Métodos Matemáticos
Tarea 04: Ecuaciones Diferenciales Ordinarias

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5 Junio 2013

Problema 1 *Radioactive decay*

Radioactive nuclei according to the law

$$\frac{dN}{dt} = -\lambda N,$$

N being the concentration of a given nuclide, and λ the particular decay constant. In a radioactive series of two different nuclides, with concentrations $N_1(t)$ and $N_2(t)$, we have

$$\frac{dN_1}{dt} = -\lambda N_1, \quad \frac{dN_2}{dt} = \lambda N_1 - \lambda_2 N_2.$$

Find $N_2(t)$ for the conditions $N_1(0) = N_0$ and $N_2(0) = 0$.

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Problema 2 *First order ODE*

Solve the following ODE's,

(a) $x\dfrac{dy}{dx} + y^2 - 1 = 0;$

(b) & $\dfrac{dy}{dx} = (x+y)^2.$

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Problema 3 *First order ODE II*

Solve the following ODE,

$$\left(\text{Sen}x - 4y^3\right)\frac{dy}{dx} + y\text{Cos}x + 3x^2 = 0.$$

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Problema 4 *First order ODE III*

Solve the following ODE,

$$\frac{dy}{dx} + y - xy^3 = 0.$$

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Problema 5 *Second order ODE*

Solve the equation,

$$4x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 - 1)y = 0.$$

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Problema 6 *Second order ODE II*

Solve the following equation,

$$x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0.$$

Hint: the second solution can be expressed in integral form.

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Problema 7 *Second order ODE III*

(a) Show that,

$$4x^2\frac{d^2y}{dx^2} + (1 - \alpha^2)y = 0,$$

has two solutions:

$$y_1(x) = a_0x^{(1+\alpha)/2}, \quad y_2(x) = a_0x^{(1-\alpha)/2}.$$

(b) Find the two linearly independent solutions when $\alpha = 0$.

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Problema 8 *Inhomogeneous second order ODE*

Find the general solution to the following inhomogeneous ODE,

$$(1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1 - x)^3$$

following the next steps,

(a) Solve the homogeneous part and find both linearly independent solutions considering the general case for the proposed solution: $y(x) = \sum_{i=0}^{\infty} a_j x^{i+s}$.

(b) Find the second solution of the homogeneous part with the series method, thus considering $y_1(x)$ as a series.

(c) Find the particular solution $y_p(x)$ for the inhomogeneous part with the variation of parameters' method.

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