Mecánica Clásica
Tarea 03: Fuerzas Centrales

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Problema 1 *Kepler's problem*

Using the potential $V(r) = -k/r$ in the Kepler's problem, find the following:

(a) the equation of motion $\theta(t)$ for a parabolic motion ($e = 1$),

(b) the equation of motion $\theta(t)$ for an elliptical motion ($e < 1$),

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Problema 2 *Shifted force's center*

(a) Show that if a particle describes a circular orbit under the influence of an attractive central force directed toward a point on the circle, then the force varies as the inverse-fifth power of the distance.

(b) Show that for the orbit described the total energy of the particle is zero.

(c) Find the period of motion.

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Problema 3 *Orbits' intersection*

(a) For circular and parabolic orbits in an atractive $1/r$ potential having the same angular momentum, show that the perihelion distance of the parabola is one-half the radius of the circle.

(b) Prove that in the same central force as in the part (a) the speed of a particle at any point in the parabolic orbit is $\sqrt{2}$ times the speed in a circular orbit passing trough the same point.

Note: perihelion is the closest orbit's point to the center of the potential.

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Problema 4 *Precession on a Kepler orbit*

Find the general orbit, $r(\theta)$, for a particle moving in a perturbed gravitational potential:

$$V = -\frac{k}{r} + \frac{\beta}{r^2}.$$

Show that if $k > 0$ and $\beta \ll k(1-\epsilon^2)$, the orbit may be thought of as an ellipse of eccentricity ϵ precessing slowly with angular velocity:

$$\omega = \frac{2\pi\beta}{\tau k a(1-\epsilon^2)},$$

where a is the semimajor axis and τ is the orbital period.

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Problema 5 *Cross section*

Examine the scattering produced by a repulsive central force $f = k/r^3$. Show that the Cross section is given by,

$$\frac{d\sigma}{d\Omega} d\Theta = \frac{k}{2E} \frac{(1-x)dx}{x^2(2-x)^2 \mathrm{Sen}\, \pi x},$$

where x is the ratio of Θ/π and E is the energy.

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Problema 6 *Unknow potential*

Find the central potential whose scattering cross section is given by

$$\frac{d\sigma(\Theta)}{d\Omega} = \alpha\pi^2 \frac{\pi - \Theta}{(2\pi - \Theta)^2 \Theta^2 \mathrm{Sen}\Theta}.$$

where α is a constant.

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