

Métodos Matemáticos
Tarea 05: Variable Compleja

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Nombre del Estudiante: _____

Problema 1 *Analytic test*

Prove that the function,

$$f(z) = z^{1/2},$$

is analytic in the whole complex plane, except in zero.

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Problema 2 *Finding complex roots*

Solve,

$$\operatorname{Sen} z = 2.$$

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Problema 3 *Polar Cauchy-Riemann conditions*

a) Show that Cauchy-Riemann conditions in polar coordinates are given by

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{1}{r} \frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r},$$

b) Show that,

$$f'(z) = \frac{df(z)}{dz} = e^{-i\theta} \left(\frac{\partial u}{\partial r} + i \frac{\partial v}{\partial r} \right)$$

Where $f(z) = u(r, \theta) + iv(r, \theta)$.

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Problema 4 *Contour integrals*

Find the value of the following integral,

$$\oint_C \frac{dz}{z^2 + z},$$

without the use of the Cauchy integral formula, where the contour C is a circle centered at the origin with $R > 1$.

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Problema 5 *Cauchy integral formula*

Evaluate

$$\oint_C \frac{dz}{z(2z + 1)},$$

where the contour C is a circle centered at the origin with $R > 1$.

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Problema 6 *Calculus of residue*

Calculate the residue of the following function around $z_0 = 0$,

$$f(z) = \frac{e^{1/z}}{z - 1},$$

through Laurent series, or the method of limits.

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Problema 7 *Real integral*

Evaluate the following integral by the residues method,

$$\int_{-\infty}^{\infty} \frac{dx}{x(x^2 - 4x + 5)}.$$

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Problema 8 *Trigonometric functions integral*

Evaluate the following integral by the residues method,

$$\int_0^{2\pi} e^{2\cos\theta} d\theta.$$

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