

Mecánica Clásica

Tarea 1: Mecánica Newtoniana

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Nombre del Estudiante: _____

Problema 1 *Equation of motion*

A force given by:

$$F = -F_0 e^{-x/\lambda},$$

where F_0 and λ are positive constants, acts on a particle that is initially at $x_0 = 0$ and moving with velocity $v_0 > 0$. Determine its velocity $v(x)$ and sketch the three possible graphs of $v(x)$ versus x .

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Problema 2 *Falling particle vs air resistance*

A particle is released from rest ($y = 0$) and falls under the influence of gravity and air resistance. Find the relationship between the velocity v and the distance of falling y when the air resistance is equal to a) αv , and b) βv^2 .

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Problema 3 *Elusive force*

The speed of a particle of mass m varies with the distance x as $v(x) = \alpha x^{-n}$. Assume $v(x=0) = 0$ at $t = 0$.

- (a) Find the force $F(x)$ responsible.
- (b) Determine $x(t)$.
- (c) Determine $F(t)$.

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Problema 4 *Charged particle in an electromagnetic field*

The motion of a charged particle in an electromagnetic field can be obtained from the Lorentz equation for the force on a particle in such a field. If the electric field vector is \mathbf{E} and the magnetic field vector is \mathbf{B} , the force on a particle of mass m that carries a charge q and has a velocity \mathbf{v} is given by,

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B},$$

where it is assumed that $v \ll c$.

- (a) If there is no electric field and if the particle enters the magnetic field in a direction perpendicular to the lines of magnetic flux, show that the trajectory is a circle with radius

$$r = \frac{mv}{qB} = \frac{v}{\omega_c},$$

where $\omega_c \equiv qB/m$ is the *cyclotron frequency*.

- (b) Choose the z -axis to lie in the direction of \mathbf{B} and let the plane containing \mathbf{E} and \mathbf{B} be the yz -plane. Thus,

$$\mathbf{B} = B\mathbf{k}, \quad \mathbf{E} = E_y\mathbf{j} + E_z\mathbf{k}.$$

Show that the z -component of the motion is given by

$$z(t) = z_0 + \dot{z}_0 t + \frac{qE_z}{2m} t^2,$$

where $z(0) \equiv z_0$ and $\dot{z}(0) \equiv \dot{z}_0$.

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Problema 5 *Rocket vertical ascent*

Consider a single-stage rocket taking off from Earth on vertical ascent under gravity (no horizontal component), neglecting air resistance, and assuming that the acceleration of gravity is constant with height, find:

- The velocity equation of motion $v = v(m)$, where m is the variable mass of the rocket.
- The height of the rocket at burnout.
- How much farther in height will the rocket go after burnout?

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Problema 6 *Gun's maximum range*

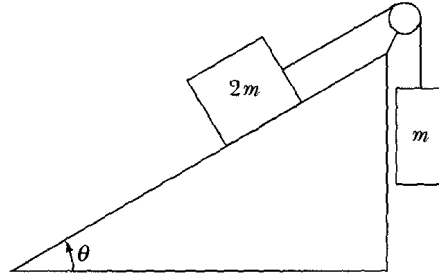
A projectile is fired with a velocity v_0 such that it passes through two points both a distance h above the horizontal. Show that if the gun is adjusted for maximum range, the separation of the points is,

$$d = \frac{v_0}{g} \sqrt{v_0^2 - 4gh}.$$

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Problema 7 *Connected masses*

Two blocks of unequal mass are connected by a string over a smooth pulley. If the coefficient of kinetic friction is μ_k , what angle θ of the incline allows the masses to move at a constant speed?



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Problema 8 *Analysis of a potential*

Consider a particle moving in the region $x > 0$ under the influence of the potential

$$U(x) = U_0 \left(\frac{a}{x} + \frac{x}{a} \right),$$

where $U_0 = 1$ J and $a = 2$ m. Plot the potential, find the equilibrium points, and determine whether they are maxima or minima.

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