Métodos Matemáticos
Tarea 01: Análisis Vectorial

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Nombre del Estudiante: ______________________________

Problema 1 *Vectorial relationships*

(a) Find a vector **A** that is perpendicular to

$$\mathbf{U} = 2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}},$$
$$\mathbf{V} = \hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}},$$

(b) What is **A** if, in addition to this requirement, we demand that it have unit magnitude?

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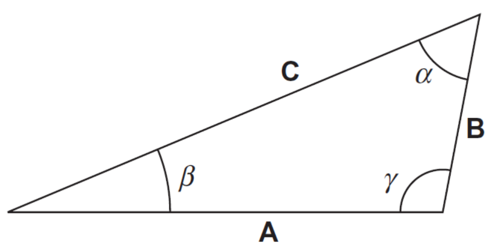
Problema 2 *Law of sines*

Derive the law of sines (see figure):

$$\frac{\text{Sen}\alpha}{|\mathbf{A}|} = \frac{\text{Sen}\beta}{|\mathbf{B}|} = \frac{\text{Sen}\gamma}{|\mathbf{C}|}.$$



. . . .

Problema 3 *Gradient*

Given the following vector,

$$\mathbf{r}_{12} = (x_1 - x_2)\hat{\boldsymbol{i}} + (y_1 - y_2)\hat{\boldsymbol{j}} + (z_1 - z_2)\hat{\boldsymbol{k}},$$

show that $\nabla_1 r_{12}$ (the gradient with respect to x_1, y_1, and z_1 of the magnitude $r_{12} = |\mathbf{r}_{12}|$) is a unit vector in the direction of $\mathbf{r}_{12}$.

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Problema 4 *Solenoidal vector field*

If $\mathbf{A}$ is irrotational, show that $\mathbf{A} \times \mathbf{r}$ is solenoidal.

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Problema 5 *Vectorial identity*

Verify the following identity,

$$\mathbf{A} \times (\nabla \times \mathbf{A}) = \frac{1}{2}\nabla(A^2) - (\mathbf{A} \cdot \nabla)\,\mathbf{A}.$$

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Problema 6 *Work*

Find the work done moving on a unit circle in the xy-plane given by,

$$\oint \mathbf{F} \cdot d\mathbf{r}, \quad \forall \quad \mathbf{F} = -\frac{y}{x^2+y^2}\hat{\boldsymbol{i}} + \frac{x}{x^2+y^2}\hat{\boldsymbol{j}},$$

on the following paths:

(a) Counterclockwise from 0 to π,

(b) Clockwise from 0 to $-\pi$,

. . . .

Problema 7 *Surface integral*

Evaluate the following integral,

$$\frac{1}{3}\int_S \mathbf{r} \cdot d\boldsymbol{\sigma}$$

over the unit cube defined by the origin $(0,0,0)$ and the unit intercepts on the positive x, y, and z axes.

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Problema 8 *Rotational field*

If $\mathbf{B} = \nabla \times \mathbf{A}$, show that,

$$\oint_S \mathbf{B} \cdot d\boldsymbol{\sigma} = 0$$

for any closed surface S.

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Problema 9 *Gauss' theorem*

Verify Gauss' theorem for the following field $\mathbf{A}$,

$$\mathbf{A} = xy\hat{\boldsymbol{i}} - x^2\hat{\boldsymbol{j}} + (x+y)\hat{\boldsymbol{k}}$$

where the surface S is the tetrahedron created in the first octant by the plane $2x+2y+z=6$.

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Problema 10 *Stokes' theorem*

Show, by using Stokes' theorem that the gradient of a scalar field is irrotational:

$$\nabla \times (\nabla\phi(\mathbf{r})) = 0.$$

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