Física Estadística I
Tarea 01: Termodinámica Clásica

Dr. Omar De la Peña Seaman

27 Enero 2020

Nombre del Estudiante: ______________________________

Problema 1 *Diesel engine*

For an ideal Diesel cycle (see figure),

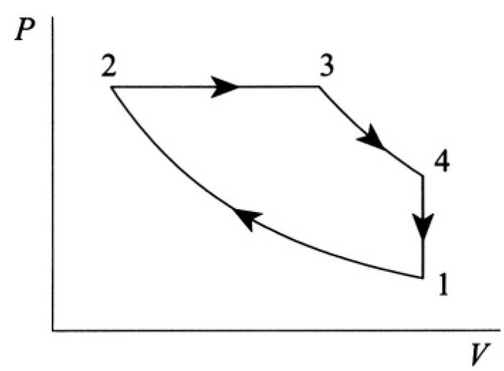
(a) Demonstrate that the internal energy is a conserved quantity, that is, $\Delta U_{tot} = 0$.

(b) Show that the thermal efficiency of this cycle is given by,

$$\eta = 1 - \frac{1}{\gamma}\frac{(1/r_E)^{\gamma} - (1/r_c)^{\gamma}}{(1/r_E) - (1/r_c)},$$

where the ratio $r_c = V_1/V_2$ is called the *compression ratio* and the ratio $r_E = V_1/V_3$ is called the *expansion ratio* for a diesel engine.

(c) Take $r_c = 20$, $r_E = 5$, and $\gamma = 1.4$ and calculate the thermal efficiency.



Hints:

1. The processes $1 \to 2$ and $3 \to 4$ are adiabatic.
2. For an adiabatic process it holds $T/T_0 = (V_0/V)^{\gamma-1}$, where $\gamma = C_p/C_v$.
3. For ideal gases, $\gamma = 5/3$ and $C_p - C_v = Nk$.

. . . .

Problema 2 *Helmholtz free energy*

A substance has the following properties,

(i) At a constant temperature T_0 the work done by it on expansion from V_0 to V is,

$$W = -KT_0 \ln\left(\frac{V}{V_0}\right).$$

(ii) The entropy is given by,

$$S = k\frac{V}{V_0}\left(\frac{T}{T_0}\right)^a,$$

where V_0, T_0, and a are fixed constants.

(a) Show that the Helmholtz free energy is the following:

$$F(T,V) = -kT_0 \ln\left(\frac{V}{V_0}\right) + k\left(\frac{V}{V_0}\right)\frac{T_0}{a+1}\left[1-\left(\frac{T}{T_0}\right)^{a+1}\right].$$

(b) Find the following equation of state:

$$pV = kT_0\left[1-\frac{V}{(a+1)V_0}\left\{1-\left(\frac{T}{T_0}\right)^{a+1}\right\}\right].$$

(c) Find that the work done at an arbitrary constant temperature T, from point 1 to point 2 is given by:

$$W = -kT_0 \ln\left(\frac{V_2}{V_1}\right) + \frac{kT_0}{(a+1)V_0}\left[1-\left(\frac{T}{T_0}\right)^{a+1}\right](V_2 - V_1).$$

. . . .

Problema 3 *van der Waals model*

A monoatomic gas obeys the van der Waals equation,

$$\left[p + a(N/V)^2\right](V - Nb) = NkT,$$

(a) Prove, using thermodynamics identities that:

$$\text{For any gas:} \quad \left.\frac{\partial C_V}{\partial V}\right|_T = T\left.\frac{\partial^2 p}{\partial T^2}\right|_V,$$

$$\text{van der Waals:} \quad \left.\frac{\partial C_V}{\partial V}\right|_T = 0.$$

(b) Show that the entropy $S(T,V)$ of the van der Waals gas is the following, considering $C_V = 3Nk/2$:

$$S(T,V) = Nk\ln\left[(V-Nb)T^{3/2}\right] + \text{cte}.$$

(c) Calculate the internal energy $U(T,V)$ and show it is as:

$$U(T,V) = \frac{3}{2}NkT - \frac{aN^2}{V} + \text{cte.}$$

(d) Demonstrate that the final temperature when the gas is adiabatically compressed from (T_1, V_1) to a final volume V_2 is,

$$T_2 = T_1 \left(\frac{V_1 - Nb}{V_2 - Nb}\right)^{2/3}$$

and that the work done on this compression process is:

$$W_{1\to 2} = \frac{3}{2}NkT_1 \left[\left(\frac{V_1 - Nb}{V_2 - Nb}\right)^{2/3} - 1\right] - \frac{aN^2}{V_1 V_2}(V_2 - V_1).$$

. . . .

Problema 4 *Heat capacities relations*

Show that,

$$C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V, \qquad C_p = -T\left(\frac{\partial^2 G}{\partial T^2}\right)_p,$$

where F is the Helmholtz free energy, and G is the Gibbs free energy.

. . . .

Problema 5 *Ideal gas on a thermodynamic cycle*

An ideal gas undergoes a reversible transformation along the path $p = aV^b$, where a and b are constants, with $a > 0$. Find that the heat capacity C along this path is given by:

$$C = C_p - \frac{b}{b+1}Nk = C_v + \frac{1}{b+1}Nk$$

Hint: the general C is defined as $C = TdS/dT$.

. . . .

Problema 6 *Heat capacities relations II*

Show that, for an ideal gas:

(a) $$F = \int C_V dT - T\int \frac{C_V}{T}dT - NkT\ln V - A_1 T + A_2,$$

(b) $$G = \int C_p dT - T\int \frac{C_p}{T}dT + NkT\ln p - B_1 T + B_2,$$

where A_1, A_2, B_1, and B_2 are constants.

. . . .