Mecánica Clásica
Tarea 03: Cálculo de Variaciones

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Problema 1 *Euler equations*

Derive the expression for the Euler equation when the functional f depend also on the second-order derivative of the function $y(x)$:

$$J = \int_1^2 f(y, y', y'', x)dx,$$

where x is the independent variable.

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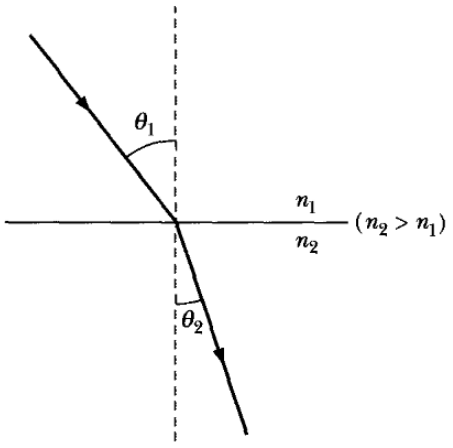
Problema 2 *Refraction law*

Consider light passing from one medium with index of refraction n_1 into another medium with index of refraction n_2. Using the principle that the path taken between two points by a ray of light is the path that can be traversed in the least time (Fermat's principle), derive the law of refraction:

$$n_1 \text{Sen}\,\theta_1 = n_2 \text{Sen}\,\theta_2.$$



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Problema 3 *Geodesics*

Find the shortest distance between two points on:

(a) a cilindrical surface,

(b) a spherical surface.

Hint: the shortest distance on a plane is a straight line.

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Problema 4 *Catenary*

An inextensible but flexible chain (*catenary* in Latin) or rope of specified length L hangs between two fixed points, (x_1, y_1) and (x_2, y_2), under the influence of gravity in the $x - y$ plane. What is the curve describing the chain's shape?
Hint: the quantity to minimize is the potential energy $U = \rho g \int y ds$, where ρ is the linear density of the rope, g the gravity, and y the high of the leght differential element ds.

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Problema 5 *Constrains*

Find the extremal of the function,

$$J = \int_0^\pi \left(y'^2 - y^2\right) dx,$$

with boundary conditions $y(0) = 0$, $y(\pi) = 1$ and subject to the constrain:

$$\int_0^\pi y dx = 1.$$

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