Estado Sólido Avanzado
Tarea 04: Propiedades dieléctricas

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Nombre del Estudiante: ____

Problema 1 *Dielectric function for an ionic crystal*

From the equation of an ionic crystal:

$$\ddot{u} + \gamma \dot{u} = -\omega_0^2 u + \frac{e^*}{\mu} E,$$

and the polarization:

$$P = \frac{N}{V} e^* u + \varepsilon_0 \frac{N}{V} \alpha E, \quad \& \quad P(\omega) = \varepsilon_0 (\varepsilon(\omega) - 1) E(\omega)$$

obtain the following,

(a) the dielectric function,

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_0^2 \left(\varepsilon_{st} - \varepsilon_\infty\right)}{\omega_0^2 - \omega^2 - i\gamma\omega},$$

with:

$$\varepsilon_{st} = \frac{N}{V} \frac{e^{*2}}{\varepsilon_0 \mu \omega_0^2} + \frac{N}{V} \alpha + 1 \quad \& \quad \varepsilon_\infty = \frac{N}{V} \alpha + 1.$$

(b) The values of ω where $\varepsilon(\omega) = 0$.
Hint: when $\gamma \to 0$, those frequency values are $\omega = \pm\omega_0$ and $\omega = \pm\omega_0 \left(\varepsilon_{st}/\varepsilon_\infty\right)^{1/2}$.

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Problema 2 *Absorption coefficient for an optically dense media*

For an optically dense media, and in the case of transmission through a sufficiently thin film:

$$\left| \frac{\tilde{n}\omega}{c} d \right| \ll 1,$$

demonstrate that the transmision intensity of the electromagnetic radiation, in a linear approximation, is given by:

$$I \propto EE^* = I_0 \left(1 - \frac{\epsilon_2 \omega}{c} d + \dots\right)$$

where the quantity $K(\omega) = \omega \epsilon_2(\omega)/c$ is also called the absorption coefficient.

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Problema 3 *Polaritonic dispersion*

From the polaritonic dispersion for tranversal modes,

$$\omega^2 = \frac{1}{\varepsilon(\omega)} c^2 q^2$$

obtained for an ionic crystal, without taking into account the damping ($\gamma = 0$), deduce:

(a) The following expresion for $\omega(q)$,

$$\left(\frac{\omega}{\omega_0}\right)^2 = \frac{1}{2}\left[\left(\frac{\omega_L}{\omega_0}\right)^2 + \frac{q^2c^2}{\varepsilon_\infty \omega_0^2}\right] \pm \frac{1}{2}\left\{\left[\left(\frac{\omega_L}{\omega_0}\right)^2 + \frac{q^2c^2}{\varepsilon_\infty \omega_0^2}\right]^2 - 4\frac{q^2c^2}{\varepsilon_\infty \omega_0^2}\right\}^{1/2},$$

(b) The following behavior of $\omega(q)$:

$$\text{large } \frac{c^2q^2}{\omega_0^2\varepsilon_\infty} : \ \omega_+ \to \frac{cq}{\sqrt{\varepsilon_\infty}} \ \ \& \ \ \omega_- \to \omega_0 = \omega_T,$$

$$\text{small } \frac{c^2q^2}{\omega_0^2\varepsilon_\infty} : \ \omega_+ \to \omega_L \ \ \& \ \ \omega_- \to \frac{cq}{\sqrt{\varepsilon_{st}}}.$$

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Problema 4 *Electric field from an uniform polarized dielectric sphere*

Demostrate that the electric field generated by an uniform polarized dielectric sphere of radius a:

(a) Outside region of the sphere ($z > a$) is:

$$E_{out} = -\frac{2Pa^3}{3\varepsilon_0 z^3},$$

(b) And in the inside region ($z < a$):

$$E_{in} = \frac{P}{3\varepsilon_0},$$

where $\mathbf{P} = P\hat{\mathbf{z}}$ is the total polarization.

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