Mecánica Clásica
Tarea 04: Dinámica Lagrangiana

Dr. Omar De la Peña Seaman

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Problema 1 *Pendulum in a rail-road car*

Find the frequency of small oscillations of a simple pendulum placed in a rail-road car that has a constant acceleration a in the x-direction, with $x(0) = 0$ and $\dot{x}(0) = v_0$.

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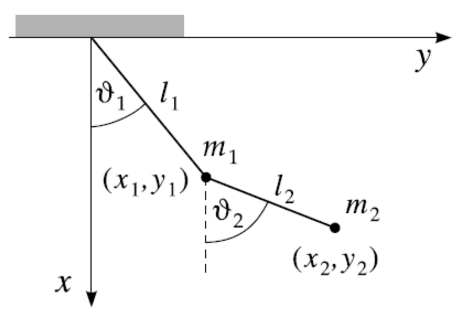
Problema 2 *Double Pendulum*

A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. If the two pendula have equal lengths and have bobs of equal mass and if both pendula are confined to move in the same plane, find:

1. Lagrange's equations of motion without assuming small angles,
2. Lagrange's equations of motion for small angles.



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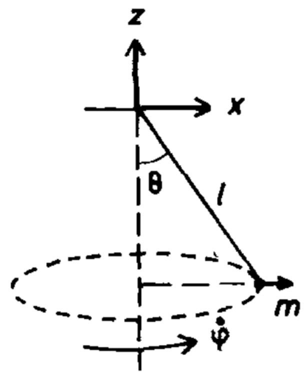
Problema 3 *Spherical pendulum*

A spherical pendulum consists of a point mass m tied by a string of lenght l to a fixed point, so that it is constrained to move on a spherical surface.

1. With what angular velocity will it move on a circle, with the string making a constant angle θ_0 with the vertical?

2. For the case in which the amplitude of the oscillations about θ_0 is small, solve for the frequency of these oscillations.



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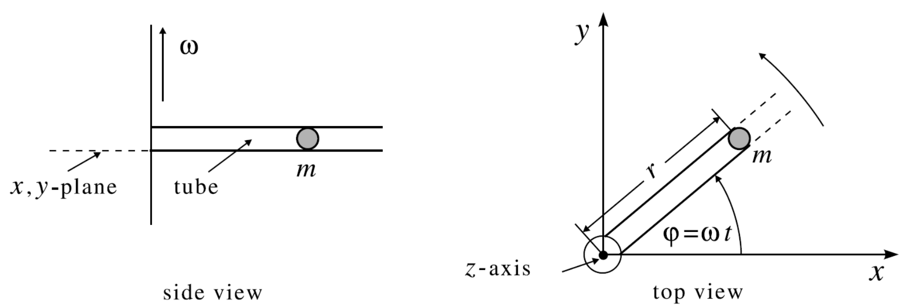
Problema 4 *Sphere in a rotating tube*

A sphere moves in a tube that rotates in the $x-y$ plane about the z axis with constant angular velocity ω. Find the equation(s) of motion and solve it(them).



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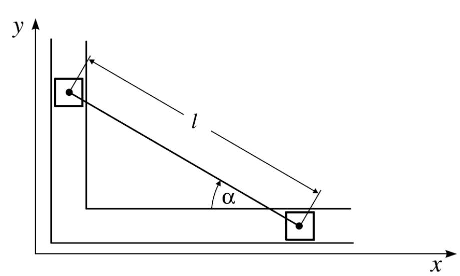
Problema 5 *Two blocks connected by a bar*

Two blocks of equal mass that are connected by a rigid bar of lenght l move without friction along a given path. The attraction of the earth acts along the negative y axis. Find the equation of motion of the system and show that:

$$t - t_0 = \int \frac{d\alpha}{\sqrt{2\left[c - (g/l)\text{Sen}\,\alpha\right]}}, \quad \text{where } c \text{ and } t_0 \text{ are constants.}$$



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Problema 6 *Pendulum with accelerated support point*

A simple pendulum of lenght b and bob with mass m is attached to a massless support moving horizontally with constant acceleration a. Determine:

1. the equations of motion,
2. the frequency for small oscillations about the equilibrium angle θ_0.

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Problema 7 *Slippery mass*

A particle of mass m starts at rest on top of a smooth fixed hemisphere of radius a. Find the force of constraint, and determine the angle at which the particle leaves the hemisphere.

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Problema 8 *Point transformations to Lagrangian*

Assume the Lagrngian for a cetain one-dimensional motion is given by:

$$L = e^{\gamma t}\left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2\right),$$

where γ, m, and k are positive constants.

1. Find the Lagrange equation,
2. Are there any constants of motion?
3. Find the general solution to the motion equation.

Suppose a point transformation is made to another generalized coordinate S, given by:

$$S = \exp\left(\frac{\gamma t}{2}\right) q,$$

4. Find the Lagrangian in terms of S.
5. Find the Lagrange equation.
6. Are there any constants of motion?

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