Estado Sólido I
Tarea 4: Modelo del Gas de Electrones Libres

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Nombre del Estudiante: ____

Problema 1 *Kinetic energy, pressure, and bulk modulus of an electron gas*

(a) Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is

$$U_0 = \frac{3}{5} N \epsilon_F.$$

(b) Derive that the relation connecting the pressure and volume of an electron gas at 0 K is $p = (2/3) U_0/V$.

(c) Show that the bulk modulus $B = -V(\partial p/\partial V)$ of an electron gas at 0 K is $B = 5p/3 = 10U_0/9V$.

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Problema 2 *Chemical potential in 1D and 2D*

Show that the chemical potential of a Fermi gas in $1D$ and $2D$ are given by:

$$1D: \quad \mu(T) = \epsilon_F \left[1 + \frac{\pi^2}{12} \left(\frac{k_B T}{\epsilon_F} \right)^2 \right] \quad \forall \quad \epsilon_F = \frac{\hbar^2 \pi^2}{2m} n^2,$$

$$2D: \quad \mu(T) = k_B T \ln \left[\exp \left(\frac{\pi n \hbar^2}{m k_B T} \right) - 1 \right],$$

for n electrons per unit length or area.

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Problema 3 *Thermal expansion influence on the free-electron gas*

Consider a sample of volume V containing N free electrons, where V experiences a thermal expansion given by $V = V_0(1 + \alpha T)$, with α as the volumetric thermal expansion coefficient. Considering such expansion on the volume, obtain the following quantities:

(a) The chemical potential $\mu(T)$,

$$\mu(T) = \epsilon_F (1+\alpha T)^{-2/3} \left[1 - \frac{\pi^2}{12} \frac{(k_B T)^2}{\epsilon_F^2} \right].$$

with ϵ_F the Fermi energy.

(b) The kinetic energy $U(T)$,

$$U(T) = \frac{3}{5} N \epsilon_F (1+\alpha T)^{-2/3} \left[1 + \frac{5\pi^2}{12} \frac{(k_B T)^2}{\epsilon_F^2} \right].$$

(c) The specific heat $c = C/N$, considering that $\alpha T \ll 1$,

$$c(T) = -\frac{2}{5} \alpha k_B T_F + \frac{2}{3} \alpha^2 k_B T_F T + \frac{\pi^2}{2} \frac{k_B T}{T_F}$$

where $T_F = \epsilon_F / k_B$ is the Fermi temperature.

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Problema 4 *Frequency dependence of the electrical conductivity*

Use the equation $m(dv/dt + v/\tau) = -eE$ for the electron drift velocity v and consider an oscillating electric field $E(t) = Ee^{i\omega t}$. Then find the following:

(a) The velocity v as a function of time and frequency.

(b) The conductivity at frequency ω, given by:

$$\sigma(\omega) = \sigma(0) \left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2} \right),$$

where $\sigma(0) = ne^2\tau/m$.

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