Física del Estado Sólido
Tarea 04: Semiconductores, Magnetismo

Dr. Omar De la Peña Seaman

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Problema 1 *$p-$doped semiconductor*

A semiconductor with a band gap energy of $\epsilon_g = 1$ eV and equal hole and electron effective masses $m_e^* = m_h^* = m_0$ (m_0 is free electron mass) is $p-$doped with an acceptor concentration of $p = 10^{18}$ cm^{-3}. The acceptor energy leved is located 0.2 eV above the valence band edge of the material.

(a) Show that the intrinsic conduction in this material is negligible at 300 K.

(b) Calculate the conductivity σ of the material at room temperature (300 K), given a hole mobility of $\mu_p = 100$ cm^2/Vs at 300 K.

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Problema 2 *Hydrogen diamagnetism*

Calculate the diamagnetic susceptibility of hydrogen atom, with the ground state wavefunction:

$$\psi = \frac{1}{(a_0^3\pi)^{1/2}} e^{-r/a_0}$$

where $a_0 = 0.529$Å is the Bohr radius.

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Problema 3 *Partially filled shell: paramagnetism*

(a) The angular momentum conmutation relations are summarized in the vector operator identities,

$$\mathbf{L} \times \mathbf{L} = i\mathbf{L}, \quad \mathbf{S} \times \mathbf{S} = i\mathbf{S}.$$

Deduce from this identities and the fact that all components of $\mathbf{L}$ conmute with all components of $\mathbf{S}$ that:

$$[\mathbf{L} + g_0\mathbf{S}, \hat{\mathbf{n}} \cdot \mathbf{J}] = i\hat{\mathbf{n}} \times (\mathbf{L} + g_0\mathbf{S})$$

for any unit vector $\hat{\mathbf{n}}$.

(b) A state $|0\rangle$ with zero total angular momentum satisfies,

$$\mathbf{J}_x |0\rangle = \mathbf{J}_y |0\rangle = \mathbf{J}_z |0\rangle = 0.$$

Deduce from the conmutation relationship on (a) that:

$$\langle 0|\mathbf{L} + g_0\mathbf{S}|0\rangle = 0,$$

even though $\mathbf{L}^2$ and $\mathbf{S}^2$ need not to vanish in the state $|0\rangle$, and $(\mathbf{L} + g_0\mathbf{S})|0\rangle$ need not be zero.

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Problema 4 *Pauli paramagnetism: temperature corrections*

Show that if T is small compared with the Fermi temperature, the temperature-dependent correction to the Pauli susceptibility, $\chi(0) = \mu_B^2 g(\varepsilon_F)$, is given by

$$\chi(T) = \chi(0)\left(1 - \frac{\pi^2}{6}(k_B T)^2\left[\left(\frac{g'}{g}\right)^2 - \frac{g''}{g}\right]\right),$$

where g, g', and g'' are the density of levels and its derivatives at the Fermi energy. Show that for free electrons this reduces to,

$$\chi(T) = \chi(0)\left(1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right).$$

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Problema 5 *Ferromagnetic condition*

From the expresion of relative excess R in terms of the Fermi distribution function partial derivatives,

$$R = -\frac{1}{N}\sum_k IR\frac{\partial f(\mathbf{k})}{\partial \tilde{E}(\mathbf{k})} - \frac{1}{4!N}\sum_k (IR)^3\frac{\partial^3 f(\mathbf{k})}{\partial \tilde{E}(\mathbf{k})^3},$$

demostrate that,

$$\frac{\partial f(\mathbf{k})}{\partial \tilde{E}(\mathbf{k})} < 0 \;\;\&\;\; \frac{\partial^3 f(\mathbf{k})}{\partial \tilde{E}(\mathbf{k})^3} > 0,$$

where,

$$f(\mathbf{k}) = \frac{1}{e^{(\tilde{E}(\mathbf{k}) - E_F)\beta} + 1} \;\;\forall\;\; \beta = \frac{1}{k_B T}.$$

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Problema 6 *Exact ground−state energy of a simple antiferromagnet*

Show that the ground−state energy of the four spin antiferromagnetic nearest−neighbor Heisenberg linear chain,

$$\mathcal{H} = J\left(\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_4 + \mathbf{S}_4 \cdot \mathbf{S}_1\right),$$

is

$$E_0 = -4JS^2\left(1 + \frac{1}{2S}\right).$$

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