

Física Estadística I

Tarea 03: Mecánica Estadística Cuántica

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Nombre del Estudiante: _____

Problema 1 *An electron in a magnetic field*

If we have an applied magnetic field $\mathbf{B} = B\hat{z}$ (z -direction), then the Hamiltonian of an electron–spin takes the following form:

$$\hat{H} = -\mu_B B \hat{\sigma}_z.$$

In the representation that makes $\hat{\sigma}_z$ diagonal:

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

The density matrix $\hat{\rho}$ is given by:

$$\hat{\rho} = \frac{1}{e^{\beta\mu_B B} + e^{-\beta\mu_B B}} \begin{bmatrix} e^{\beta\mu_B B} & 0 \\ 0 & e^{-\beta\mu_B B} \end{bmatrix}.$$

1. Evaluate the density matrix $\hat{\rho}$ of the electron–spin in the representation that makes $\hat{\sigma}_x$ diagonal, and show that is given by:

$$\hat{\rho}' = \frac{1}{e^{\beta\mu_B B} + e^{-\beta\mu_B B}} \begin{bmatrix} \text{Cosh}(\beta\mu_B B) & \text{Senh}(\beta\mu_B B) \\ \text{Senh}(\beta\mu_B B) & \text{Cosh}(\beta\mu_B B) \end{bmatrix}.$$

2. Show that the value $\langle \hat{\sigma}'_z \rangle$, resulting from this representation, is expressed as:

$$\langle \hat{\sigma}'_z \rangle = \text{Tgh}(\beta\mu_B B).$$

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