Mecánica Clásica
Tarea 02: Principios variacionales

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Nombre del Estudiante: ____

Problema 1 *Minimum surface*

A curve $y(x)$ in the $x-y$ plane connecting the points (x_1, y_1) and (x_2, y_2) is revolved around the x axis and generates a surface of revolution. Show that the curve which generates the surface with the least area can be expressed as

$$y(x) = C_1 \mathrm{Cosh}\left(\frac{x - C_2}{C_1}\right),$$

where C_1 and C_2 are constants.

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Problema 2 *Parabola*

Show that the extremal of the isoperimetric problem,

$$I\left[y(x)\right] = \int_{x_1}^{x_2} {y'}^2 dx$$

subject to the condition,

$$J\left[y(x)\right] = \int_{x_1}^{x_2} y dx = k = \mathrm{cte},$$

is a parabola. Determine the equation of the parabola passing through the points $P_1(1,3)$ and $P_1(4,24)$ with $k = 36$.

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Problema 3 *Rolling hoop*

A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown in the figure. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange multipliers to find the point at which the hoop falls off the cylinder.

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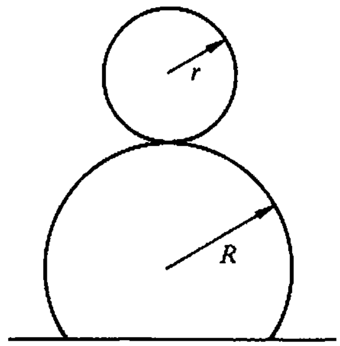

Problema 4 *Rotating massless hoop*

A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω. Obtain the Lagrange equations of motion assuming the only external forces arise from gravity.

1. What are the constants of motion?
2. Show that if ω is greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the hoop at a point other than the bottom, but that if $\omega < \omega_0$, the only stationary point for the particle is at the bottom of the hoop.
3. What is the value of ω_0?

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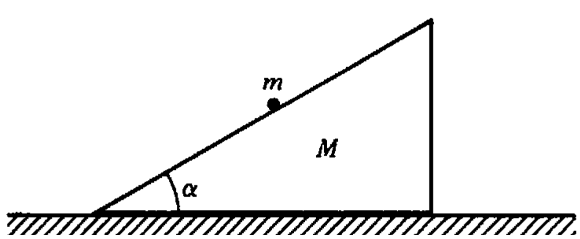
Problema 5 *Mass on a moving wedge*

A particle of mass m slides without friction on a wedge of angle α and mass M that can move without friction on a smooth horizontal surface, as show in the figure.

1. Treating the constraint of the particle on the wedge by the method of Lagrange multipliers, find the equations of motion for the particle on the wedge.
2. Obtain an expression for the forces of constraint.
3. What are the constants of motion for the system?



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Problema 6 *Masses on circular paths*

Consider two particles of masses m_1 and m_2. Let m_1 be confined to move on a circle of radius a in the $z = 0$ plane, centered at $x = y = 0$. Let m_2 be confined to move on a circle of radius b in the $z = c$ plane, centered at $x = y = 0$, with $b < a$. A light (massless) spring of spring-constant k is attached between the two particles. Using Lagrange multipliers:

1. Find the Lagrangian for the system.
2. Obtain the equations of motion.

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