Mecánica Clásica
Tarea 09: Hamilton-Jacobi

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27 Noviembre 2019

Problema 1 *Sliding rod*

One end of a uniform rod of lenght $2l$ and mass m rests against a smooth horizontal floor and the other against a smooth vertical surface (both surfaces without friction). Assuming that the rod is constrained to move under gravity with its ends always in contact with the surfaces, use the Hamilton-Jacobi equations to reduce the soluction of the problem to the integral form (quadratures).

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Problema 2 *Roller coaster*

A particle is constrained to move on a roller coaster, the equation of whose curve is

$$z = A\text{Cos}^2\frac{2\pi x}{\lambda}.$$

There is the usual constant downward force of gravity. Obtain the trajectories in space face, that is $p = p(x)$.

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Problema 3 *General oscillations*

A particle of mass m moves in one dimension subject to the potential

$$V = \frac{a}{\text{Sen}^2(x/x_0)}.$$

1. Obtain the integral expresion for Hamilton's caracteristic function. Under what conditions can action-angle variables be used?
2. Assuming these are met, find the frequency of oscillation by the action-angle method.
3. Check the result in the limit of oscillations of small amplitude.

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Problema 4 *Parametric oscillating movement*

A particle of mass m is constrained to move on a curve in the vertical plane defined by the parametric equations

$$y = l(1 - \mathrm{Cos}\, 2\phi), \quad x = l(2\phi + \mathrm{Sen}\, 2\phi).$$

There is the usual constant gravitational force acting in the vertical y direction.
By the method of action-angle variables, find the frequency of oscillation such that the maximum of ϕ is equal to $\pi/4$.
Hint: $\oint pdq = 2\int_{-\phi_0}^{\phi_0} pdq$.

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