Mecánica Clásica
Tarea 4: Dinámica Lagrangiana y Hamiltoniana

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Nombre del Estudiante: _____

Problema 1 *Pendulum in a rail-road car* **(15 pts.)**

Find the frequency of small oscillations of a simple pendulum placed in a rail-road car that has a constant acceleration a in the x-direction, with $x(0) = 0$ and $\dot{x}(0) = v_0$.

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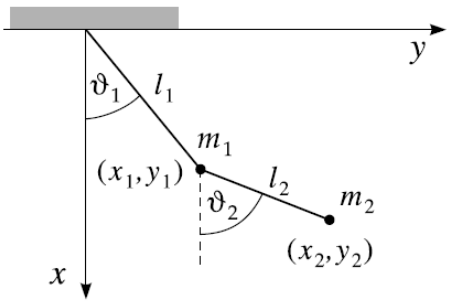
Problema 2 *Double Pendulum* **(20 pts.)**

A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. If the two pendula have equal lengths and have bobs of equal mass and if both pendula are confined to move in the same plane, find:

1. Lagrange's equations of motion without assuming small angles.
2. Lagrange's equations of motion for small angles.



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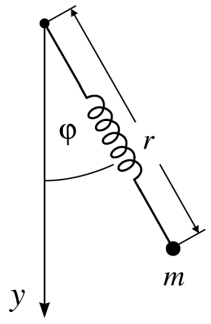
Problema 3 *String Pendulum* **(15 pts.)**

A mass m is suspended by a spring with spring constant k in the gravitational field. Besides the longitudinal spring vibration, the spring performs a plane-pendulum motion. Find the Lagrangian and derive the equations of motion.



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Problema 4 *Slippery mass* **(15 pts.)**

A particle of mass m starts at rest on top of a smooth fixed hemisphere of radius a. Find the force of constraint, and determine the angle at which the particle leaves the hemisphere.

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Problema 5 *Point transformations to Lagrangian* **(15 pts.)**

Assume the Lagrangian for a cetain one-dimensional motion is given by:

$$L = e^{\gamma t}\left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2\right),$$

where γ, m, and k are positive constants.

1. Find the Lagrange equation,
2. Are there any constants of motion?
3. Find the general solution to the motion equation.

Suppose a point transformation is made to another generalized coordinate S, given by:

$$S = \exp\left(\frac{\gamma t}{2}\right) q,$$

4. Find the Lagrangian in terms of S.
5. Find the Lagrange equation.
6. Are there any constants of motion?

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Problema 6 *Particle on a cylinder surface* **(20 pts.)**

We have a particle of mass m constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$, that is subject to a force directed toward the origin and proportional to the distance of the particle from the origin: $\mathbf{F} = -kr\hat{\mathbf{r}}$, with $r = \sqrt{x^2 + y^2 + z^2}$.
Find the following:

1. The Hamiltonian, using the Legendre transformation.
2. The canonical equations of motion of the particle.
3. The equation of motion in the z direction.

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