Métodos Matemáticos
Tarea 02: Álgebra lineal

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Nombre del Estudiante: ____

Problema 1 *Spin-1 particle matrices*

One description of spin-1 particles uses the following matrices

$$\mathbf{M}_x = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},\ \mathbf{M}_y = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix},\ \mathbf{M}_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Show that,

(a) $[\mathbf{M}_x, \mathbf{M}_y] = i\mathbf{M}_z$, and so on (cyclic permutation of indices). Using the Levi-Civita symbol we may write,

$$[\mathbf{M}_i, \mathbf{M}_j] = i\sum_k \epsilon_{ijk}\mathbf{M}_k.$$

(b) $\mathbf{M}^2 \equiv \mathbf{M}_x^2 + \mathbf{M}_y^2 + \mathbf{M}_z^2 = 2\,\mathbf{1}_3$, where $\mathbf{1}_3$ is the 3×3 unit matrix.

(c) $[\mathbf{M}^2, \mathbf{M}_i] = 0$,
$[\mathbf{M}_z, \mathbf{L}^+] = \mathbf{L}^+$,
$[\mathbf{L}^+, \mathbf{L}^-] = 2\mathbf{M}_z$,
where $\mathbf{L}^+ \equiv \mathbf{M}_x + i\mathbf{M}_y$ & $\mathbf{L}^- \equiv \mathbf{M}_x - i\mathbf{M}_y$.

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Problema 2 *Hermitian matrices*

If **A** and **B** are Hermitian matrices, show that the following matrices are also Hermitian,

$$\mathbf{C} = \mathbf{AB} + \mathbf{BA}, \quad \mathbf{D} = i(\mathbf{AB} - \mathbf{BA})$$

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Problema 3 *Eigenvalue problem*

Find the eigenvalues and eigenvectors of the following matrices,

$$\mathbf{A} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

Construct the similarity transformation of the above matrices.

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Problema 4 *Angular momentum component matrices*

The matrices representing the angular momentum components $\mathbf{L}_x$, $\mathbf{L}_y$, and $\mathbf{L}_z$ are all Hermitian. Show that the eigenvalues of $\mathbf{L}^2$, where $\mathbf{L}^2 = \mathbf{L}_x^2 + \mathbf{L}_y^2 + \mathbf{L}_z^2$, are real and nonnegative.

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Problema 5 *Unitary matrix*

Show that all the eigenvalues of a unitary matrix $\mathbf{U}$ have unit magnitude.

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Problema 6 *Direct product*

Given the follwing matrices,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

find the direct product and show that it does not commute.

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Problema 7 *Eigenvalue problem II*

$\mathbf{A}$ has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -1$ and corresponding eigenvectors,

$$\mathbf{x}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{x}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Construct the matrix $\mathbf{A}$.

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Problema 8 *Function of matrices*

An $n \times n$ matrix $\mathbf{A}$ has n eigenvalues A_i. If $\mathbf{B} = \exp(\mathbf{A})$, show that $\mathbf{B}$ has the same eigenvectors as $\mathbf{A}$ with the corresponding eigenvalues B_i given by $B_i = \exp(A_i)$.

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