Mecánica Clásica
Tarea 08: Transformaciones Canónicas

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Problema 1 *Canonical transformation*

The transformation equations between two sets of coordinates are,

$$Q = \ln(1 + q^{1/2}\mathrm{Cos}\, p),$$
$$P = 2(1 + q^{1/2}\mathrm{Cos}\, p)q^{1/2}\mathrm{Sen}\, p.$$

1. Show directly from these transformation equations that (Q, P) are canonical variables if q and p are.
2. Show that the function that generates this transformation is:

$$F_3 = -(e^Q - 1)^2 \mathrm{Tg}\, p.$$

. . . .

Problema 2 *Canonical transformation II*

Prove that the transformation

$$Q_1 = q_1^2, \qquad Q_2 = q_2 \mathrm{Sec}\, p_2,$$
$$P_1 = \frac{p_1 \mathrm{Cos}\, p_2 - 2q_2}{2q_1 \mathrm{Cos}\, p_2}, \qquad P_2 = \mathrm{Sen}\, p_2 - 2q_1,$$

is canonical by any method you choose. Find a suitable generating function that will lead to this transformation.

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Problema 3 *Damping forces*

A particle of mass m moves in one dimension q in a potential energy field $V(q)$ and is retarded by a damping force $-2m\gamma\dot{q}$ proportional to its velocity such that its Lagrangian is given by:

$$L = \exp(2\gamma t)\left[\frac{1}{2}m\dot{q}^2 - V(q)\right].$$

1. Show that the Hamiltonian is

$$H = \frac{p^2 \exp(-2\gamma t)}{2m} + V(q)\exp(2\gamma t),$$

   where $p = m\dot{q}\exp(2\gamma t)$ is the momentum conjugate to q.

2. For the generating function:

$$F_2(q, P, t) = \exp(\gamma t) q P,$$

   find the transformed Hamiltonian $K(Q, P, t)$.

3. For an oscillator potential

$$V(q) = \frac{1}{2} m\omega^2 q^2$$

   show that the transformed Hamiltonian yields a constant of the motion:

$$K = \frac{P^2}{2m} + \frac{1}{2} m\omega^2 Q^2 + \gamma Q P.$$

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Problema 4 *Constants of motion*

A system of two degrees of freedom is described by the Hamiltonian

$$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2.$$

Show that

$$F_1 = \frac{p1 - a q_1}{q_2}, \quad \& \quad F_2 = q_1 q_2$$

are constants of motion.

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Problema 5 *Noether theorem*

For the following Lagrangian,

$$L = \dot{q}_1 \dot{q}_2 - \omega^2 q_1 q_2,$$

1. Describe the motion of the system. That is, find the Lagrangian equations of motion.

2. Shows that L is invariant under the family of point transformation

$$Q_1 = e^{\epsilon} q_1, \quad Q_2 = e^{-\epsilon} q_2.$$

3. Find the Noether constant associated with this group of transformation, and demostrate that it is actually a constant of motion.

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