Estado Sólido I
Tarea 4: Modelo del Gas de Electrones Libres

Dr. Omar De la Peña Seaman

7 mayo 2025

Nombre del Estudiante: ______________________________

Problema 1 *Kinetic energy, pressure, and bulk modulus of an electron gas*

(a) Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is

$$U_0 = \frac{3}{5} N \epsilon_F.$$

(b) Derive that the relation connecting the pressure and volume of an electron gas at 0 K is $p = (2/3)U_0/V$.

(c) Show that the bulk modulus $B = -V(\partial p/\partial V)$ of an electron gas at 0 K is $B = 5p/3 = 10U_0/9V$.

. . . .

Problema 2 *Cohesive energy of the free electron Fermi gas*

We define the dimensionless lenght r_s as r_0/a_H, where r_0 is the radius of a sphere that contains one electron, and a_H is the Bohr radius given by $\hbar^2/e^2m$.

(a) Show that the average kinetic energy per electron in a free electron Fermi gas at 0 K is $2.21/r_s^2$, where the energy is expressed in rydbergs with 1 Ry $= me^4/2\hbar^2$.

(b) Show that the coulomb energy of a point positive charge e interacting with the uniform electron distribution of one electron in the volume of a sphere of radius r_0 (given by $\rho = -e/V$) is $-3e^2/2r_0$ or $-3/r_s$ in rydbergs. You can consider the point particle located at the origin of the sphere, and use the following expresion for the interaction:

$$V_C^+ = \int \frac{e\rho}{|\mathbf{r}|} d\mathbf{r}.$$

(c) Show that the coulomb self-energy of the electron distribution of the sphere is $3e^2/5r_0$ or $6/5r_s$ in rydbergs, using the following expression:

$$V_C^{self} = \int \frac{q(\mathbf{r})\rho}{|\mathbf{r}|} d\mathbf{r},$$

where $q(\mathbf{r})$ is the charge of the electron distribution at a given $\mathbf{r}$.

(d) Construct the total energy of the system, by summing the kinetic and both coulomb potential energies. Show that the equilibrium value for r_s is 2.46, and find the cohesive energy value (the minimum of energy).

. . . .

Problema 3 *Chemical potential in 1D and 2D*

Obtain the following expressions for the chemical potential of,

(a) A $1D$ system:

$$\mu(T) = \epsilon_F \left[1 + \frac{\pi^2}{12} \left(\frac{k_B T}{\epsilon_F} \right)^2 \right] \quad \forall \quad \epsilon_F = \frac{\hbar^2 \pi^2}{2m} n^2.$$

(b) A $2D$ system:

$$\mu(T) = k_B T \ln \left[\exp \left(\frac{\pi n \hbar^2}{m k_B T} \right) - 1 \right].$$

where n corresponds to the number of electrons per unit length or area.

. . . .

Problema 4 *Frequency dependence of the electrical conductivity*

Use the equation $m(dv/dt + v/\tau) = -eE(t)$ for the electron drift velocity v and consider an oscillating electric field $E(t) = Ee^{i\omega t}$. Then find the following:

(a) The velocity v as a function of time and frequency.

(b) The conductivity at frequency ω, given by:

$$\sigma(\omega) = \sigma(0) \left(\frac{1 - i\omega\tau}{1 + (\omega\tau)^2} \right),$$

where $\sigma(0) = ne^2\tau/m$.

. . . .