Mecánica Clásica
Tarea 06: Oscilaciones

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Nombre del Estudiante: ______________________________

Problema 1 *General oscillating system*

Consider a particle of mass m with two degrees of freedom (x_1, x_2) that obeys the Lagrangian,

$$L = \frac{1}{2}m(\dot{x}_1^2 + \dot{x}_2^2) - \frac{1}{2}V_{ij}x_i x_j$$

where the V_{ij} are constants. Assuming that $V_{12} > V_{22} > 0$ and $(V_{11} - V_{22}) \ll V_{12} = V_{21}$, find the eigenvalues and eigenvectors of the system.
Hint: Express the results to first order in $\epsilon = (V_{11} - V_{22})/8V_{12}$ (which is very small!!)

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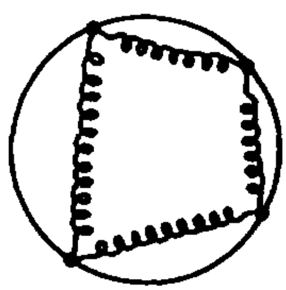
Problema 2 *Circular molecule*

Four identical masses are connected by four identical springs, and constrained to move on a frictionless circle of radius b.



1. Calculate the normal frequencies of small oscillations.
2. Determine the corresponding normal modes.

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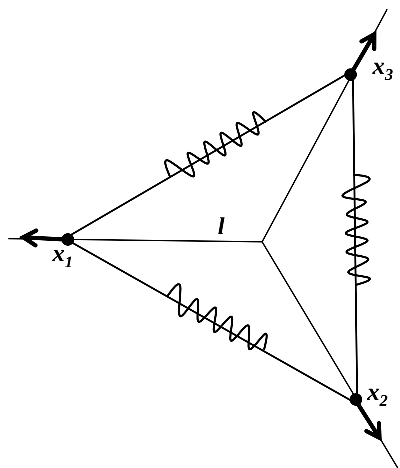
Problema 3 *Triangular molecule*

Three bodies of equal mass m and indicated by $i = 1, 2, 3$ are constrained to perform small oscilations along different coplanar axes forming $120°$ angles at their common intersection (see figure). Identical coupling springs hold these bodies near equilibrium positions which are at a distance l from the intersection on each axis.



1. Show that the equations of motion of the three bodies are represented by the coupled system,
$$m\frac{d^2x_i}{dt^2} = -Kx_i - k(x_1 + x_2 + x_3),$$
where $x_i(t) + l$ indicated their respective distances from the intersection, and $K = k = (3/4)q$, where q is the spring constant.

2. Find the normal frequencies.

3. Calculate the normal modes of the system.

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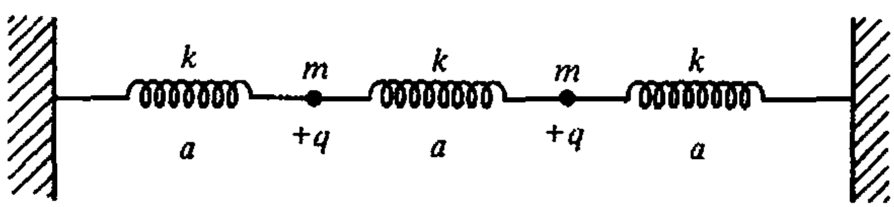
Problema 4 *Oscillating charges*

Two mass points of equal mass m are connected to each other and to fixed points by three equal springs of force constant k, as shown below,



The equilibrium lenght of each spring is a. Each mass point has a positive charge $+q$, and they repel each other according to Coulomb law. Find the eigenfrequencies of the system.

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