

Métodos Matemáticos
Tarea 03: Series infinitas

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Nombre del Estudiante: _____

Problema 1 *Doppler effect*

Using binomial expansions, compare the three Doppler shift formulas:

- (a) $\nu' = \nu \left(1 \pm \frac{v}{c}\right)^{-1}$ moving source;
- (b) $\nu' = \nu \left(1 \pm \frac{v}{c}\right)$ moving observer;
- (c) $\nu' = \nu \left(1 \pm \frac{v}{c}\right) \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ relativistic.

Note: the relativistic formula agrees with the classical formulas if terms of order v^2/c^2 can be neglected.

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Problema 2 *l'Hôpital's rule*

Prove l'Hôpital's rule,

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)}$$

by the use of series, when $f(x_0)$ and $g(x_0)$ are both zero.

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Problema 3 *Maclaurin expansion*

Use Maclaurin series to solve the following,

$$\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{1 - \cos^2 x} \right).$$

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Problema 4 *Klein-Nishina formula*

The Klein-Nishina formula for the scattering of photons by electrons contains a term of the form,

$$f(\epsilon) = \frac{1 + \epsilon}{\epsilon^2} \left[\frac{2 + 2\epsilon}{1 + 2\epsilon} - \frac{\ln(1 + 2\epsilon)}{\epsilon} \right].$$

Here $\epsilon = h\nu/mc^2$, the ratio of the photon energy to the electron rest mass energy. Find $\lim_{\epsilon \rightarrow 0} f(\epsilon)$.

Hint: expand all possible series up to $O(\epsilon^3)$.

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Problema 5 *Sawtooth wave*

A different sawtooth wave is described by

$$f(x) = \begin{cases} -\frac{1}{2}(\pi + x), & -\pi \leq x < 0 \\ +\frac{1}{2}(\pi - x), & 0 < x \leq \pi. \end{cases}$$

Show that, $f(x) = \sum_{n=1}^{\infty} (\text{Sen}nx/n)$.

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Problema 6 *Fourier series*

Represent the following function $f(x)$ as a Fourier series:

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \text{Sen}x, & 0 < x < \pi. \end{cases}$$

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Problema 7 *Fourier transform*

Find the Fourier transform of the following function,

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1. \end{cases}$$

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Problema 8 *Shifted triangular pulse*

Given

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \\ 0, & x \geq 2 \end{cases}$$

find the exponential Fourier and cosine Fourier transforms of $f(x)$.

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