Estado Sólido Avanzado
Tarea 03: Magnetismo

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Problema 1 *Hydrogen diamagnetism*

Calculate the diamagnetic susceptibility of hydrogen atom, with the ground state wavefunction:

$$\psi = \frac{1}{(a_0^3\pi)^{1/2}} e^{-r/a_0}$$

where $a_0 = 0.529$ Å is the Bohr radius.

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Problema 2 *Partially filled shell: paramagnetism*

(a) The angular momentum conmutation relations are summarized in the vector operator identities,

$$\mathbf{L} \times \mathbf{L} = i\mathbf{L}, \quad \mathbf{S} \times \mathbf{S} = i\mathbf{S}.$$

Deduce from this identities and the fact that all components of $\mathbf{L}$ conmute with all components of $\mathbf{S}$ that:

$$[\mathbf{L} + g_0\mathbf{S}, \hat{\mathbf{n}} \cdot \mathbf{J}] = i\hat{\mathbf{n}} \times (\mathbf{L} + g_0\mathbf{S})$$

for any unit vector $\hat{\mathbf{n}}$.

(b) A state $|0\rangle$ with zero total angular momentum satisfies,

$$\mathbf{J}_x |0\rangle = \mathbf{J}_y |0\rangle = \mathbf{J}_z |0\rangle = 0.$$

Deduce from the conmutation relationship on (a) that:

$$\langle 0|(\mathbf{L} + g_0\mathbf{S})|0\rangle = 0,$$

even though $\mathbf{L}^2$ and $\mathbf{S}^2$ need not to vanish in the state $|0\rangle$, and $(\mathbf{L} + g_0\mathbf{S})|0\rangle$ need not be zero.

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Problema 3 *Pauli paramagnetism: temperature corrections*

Show that if T is small compared with the Fermi temperature, the temperature-dependent correction to the Pauli susceptibility, $\chi(0) = \mu_B^2 g(\varepsilon_F)$, is given by

$$\chi(T) = \chi(0)\left(1 - \frac{\pi^2}{6}(k_B T)^2 \left[\left(\frac{g'}{g}\right)^2 - \frac{g''}{g}\right]\right),$$

where g, g', and g'' are the density of states and its derivatives at the Fermi energy. Show that for free electrons this reduces to,

$$\chi(T) = \chi(0)\left(1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right).$$

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Problema 4 *Derivatives of the Fermi-Dirac distribution function*

For the Fermi-Dirac distribution function,

$$f(E) = \frac{1}{e^{(E-E_F)\beta} + 1} \quad \forall \quad \beta = \frac{1}{k_B T},$$

where was considered $\mu \approx E_F$, demostrate that:

$$\frac{\partial f}{\partial E} < 0 \quad \& \quad \frac{\partial^3 f}{\partial E^3} > 0,$$

for every value of T.

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Problema 5 *Mean field theory near the critical point*

(a) Deduce that as T aproaches T_c from below, the spontaneous magnetization of a ferromagnet vanishes as $(T_c - T)^{1/2}$ according to mean field theory.
Note: Experimentally, the behavior is like $(T_c - T)^\beta$ with $\beta = 0.33 - 0.37$.

(b) Deduce that at T_c, the magnetization density $M(H, T_c)$ vanishes as $H^{1/3}$ in mean field theory.
Note: Experiments and calculations indicate an exponent closer to 1/5.

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