Mecánica Clásica
Tarea 07: Ecuaciones de Hamilton

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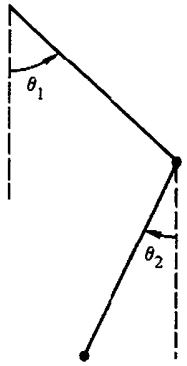
Problema 1 *Double pendulum*

Formulate the double-pendulum problem, as illustrated in the figure (with $m_1 \neq m_2$ and $l_1 \neq l_2$), in terms of the Hamiltonian and Hamilton's equations of motion. Obtain the Hamiltonian directly from L as a Legendre transformation.



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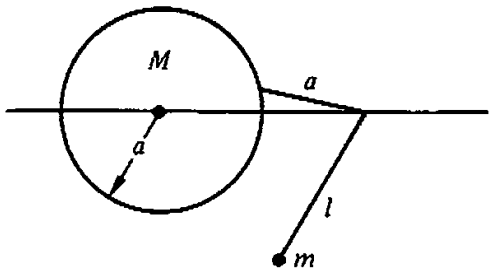
Problema 2 *Coupled pendulum*

The point of suspension of a plane simple pendulum of mass m and lenght l is constrained to move along a horizontal track and is connected to a point on the circumference of an uniform flywheel of mass M and radius a through a massless connecting rod also of lenght a (see figure). Find the Hamiltonian for the combined system and determine Hamilton's equation of motion.



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Problema 3 *Bar and spring*

Find the Hamiltonian and the Hamilton's canonical equations of motion for a uniform bar of mass M and lenght $2l$ that is suspended from one end by a spring of force constant k. The bar can swing freely only in one vertical plane, and the spring is constrained to move only in the vertical direction.

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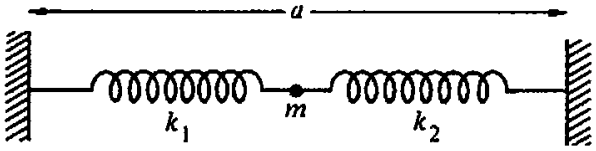
Problema 4 *Two springs*

A particle of mass m can move in one dimension under the influence of two springs connected to fixed points a distance a apart. The springs obey Hooke's law and have zero unstretched lengths and force constants k_1 and k_2, respectively.



1. Using the position of the particle from one fixed point as the generalized coordinate, find the Lagrangian and the corresponding Hamiltonian. Is the Hamiltonian the total energy? Is the Hamiltonian conserved?

2. Introduce a new coordinate Q defined by

$$Q = q - b\mathrm{Sen}\,\omega t, \qquad b = \frac{k_2 a}{k_1 + k_2}.$$

   What is the Lagrangian in terms of Q? What is the corresponding Hamiltonian? Is the Hamiltonian the total energy? Is the Hamiltonian conserved?

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