## Estado Sólido I Tarea 4: Modelo del Gas de Electrones Libres

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Nombre del Estudiante:

**Problema 1** Kinetic energy, pressure, and bulk modulus of an electron gas

(a) Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is

$$U_0 = \frac{3}{5} N \epsilon_F.$$

- (b) Derive that the relation connecting the pressure and volume of an electron gas at 0 K is  $p = (2/3)U_0/V$ .
- (c) Show that the bulk modulus  $B = -V(\partial p/\partial V)$  of an electron gas at 0 K is  $B = 5p/3 = 10U_0/9V$ .

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**Problema 2** Cohesive energy of the free electron Fermi gas

We define the dimensionless lenght  $r_s$  as  $r_0/a_H$ , where  $r_0$  is the radius of a sphere that contains one electron, and  $a_H$  is the Bohr radius given by  $\hbar^2/e^2m$ .

- (a) Show that the average kinetic energy per electron in a free electron Fermi gas at 0 K is  $2.21/r_s^2$ , where the energy is expressed in rydbergs with 1 Ry =  $me^4/2\hbar^2$ .
- (b) Show that the coulomb energy of a point positive charge e interacting with the uniform electron distribution of one electron in the volume of a sphere of radius  $r_0$  (given by  $\rho = -e/V$ ) is  $-3e^2/2r_0$  or  $-3/r_s$  in rydbergs. You can consider the point particle located at the origin of the sphere, and use the following expression for the interaction:

$$V_C^+ = \int \frac{e\rho}{|\mathbf{r}|} d\mathbf{r}$$

(c) Show that the coulomb self-energy of the electron distribution of the sphere is  $3e^2/5r_0$  or  $6/5r_s$  in rydbergs, using the following expression:

$$V_C^{self} = \int \frac{q(\mathbf{r})\rho}{|\mathbf{r}|} d\mathbf{r},$$

where  $q(\mathbf{r})$  is the charge of the electron distribution at a given  $\mathbf{r}$ .

(d) Construct the total energy of the system, by summing the kinetic and both coulomb potential energies. Show that the equilibrium value for  $r_s$  is 2.46, and find the cohesive energy value (the minimum of energy).

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