Física Estadística I
Tarea 02: Mecánica Estadística Clásica

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Nombre del Estudiante: ______________________________

Problema 1 *N three-dimensional harmonic oscillators*

For a collection of N three-dimensional quantum harmonic oscillators of frequency ω and total energy E, find that the number of microstates Ω, entropy S, and the temperature T are given by:

$$\begin{aligned}
\Omega(E,N) &= \frac{(E/\hbar\omega + 3N/2 - 1)!}{(3N-1)!\,(E/\hbar\omega - 3N/2)!},\\
S(E,N) &= Nk_B\left[-3\ln 3 + \left(\frac{E}{N\hbar\omega} + \frac{3}{2}\right)\ln\left(\frac{E}{N\hbar\omega} + \frac{3}{2}\right) - \left(\frac{E}{N\hbar\omega} - \frac{3}{2}\right)\ln\left(\frac{E}{N\hbar\omega} - \frac{3}{2}\right)\right],\\
T &= \frac{\hbar\omega}{k_B}\left[\ln\frac{(E/N\hbar\omega + 3/2)}{(E/N\hbar\omega - 3/2)}\right]^{-1}.
\end{aligned}$$

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Problema 2 *Classical spin system*

1. We have a system of three classical non-interacting spins that can be aligned up $\uparrow$, or down $\downarrow$. The spins are under the influence of an applied magnetic field towards the down direction. Such field gives an ϵ energy for the $\uparrow$ configuration, and an $-\epsilon$ energy for the $\downarrow$ configuration. Using the microcanonical ensemble, obtain the probability of the following configurations,

   (a) $(\downarrow\uparrow\uparrow)$, if the total energy is ϵ.

   (b) $(\downarrow\downarrow\downarrow)$, if the total energy is ϵ.

   (c) $(\downarrow\downarrow\downarrow)$, if the total energy is -3ϵ.

   Where the probability is defined as:

   $$P = \frac{\text{possible cases}}{\text{total cases}}.$$

2. Now, using the canonical ensemble, find the probability of two configurations: $(\uparrow\uparrow\uparrow)$ and $(\downarrow\downarrow\downarrow)$, which we are calling $P_{\uparrow\uparrow\uparrow}$ and $P_{\downarrow\downarrow\downarrow}$, respectively. Analyze the ratio $P_{\uparrow\uparrow\uparrow}/P_{\downarrow\downarrow\downarrow}$ on the following cases,

(a) $T = \epsilon/k_B$,
(b) $T = 0$,
(c) $T \to \infty$.

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Problema 3 *Zipper problem*

A zipper has N links, each link has a state in which it is closed with energy 0 and a state in which it is open with energy ϵ. We require that the zipper only unzip from one side (say from the left) and that the n link can only open if all links to the left of it $(1, 2, \ldots, n-1)$ are already open.

1. Find that the canonical partition function is:
$$Z(T, V, N) = \frac{1 - e^{-\beta\epsilon(N+1)}}{1 - e^{-\beta\epsilon}}.$$
2. Show that the average number of open links $\langle n_a \rangle$ is given by:
$$\langle n_a \rangle = \frac{e^{-\beta\epsilon}}{1 - e^{-\beta\epsilon}} - \frac{(N+1)e^{-\beta\epsilon(N+1)}}{1 - e^{-\beta\epsilon(N+1)}}.$$
3. Obtain $\langle n_a \rangle$ for low-temperature ($k_B T \ll \epsilon$) and high-temperature ($k_B T \gg \epsilon$) limits:
$$\begin{aligned} &\text{if } \; k_B T \ll \epsilon \quad \Rightarrow \quad \langle n_a \rangle \approx e^{-\beta\epsilon}, \\ &\text{if } \; k_B T \gg \epsilon \quad \Rightarrow \quad \langle n_a \rangle \approx N/2. \end{aligned}$$

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Problema 4 *N harmonic oscillators in $1D$*

For a collection of N harmonic oscillators in $1D$ of frequency ω, discussing the system:

1. Classically, find that the canonical partition function, and the entropy are given by:
$$\begin{aligned} Z(T, V, N) &= \left(\frac{k_B T}{\hbar\omega}\right)^N, \\ S(T, V, N) &= N k_B \left[\ln\left(\frac{k_B T}{\hbar\omega}\right) + 1\right], \\ S(E, V, N) &= N k_B \left[\ln\left(\frac{E}{N\hbar\omega}\right) + 1\right]. \end{aligned}$$
2. Quantum mechanically, obtain also the canonical partition function, and the entropy, described as:
$$\begin{aligned} Z(T, V, N) &= \left[\frac{1}{2\mathrm{Sinh}\left(\hbar\omega/2k_B T\right)}\right]^N, \\ S(T, V, N) &= N k_B \left[\frac{\hbar\omega/k_B T}{e^{\hbar\omega/k_B T} - 1} - \ln\left(1 - e^{-\hbar\omega/k_B T}\right)\right], \\ S(E, V, N) &= N k_B \left[\left(\frac{E}{N\hbar\omega} + \frac{1}{2}\right)\ln\left(\frac{E}{N\hbar\omega} + \frac{1}{2}\right) - \left(\frac{E}{N\hbar\omega} - \frac{1}{2}\right)\ln\left(\frac{E}{N\hbar\omega} - \frac{1}{2}\right)\right]. \end{aligned}$$

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