Física del Estado Sólido
Tarea 02: Modelo del Gas de Electrones Libres, Electrón en un Potencial Periódico

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Problema 1 *Kinetic energy, pressure, and bulk modulus of an electron gas*

(a) Show that the kinetic energy of a three-dimensional gas of N free electrons at 0K is

$$U_0 = \frac{3}{5} N \epsilon_F.$$

(b) Derive a relation connecting the pressure and volume of an electron gas at 0 K. *Hint:* The result may be written as $p = (2/3)U_0/V$.

(c) Show that the bulk modulus $B = -V(\partial p/\partial V)$ of an electron gas at 0 K is $B = 5p/3 = 10U_0/9V$.

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Problema 2 *Chemical potential in two dimensions*

Show that the chemical potential of a Fermi gas in two dimensions is given by:

$$\mu(T) = k_B T \ln \left[\exp(\pi n \hbar^2 / m k_B T) - 1 \right],$$

for n electrons per unit area.
Hint: The density of orbitals of a free electron gas in two dimensions is independent fo energy: $D(\epsilon) = m/\pi\hbar^2$, per unit area of specimen.

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Problema 3 *Frequency dependence of the electrical conductivity*

Use the equation $m(dv/dt + v/\tau) = -eE$ for the electron drift velocity v to show that the conductivity at frequency ω is

$$\sigma(\omega) = \sigma(0) \left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2} \right),$$

where $\sigma(0) = ne^2\tau/m$.

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Problema 4 *Kinetic energy of free electron Fermi gas*

We define the dimensionless lenght r_s as r_0/a_H, where r_0 is the radius of a sphere that contains one electron, and a_H is the Bohr radius $\hbar^2/e^2m$. Show that the average kinetic energy per electron in a free electron Fermi gas at 0 K is $2.21r_s^2$, where the energy is expressed in rydbergs with 1 Ry $= me^4/2\hbar^2$.

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Problema 5 *Square lattice, free electron energies*

(a) Show for a simple square lattice (two dimensions) that the kinetic energy of a free electron at a corner of the first Brioullin zone is higher than that of an electron at midpoint of a side face of the zone by a factor of 2.

(b) What is the corresponding factor for a simple cubic lattice (three dimensions)?

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Problema 6 *Square lattice*

Consider a square lattice in two dimensions with the crystal potential

$$U(x,y) = -4U\mathrm{Cos}(2\pi x/a)\mathrm{Cos}(2\pi y/a).$$

Apply the secular equation,

$$\left(\frac{\hbar^2 k^2}{2m} - \epsilon\right) C_{\mathbf{k}} + \sum_{\mathbf{G}} C_{\mathbf{k}-\mathbf{G}} V_{\mathbf{G}} = 0,$$

and find approximately the energy gap at the corner point $(\pi/a, \pi/a)$ of the Brillouin zone. It will suffice to solve a 2×2 determinant equation.

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Problema 7 *Kronig-Penney model*

For the square-well periodic potential (Kronig-Penney potential),

(a) Find the general trascendental equation that comes as a result of applying the boundary and periodic conditions to the wave functions:

$$\left[\left(q^2 - \kappa^2\right)/2q\kappa\right] \mathrm{Senh}\, qb \mathrm{Sen}\, \kappa a + \mathrm{Cosh}\, qb \mathrm{Cos}\kappa a = \mathrm{Cos}\, k(a+b).$$

(b) For the delta-function approximation to this potential, show:

- $P = q^2 ba/a$ is a finite quantity (does not diverge),
- $q \gg \kappa$,
- $qb \ll 1$.

(c) For the delta-function approximation as well, deduce the approximate form of the trascendental equation (applying the approximations mentioned above):

$$\frac{P}{\kappa a}\mathrm{Sen}\,\kappa a + \mathrm{Cos}\,\kappa a = \mathrm{Cos}\,ka.$$

(d) For that approximation to the potential (delta-function) find at $k = 0$ the energy of the lowest energy band.

(e) For the same problem find the band gap at $k = \pi/a$.

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