magnetization curve. We notice that all along the interval in which we can compare with the reported measurements we obtain a good agreement. In Ref. [2] for a volume fraction $\phi = 0.25$ and for an applied field of 500 G, by using a model of perfect chains the authors estimate a sound velocity of $v=20$ m/s. For that condition we obtain a better agreement.

The simple effective media approximation we have used here provides a quantitative accurate description for the sound speed for both longitudinal modes observed in a MR slurry of spherical iron particles. However, some aspects related to the cluster size and structure must be better understood. It is of particular interest to us to obtain some insights about the link between the fractal features of the clusters and the mechanical properties of these complex materials.

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