# Mecánica Clásica <br> Tarea 05: Cuerpo Rígido II - Ecuaciones de Movimiento 

Dr. Omar De la Peña Seaman

4 Octubre 2019

Nombre del Estudiante: $\qquad$

Problema 1 Euler's equations
Derive Euler's equation of motion:

$$
I_{3} \dot{\omega}_{3}-\omega_{1} \omega_{2}\left(I_{1}-I_{2}\right)=N_{3},
$$

from the Lagrange equations,

$$
\frac{d}{d t}\left(\frac{\partial T}{\partial \dot{q}_{j}}\right)-\frac{\partial T}{\partial q_{j}}=Q_{j} \quad \forall T=\text { kinetic energy, }
$$

for the generalized coordinate $\psi$, using the Euler's angles set $\{\phi, \theta, \psi\}$.

## Problema 2 Right cylinder

What is the height-to-diameter ratio of a right cylinder such that all principal moments of inertia at the center of the cylinder are the same?

## Problema 3 Right triangle

Calculate the principal moments of inertia and principal axes about the center of mass of a flat rigid body in the shape of $45^{\circ}$ right triangle with uniform mass density.
Hint: the center of mass is located at the point where the medians intersect.

## Problema 4 Plate

A thin square plate with side lenght $a$ rotates at a constant angular frequency $\omega$ about an axis through the center tilted by an angle $\theta$ with respect to the normal of the plate.

1. Find the principal moments of inertia.
2. Find the angular momentum $\mathbf{L}$ and the torque components along the principal axes of the plate.
3. Calculate the angular momentum $\mathbf{L}$ and the torque in the laboratory (fixed) system.


## Problema 5 Triangle oscillator

Find the frequency of small oscillations for a thin homogeneous plate if the motion takes place in the plane of the plate and if the plate has the shape of an equilateral triangle and is suspended:

1. from the midpoint of one side.
2. from one vertex.
