

Estado Sólido Avanzado

Tarea 04: Propiedades dieléctricas

Dr. Omar De la Peña Seaman

28 Marzo 2019

Problema 1 *Dielectric function for an ionic crystal*

From the equation of an ionic crystal:

$$\ddot{u} + \gamma \dot{u} = -\omega_0^2 u + \frac{e^*}{\mu} E,$$

and the polarization:

$$P = \frac{N}{V} e^* u + \varepsilon_0 \frac{N}{V} \alpha E, \quad \& \quad P(\omega) = \varepsilon_0 (\varepsilon(\omega) - 1) E(\omega)$$

obtain the following,

(a) the dielectric function,

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_0^2 (\varepsilon_{st} - \varepsilon_\infty)}{\omega_0^2 - \omega^2 - i\gamma\omega},$$

with:

$$\varepsilon_{st} = \frac{N}{V} \frac{e^{*2}}{\varepsilon_0 \mu \omega_0^2} + \frac{N}{V} \alpha + 1 \quad \& \quad \varepsilon_\infty = \frac{N}{V} \alpha + 1.$$

(b) The values of ω where $\varepsilon(\omega) = 0$.

Hint: when $\gamma \rightarrow 0$, those frequency values are $\omega = \pm\omega_0$ and $\omega = \pm\omega_0 (\varepsilon_{st}/\varepsilon_\infty)^{1/2}$.

.....

Problema 2 *Polaritonic dispersion*

From the polaritonic dispersion for transversal modes,

$$\omega^2 = \frac{1}{\varepsilon(\omega)} c^2 q^2$$

obtained for an ionic crystal, without taking into account the damping ($\gamma = 0$), deduce:

(a) The following expression for $\omega(q)$,

$$\left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{2} \left[\left(\frac{\omega_L}{\omega_0}\right)^2 + \frac{q^2 c^2}{\varepsilon_\infty \omega_0^2} \right] \pm \frac{1}{2} \left\{ \left[\left(\frac{\omega_L}{\omega_0}\right)^2 + \frac{q^2 c^2}{\varepsilon_\infty \omega_0^2} \right]^2 - 4 \frac{q^2 c^2}{\varepsilon_\infty \omega_0^2} \right\}^{1/2},$$

(b) The following behavior of $\omega(q)$:

$$\begin{aligned} \text{large } \frac{c^2 q^2}{\omega_0^2 \varepsilon_\infty} : & \quad \omega_+ \rightarrow \frac{cq}{\sqrt{\varepsilon_\infty}} \quad \& \quad \omega_+ \rightarrow \omega_0 = \omega_T, \\ \text{small } \frac{c^2 q^2}{\omega_0^2 \varepsilon_\infty} : & \quad \omega_+ \rightarrow \omega_L \quad \& \quad \omega_- \rightarrow \frac{cq}{\sqrt{\varepsilon_{st}}}. \end{aligned}$$

.....

Problema 3 *Electric field from an uniform polarized dielectric sphere*

Demonstrate that the electric field generated by an uniform polarized dielectric sphere of radius a :

(a) Outside region of the sphere ($z > a$) is:

$$E_{out} = -\frac{2Pa^3}{3\varepsilon_0 z^3},$$

(b) And in the inside region ($z < a$):

$$E_{in} = \frac{P}{3\varepsilon_0},$$

where $\mathbf{P} = P\hat{\mathbf{z}}$ is the total polarization.

.....