Estado Sólido Avanzado
Tarea 04: Propiedades dieléctricas

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13 Noviembre 2013

Problema 1 *Dielectric function for an ionic crystal*

From the equation of an ionic crystal:

$$\ddot{u} + \gamma\dot{u} = -\omega_0^2 u + \frac{e^*}{\mu}E,$$

and the polarization:

$$P = \frac{N}{V}e^* u + \varepsilon_0 \frac{N}{V}\alpha E, \quad \& \quad P(\omega) = \varepsilon_0(\varepsilon(\omega) - 1)E(\omega)$$

obtain:

(a) the dielectric function,

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_0^2\left(\varepsilon_{st} - \varepsilon_\infty\right)}{\omega_0^2 - \omega^2 - i\gamma\omega},$$

with:

$$\varepsilon_{st} = \frac{N}{V}\frac{e^{*2}}{\varepsilon_0 \mu \omega_0^2} + \frac{N}{V}\alpha + 1 \quad \& \quad \varepsilon_\infty = \frac{N}{V}\alpha + 1.$$

(b) The values of ω where $\varepsilon(\omega) = 0$.
Hint: when $\gamma \to 0$ those frequency values are $\omega = \pm\omega_0$ and $\omega = \pm\omega_0\left(\varepsilon_{st}/\varepsilon_\infty\right)^{1/2}$.

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Problema 2 *Polaritonic dispersion*

From the polaritonic dispersion for tranversal modes,

$$\omega^2 = \frac{1}{\varepsilon(\omega)}c^2 q^2$$

obtained for an ionic crystal, where the damping has not been taking into account ($\gamma = 0$), deduce:

(a) the following expresion for $\omega(q)$,

$$\left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{2}\left[\left(\frac{\omega_L}{\omega_0}\right)^2 + \frac{q^2c^2}{\varepsilon_\infty\omega_0^2}\right] \pm \frac{1}{2}\left\{\left[\left(\frac{\omega_L}{\omega_0}\right)^2 + \frac{q^2c^2}{\varepsilon_\infty\omega_0^2}\right]^2 - 4\frac{q^2c^2}{\varepsilon_\infty\omega_0^2}\right\}^{1/2},$$

(b) the following behavior of $\omega(q)$:

$$\begin{aligned} \text{large } \frac{cq}{\omega_0 \varepsilon_\infty}: &\qquad \omega_+ \to \frac{cq}{\sqrt{\varepsilon_\infty}} \;\;\&\;\; \omega_+ \to \omega_0 = \omega_T, \\ \frac{cq}{\omega_0 \varepsilon_\infty} \to 0: &\qquad \omega_+ \to \omega_L \;\;\&\;\; \omega_- \to \frac{cq}{\sqrt{\varepsilon_{st}}}. \end{aligned}$$

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Problema 3 *Electric field from an uniform polarized dielectric sphere*

Demostrate that the electric field generated by an uniform polarized dielectric sphere of radius a:

(a) outside region of the sphere ($z > a$) is:

$$E_{out} = -\frac{2Pa^3}{3\varepsilon_0 z^3},$$

(b) and in the inside region ($z < a$):

$$E_{in} = \frac{P}{3\varepsilon_0},$$

where $\mathbf{P} = P\hat{\mathbf{z}}$ is the total polarization.

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