Física Estadística I
Tarea 04: Gases Ideales Cuánticos – Gas de Bose-Einstein

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Problema 1 *Entropy of an ideal gas*

Show that the entropy of an ideal gas in thermal equilibrium is given:

1. In the case of bosons by the following expression,

$$S = k_B \sum_k \left[\langle \hat{n}_k + 1 \rangle \ln \langle \hat{n}_k + 1 \rangle - \langle \hat{n}_k \rangle \ln \langle \hat{n}_k \rangle \right].$$

2. In the case of fermions as:

$$S = k_B \sum_k \left[- \langle 1 - \hat{n}_k \rangle \ln \langle 1 - \hat{n}_k \rangle - \langle \hat{n}_k \rangle \ln \langle \hat{n}_k \rangle \right].$$

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Problema 2 *Occupation number's fluctuation*

1. Derive, for all three statistics, the following expressions for the occupation number's fluctuation, $\sigma^2_{\hat{n}_k} = \langle \hat{n}_k^2 \rangle - \langle \hat{n}_k \rangle^2$:

$$\begin{aligned}
\text{Bose-Einstein:} \quad & \sigma^2_{\hat{n}_k}\Big|_{BE} = \langle \hat{n}_k \rangle + \langle \hat{n}_k \rangle^2 . \\
\text{Fermi-Dirac:} \quad & \sigma^2_{\hat{n}_k}\Big|_{FD} = \langle \hat{n}_k \rangle - \langle \hat{n}_k \rangle^2 . \\
\text{Maxwell-Boltzmann:} \quad & \sigma^2_{\hat{n}_k}\Big|_{MB} = \langle \hat{n}_k \rangle .
\end{aligned}$$

2. Show that, quite generally:

$$\sigma^2_{\hat{n}_k} = \langle \hat{n}_k^2 \rangle - \langle \hat{n}_k \rangle^2 = k_B T \left. \frac{\partial \langle \hat{n}_k \rangle}{\partial \mu} \right|_T .$$

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Problema 3 *Thermodynamic relationships for ideal Bose gas*

1. For the ideal Bose gas it was demonstrated that:

$$\frac{1}{\zeta}\left(\frac{\partial\zeta}{\partial T}\right)_V = -\frac{3}{2T}\frac{g_{3/2}(\zeta)}{g_{1/2}(\zeta)},$$

   thus, show the following:

$$\frac{1}{\zeta}\left(\frac{\partial\zeta}{\partial T}\right)_p = -\frac{5}{2T}\frac{g_{5/2}(\zeta)}{g_{3/2}(\zeta)}.$$

2. Hence, also show that,

$$\gamma = \frac{C_p}{C_V} = \frac{(\partial\zeta/\partial T)_p}{(\partial\zeta/\partial T)_V} = \frac{5}{3}\frac{g_{5/2}(\zeta)g_{1/2}(\zeta)}{[g_{3/2}(\zeta)]^2}.$$

3. Check that, as T aproaches T_c from above, both γ and C_p diverge as $(T - T_c)^{-1}$.

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